ON THE GUPTA-BLEULER AND DIRAC-FOCK QUANTIZATION SCHEMES REVISITING

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Abstract

I revisited the Gupta-Bleuler and Dirac-Fock quantization schemes in quantum electrodynamics, guaranteeing the energy positivity of photon quanta via the weak Lorenz condition, imposed on the related Fock space. Having reanalyzed the Dirac-Fock-Podolsky Lagrangian formalism of the electrodynamics, I have stated both the energy positivity of photon quanta and the strong Lorenz condition on the Fock space.

It is well known that the usually accepted Dirac type quantization scheme [8, 3] in quantum electrodynamics meets some mathematical problems related with the necessity to satisfy simultaneously the requirement of the local positive energy density, the Lorenz subsidiary condition, involved on the quantum states, and the condition of transversality. Moreover, the whole theory formulation must be relativistic covariant. As the corresponding photon quanta are massless, attempts to carry out the Dirac type quantization scheme to electrodynamics, being a vector field theory, encounter fundamental difficulties, resulting in meaningless expressions, at the early fiftieth of the past century there were undertaken efforts to overcome them by means of some weakly argued from the physics point of view mathematical tricks, suggested [20, 5] by Gupta and Bleuler, based both on the physically not motivated indefinite metric and the corresponding weak Lorenz subsidiary condition, imposed on the related Fock space.

In the work under regard I have also revisited the Dirac quantization scheme within the Fock space representation of quantum states applied to a modified Lagrangian description of electrodynamics. Based on this quantization reformulation, there is stated that the Lorenz subsidiary condition satisfies strongly on the whole Fock space, is time-invariant and simultaneously guaranteeing the energy density positivity of photon quanta and the related relativistic covariance.

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