Darboux transformations applied to graphene in external magnetic fields

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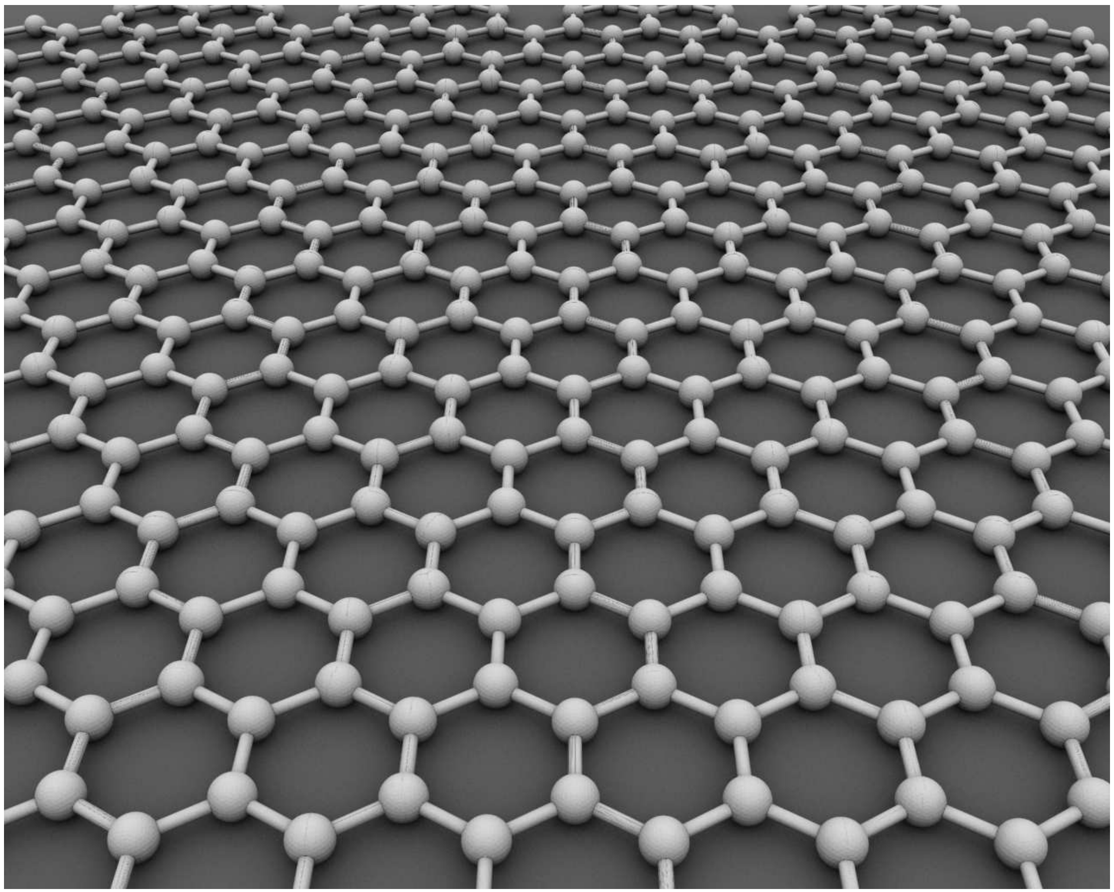
Contents

Introduction

- Graphene is a single layer of carbon atoms placed in a hexagonal configuration
- It is the thinnest, strong and flexible material ever known

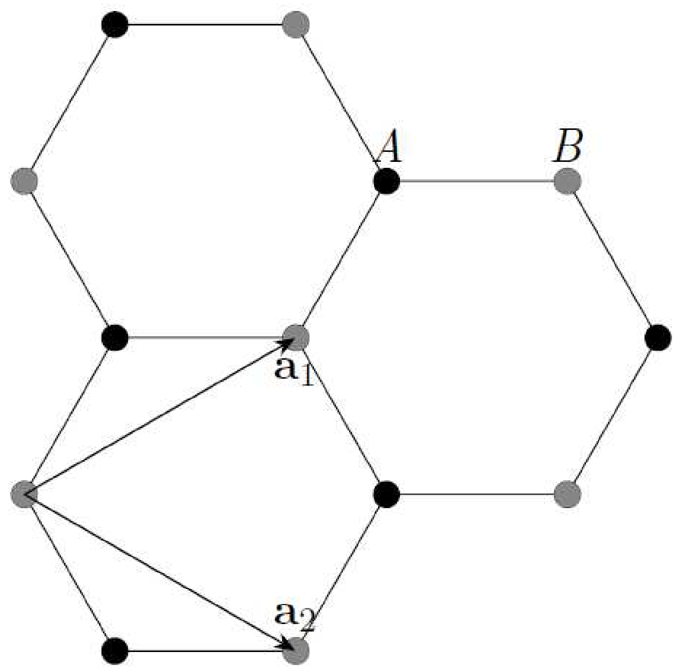


Introduction

- Several structures arise from graphene: graphite, carbon nanotubes, fullerenes
- Theoretical interest: is a two-dimensional system
- Low energy electrons in graphene behave as massles Dirac fermions
- Relativistic quantum mechanics can be imitated for velocities 300 times lower than c
- Electron confinement is produced by magnetic fields which are orthogonal to the layer
- Darboux transformation (DT) is the natural tool to address the electron motion in graphene

Dirac-Weyl equation and graphene

The graphene description is made in terms of two non-equivalent triangular lattices of atoms A and B:



The tight binding model considers that the two atoms in the unit cell of graphene interact just with its nearest neighbors

Dirac-Weyl equation and graphene

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} 0 & \gamma_0 S(\mathbf{k}) \\ \gamma_0 \bar{S}(\mathbf{k}) & 0 \end{pmatrix}$$

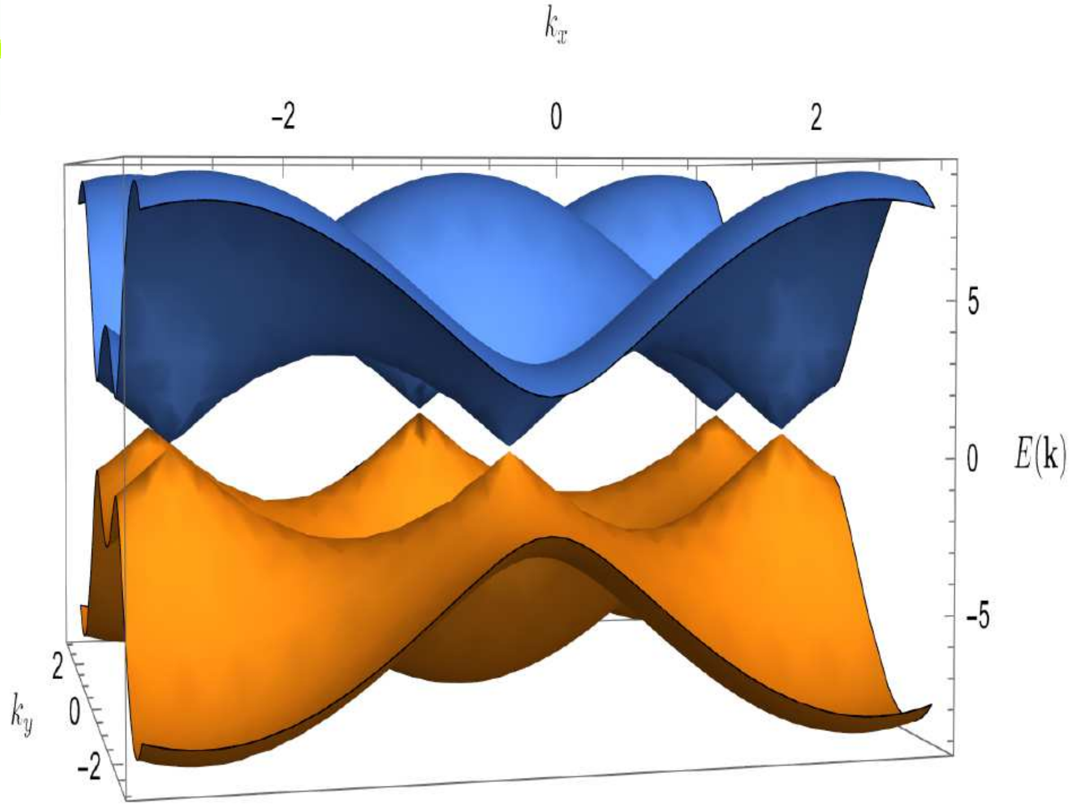
where $\gamma_0 \approx -2.97$ eV is the hopping parameter,

$$S(\mathbf{k}) = \sum_{\boldsymbol{\delta}} e^{i\mathbf{k}\cdot\boldsymbol{\delta}} = 2\text{exp}\left(\frac{ik_x a}{2\sqrt{3}}\right)\text{cos}\left(\frac{k_y a}{2}\right) + \text{exp}\left(\frac{-ik_x a}{\sqrt{3}}\right)$$

$a/\sqrt{3} = 1.42$ Å is the nearest-neighbor distance. The eigenvalues of $\mathcal{H}(\mathbf{k})$ are given by

$$\begin{aligned} E(\mathbf{k}) &= \pm\gamma_0 |S(\mathbf{k})| = \pm\gamma_0\sqrt{3 + f(\mathbf{k})} \\ f(\mathbf{k}) &= 2\text{cos}\,(k_y a) + 4\text{cos}\left(\frac{k_y a}{2}\right)\text{cos}\left(\frac{\sqrt{3}k_x a}{2}\right) \end{aligned}$$

Dirac-Weyl equation and graphene



Around the so-called Dirac point $\mathbf{K}^-$ it turns out that:

$$\mathcal{H}^-(\mathbf{p}) = \quad v_F \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix} = v_F \boldsymbol{\sigma} \cdot \mathbf{p}$$

Dirac-Weyl equation and graphene

- The Dirac-Weyl equation for low energy electrons in graphene placed in a magnetic field is

$$\mathbf{H}\Psi(x,y) = v_F\boldsymbol{\sigma}\cdot\left[\mathbf{p}+\frac{e\mathbf{A}}{c}\right]\Psi(x,y) = E\Psi(x,y)$$

  - $v_F \sim 8\times10^5 m/s$ is the Fermi velocity
  - $\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$ are the Pauli matrices
  - $\mathbf{p} = -i\hbar(\partial_x, \partial_y)^T$ is the 2-dim momentum operator
  - $-e$ is the electron charge
  - $\mathbf{A}$ is the vector potential
  - $\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A}$ is the external magnetic field

Dirac-Weyl equation and graphene

- For magnetic fields orthogonal to the graphene layer, which change just along x-direction, the vector potential in the Landau gauge reads

$$\mathbf{A} = \mathcal{A}(x)\hat{e}_y, \ \mathbf{B} = \mathcal{B}(x)\hat{e}_z, \ \mathcal{B}(x) = \mathcal{A}'(x)$$

- The translation invariance of H along y-axis suggests that

$$\Psi(x, y) = e^{iky} \begin{bmatrix} \psi^+(x) \\ i\psi^-(x) \end{bmatrix}$$

where k is the wavenumber in y direction, $\psi^{\pm}(x)$ are the electron amplitudes on the two adjacent sites A, B in the unit cell of graphene

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Dirac-Weyl equation and graphene

Thus

$$\left(\pm\frac{d}{dx}+\frac{e}{c\hbar}\mathcal{A}+k\right)\psi^{\mp}(x)=\frac{E}{\hbar\upsilon_F}\psi^{\pm}(x)$$

This resembles what happens in DT:

$$H^{\pm}\psi^{\pm}(x)=\mathcal{E}\psi^{\pm}(x)$$

$$H^{\pm}=-\frac{d^2}{dx^2}+V^{\pm},$$

$$V^{\pm}=\left(\frac{e\mathcal{A}}{c\hbar}+k\right)^2\pm\frac{e}{c\hbar}\mathcal{A}'$$

$$\mathcal{E}=\frac{E^2}{\hbar^2\upsilon_F^2}$$

DT and graphene

The following relations are fulfilled:

$H^- = L_0^+ L_0^-, \quad H^+ = L_0^- L_0^+$	factorizations
$H^+ L_0^- = L_0^- H^-, \quad H^- L_0^+ = L_0^+ H^+$	intertwining relations
$L_0^- = \frac{d}{dx} + W_0(x), \quad L_0^+ = -\frac{d}{dx} + W_0(x)$	intertwining operators
$W_0(x) = \frac{eA_y(x)}{c\hbar} + k$	superpotential
$V^-(x) = W_0^2(x) - W_0'(x)$	initial potential
$V^+(x) = V^-(x) + 2W_0'(x)$	SUSY partner potential

DT and graphene

The eigenfunctions of $H^{\pm}$ are connected to each other as:

$$\psi_n^+(x) = \frac{L_0^- \psi_{n+1}^-(x)}{\sqrt{\mathcal{E}_{n+1}^-}}$$

$$\psi_{n+1}^-(x) = \frac{L_0^+ \psi_n^+(x)}{\sqrt{\mathcal{E}_n^+}}$$

$$\psi_0^-(x) \sim e^{-\int W_0(x)dx}$$

The eigenvalues of $H^{\pm}$ are $\mathcal{E}_n^+ = \mathcal{E}_{n+1}^-, \quad \mathcal{E}_0^- = 0$. Note that

$$L_0^- \psi_0^-(x) = 0 \quad \Rightarrow \quad W_0(x) = -\frac{\psi_0^{-\prime}(x)}{\psi_0^-(x)} = -[\ln(\psi_0^-)]'$$

DT and graphene

The magnetic field applied to graphene is

$$B_0(x) = A_y'(x) = \frac{c\hbar}{e} W_0'(x) = -\frac{c\hbar}{e}\{\ln[\psi_0^-(x)]\}''$$

The eigenfunctions and eigenvalues for the Dirac electron in graphene are

$$E_0 = \hbar v_F \sqrt{\mathcal{E}_0^-} = 0, \qquad E_{n+1} = \hbar v_F \sqrt{\mathcal{E}_{n+1}^-}$$

$$\Psi_0 = e^{iky} \left[\begin{array}{c} 0 \\ i\,\psi_0^-(x) \end{array} \right], \quad \Psi_{n+1} = e^{iky} \left[\begin{array}{c} \psi_n^+(x) \\ i\,\psi_{n+1}^-(x) \end{array} \right]$$

Shape invariant case

- For some special forms of $B_0(x)$ the auxiliary potentials $V^{\pm}(x)$ become shape invariant (Infeld-Hull, Gendenshtein), i.e., when deleting the ground state of $V^{-}(x)$ the produced potential $V^{+}(x)$ can be obtained as well from V^{-} by just changing their parameters (up to an energy displacement)
- This approach was explored by Kuru, Negro, Nieto [J. Phys. Cond. Matt. 21 (2009) 455305]

Shape invariant case

- Example. For a constant homogeneous magnetic field $\mathcal{B} = B_0 > 0$ we must take $\mathbf{A} = \hat{e}_y B_0 x$
- The superpotential is $W_0(x) = \frac{\omega}{2}x + k$, $\omega = \frac{2eB_0}{c\hbar}$
- The two shape invariant potentials are $V^{\pm}(x) = \frac{\omega^2}{4}\left(x + \frac{2k}{\omega}\right)^2 \pm \frac{\omega}{2}$
- The eigenfunctions and eigenvalues of $H^{\pm}$ are

$$\mathcal{E}_0^- = 0, \quad \mathcal{E}_{n+1}^- = \mathcal{E}_n^+ = \omega(n+1), \quad n = 0, 1, 2, \ldots$$

$$\psi_n^- = \psi_n^+ = N_n\, e^{-\frac{\omega}{4}(x+\frac{2k}{\omega})^2} \;\; H_n\left[\sqrt{\frac{\omega}{2}}\left(x + \frac{2k}{\omega}\right)\right]$$

$N_n = \sqrt{\frac{1}{2^n n!}\left(\frac{\omega}{2\pi}\right)^{\frac{1}{2}}}$, H_n are Hermite polynomials

General DT

Next we follow [Midya, Fdez, J Phys A 47 (2014) 285302], [Castillo-Celeita, Fdez, J Phys A 53 (2020) 035302], based on [Mielnik, J Math Phys 25 (1984) 3387]

In order to generate new magnetic fields for which the problem is exactly solvable, let us displace up H^-:

1. $\tilde{H}_0 \equiv H^- - \epsilon_1 = -\frac{d^2}{dx^2} + V^-(x) - \epsilon_1, \quad \epsilon_1 \leq \mathcal{E}_0^- = 0$
2. From $\tilde{H}_0$ we build a new Hamiltonian H_1 through

$$H_1 L_1^+ = L_1^+ \tilde{H}_0$$

$$H_1 = -\frac{d^2}{dx^2} + V_1(x, \epsilon_1)$$

$$L_1^+ = -\frac{d}{dx} + W_1(x, \epsilon_1)$$

General DT

Thus

$$W_1^2(x,\epsilon_1) + W_1'(x,\epsilon_1) = \tilde{V}_0(x)$$
$$V_1(x,\epsilon_1) = \tilde{V}_0(x) - 2W_1'(x,\epsilon_1)$$

By changing $W_1(x,\epsilon_1) = u_1^{(0)\prime}/u_1^{(0)}$ it is obtained

$$-u_1^{(0)\prime\prime} + \tilde{V}_0(x)u_1^{(0)} = 0 \qquad \text{SE for } \tilde{H}_0$$

The generated magnetic field becomes

$$B_1(x,\epsilon_1) = \frac{c\hbar}{e}W_1'(x,\epsilon_1) = -B_0(x) + \frac{c\hbar}{e}\left\{\ln\left[\frac{u_1^{(0)}(x)}{\psi_0^-(x)}\right]\right\}''$$

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General DT

3. The auxiliary eigenvalues and eigenfunctions are

$$\tilde{\mathcal{E}}_n^{(0)} = \mathcal{E}_n^- - \epsilon_1, \quad \psi_n^-(x)$$

$$\mathcal{E}_0^{(1)} = 0, \qquad \psi_0^{(1)}(x) \sim e^{-\int W_1(x,\epsilon_1)dx} = \frac{1}{u_1^{(0)}}$$

$$\mathcal{E}_{n+1}^{(1)} = \tilde{\mathcal{E}}_n^{(0)}, \qquad \psi_{n+1}^{(1)}(x) = \frac{1}{\sqrt{\tilde{\mathcal{E}}_n^{(0)}}} L_1^+ \psi_n^-(x), \quad n = 0, 1, \ldots$$

The new solutions for graphene become

$$E_0 = \hbar v_F \sqrt{\mathcal{E}_0^{(1)}} = 0, \quad E_{n+1} = \hbar v_F \sqrt{\mathcal{E}_{n+1}^{(1)}}$$

$$\Psi_0 = e^{iky} \begin{bmatrix} 0 \\ i\, \psi_0^{(1)}(x) \end{bmatrix}, \quad \Psi_{n+1} = e^{iky} \begin{bmatrix} \psi_n^-(x) \\ i\, \psi_{n+1}^{(1)}(x) \end{bmatrix}$$

General DT

- Example. The general solution of the SE for $\tilde{V}_0 = V^-(x) - \epsilon_1$ with zero energy is ($a = -\epsilon_1/2\omega$):

$$u_1^{(0)} = e^{-\frac{\omega}{4}(x+\frac{2k}{\omega})^2}\Big\{ {}_1F_1[a, \tfrac{1}{2}, \tfrac{\omega}{2}(x + \tfrac{2k}{\omega})^2]$$

$$+2\nu_1 \frac{\Gamma(a+\frac{1}{2})}{\Gamma(a)}\sqrt{\tfrac{\omega}{2}}(x + \tfrac{2k}{\omega})\,{}_1F_1[a + \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{\omega}{2}(x + \tfrac{2k}{\omega})^2]\Big\}$$

- In particular, for $\epsilon_1 = -\omega/5$, $\nu_1 = 0$ it is obtained

$$V_1(x, \epsilon_1) = \tilde{V}_0 - 2\Big[\tfrac{\omega}{2}(x + \tfrac{2k}{\omega})\Big(- 1 + \tfrac{2}{5}\frac{{}_1F_1[\frac{11}{10}, \frac{3}{2}, \frac{\omega}{2}(x+\frac{2k}{\omega})^2]}{{}_1F_1[\frac{1}{10}, \frac{1}{2}, \frac{\omega}{2}(x+\frac{2k}{\omega})^2]}\Big)\Big]'$$

$$B_1(x, \epsilon_1) = -B_0 + \frac{2B_0}{5}\Big[(x + \tfrac{2k}{\omega})\frac{{}_1F_1[\frac{11}{10}, \frac{3}{2}, \frac{\omega}{2}(x+\frac{2k}{\omega})^2]}{{}_1F_1[\frac{1}{10}, \frac{1}{2}, \frac{\omega}{2}(x+\frac{2k}{\omega})^2]}\Big]'$$

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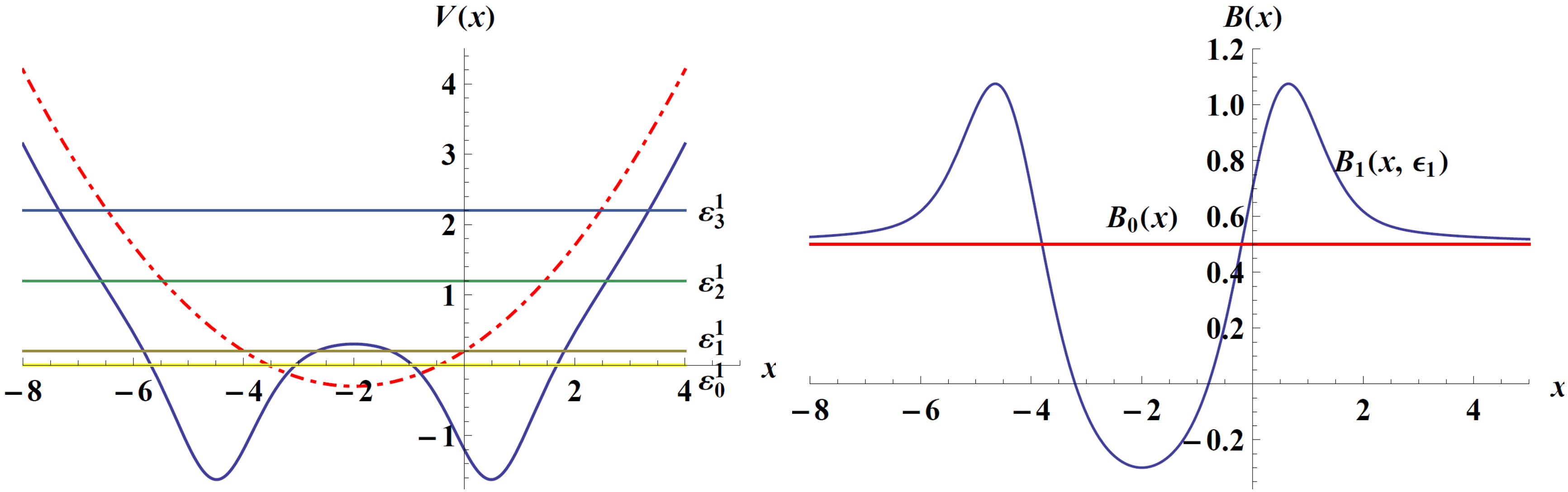
General DT



$V(x)$
4
3
2
1
−1
−8
−6
−4
−2
2
4
x
ε_3^1
ε_2^1
ε_1^1
ε_0^1
$B(x)$
1.2
1.0
0.8
0.6
0.4
0.2
−0.2
$B_1(x, \epsilon_1)$
$B_0(x)$

Iterative DT

1. Now let us displace up H_1 as

$$\tilde{H}_1 \equiv H_1 - \epsilon_2, \quad \epsilon_2 < \epsilon_1 \quad \Rightarrow \quad \tilde{\mathcal{E}}_0^{(1)} = -\epsilon_2 \geq 0$$

2. From $\tilde{H}_1$ a new Hamiltonian H_2 is constructed

$$H_2 L_2^+ = L_2^+ \tilde{H}_1$$
$$H_2 = -\frac{d^2}{dx^2} + V_2(x, \epsilon_2)$$
$$L_2^+ = -\frac{d}{dx} + W_2(x, \epsilon_2)$$

Thus

$$W_2^2(x, \epsilon_2) + W_2'(x, \epsilon_2) = \tilde{V}_1(x, \epsilon_1)$$
$$V_2(x, \epsilon_2) = \tilde{V}_1(x, \epsilon_1) - 2W_2'(x, \epsilon_2)$$

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Iterative DT

By making $W_2(x,\epsilon_2) = u_2^{(1)'}/u_2^{(1)}$:

$$-u_2^{(1)''} + \tilde{V}_1(x,\epsilon_1)u_2^{(1)} = 0$$

$u_2^{(1)}$ arises from acting L_1^+ onto an appropriate solution of H^-:

$$u_2^{(1)} \propto L_1^+ u_2^{(0)} = -\mathsf{W}(u_1^{(0)}, u_2^{(0)})/u_1^{(0)}$$
$$-u_2^{(0)''} + V^-(x)u_2^{(0)} = (\epsilon_1+\epsilon_2)u_2^{(0)}$$

The new potential and external magnetic field are

$$V_2(x,\epsilon_2) = V^-(x) - 2\{\ln[\mathsf{W}(u_1^{(0)}, u_2^{(0)})]\}'' - (\epsilon_1+\epsilon_2)$$
$$B_2(x,\epsilon_2) = \frac{c\hbar}{e}W_2'(x,\epsilon_2) = -B_1(x,\epsilon_1) + \frac{c\hbar}{e}\{\ln[\mathsf{W}(u_1^{(0)}, u_2^{(0)})]\}''$$

Iterative DT

3. The auxiliary eigenvalues and eigenfunctions are

$$\tilde{\mathcal{E}}_n^{(1)} = \mathcal{E}_n^{(1)} - \epsilon_2, \qquad \psi_n^{(1)}(x)$$

$$\mathcal{E}_0^{(2)} = 0, \quad \psi_0^{(2)}(x) \sim e^{-\int W_2(x,\epsilon_2)dx} = \frac{1}{u_2^{(1)}}$$

$$\mathcal{E}_{n+1}^{(2)} = \tilde{\mathcal{E}}_n^{(1)}, \quad \psi_1^{(2)}(x) = \frac{L_2^+ \psi_0^{(1)}(x)}{\sqrt{\tilde{\mathcal{E}}_0^{(1)}}}, \quad \psi_{n+2}^{(2)}(x) = \frac{L_2^+ \psi_{n+1}^{(1)}(x)}{\sqrt{\tilde{\mathcal{E}}_{n+1}^{(1)}}}$$

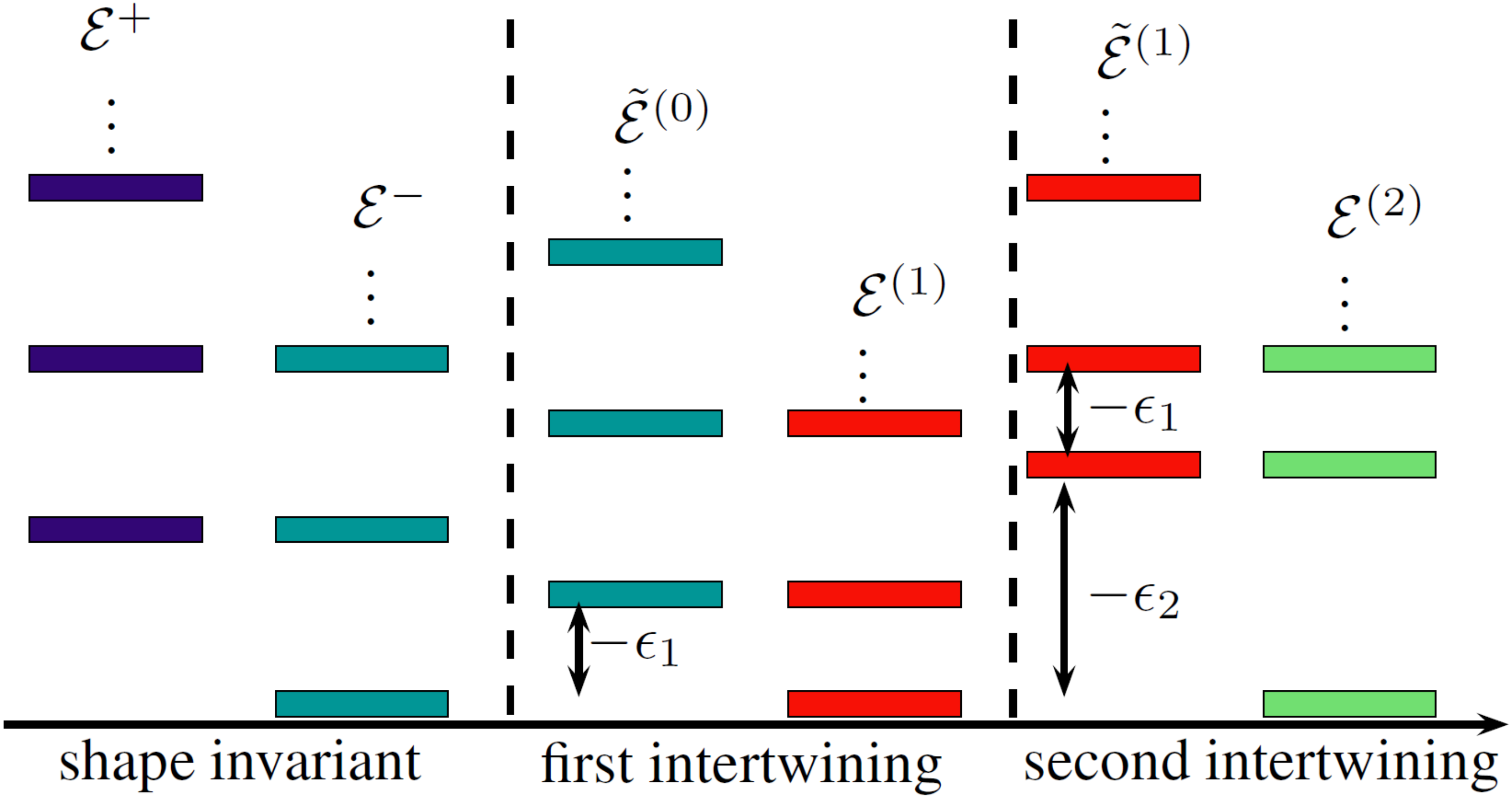
The new solutions for graphene are

$$E_0 = \hbar v_F \sqrt{\mathcal{E}_0^{(2)}} = 0, \quad E_{n+1} = \hbar v_F \sqrt{\mathcal{E}_{n+1}^{(2)}}$$

$$\Psi_0 = e^{iky} \begin{bmatrix} 0 \\ i\,\psi_0^{(2)}(x) \end{bmatrix}, \quad \Psi_{n+1} = e^{iky} \begin{bmatrix} \psi_n^{(1)}(x) \\ i\,\psi_{n+1}^{(2)}(x) \end{bmatrix}$$

The procedure can be continued at will!

Iterative DT



Iterative DT

- Example. In the second SUSY step $V_1(x)$ is moved up by $-\epsilon_2$:

$$\tilde{V}_1(x,\epsilon_1) = \frac{\omega^2}{4}\left(x+\frac{2k}{\omega}\right)^2 - \frac{\omega}{2} - 2W_1'(x,\epsilon_1) - \epsilon_1 - \epsilon_2$$

- The second-order potential and magnetic field are

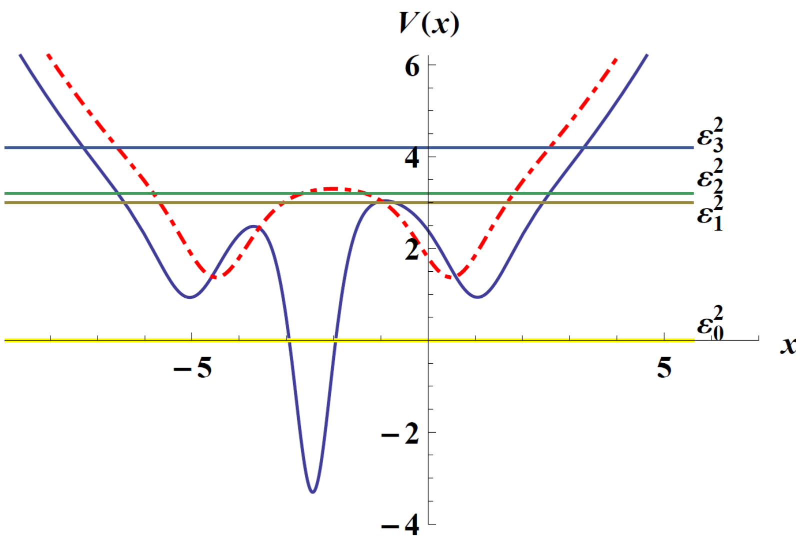
$$V_2(x,\epsilon_2) = \tilde{V}_0 - 2\{\ln[\mathrm{W}(u_1^{(0)},u_2^{(0)})]\}'' - \epsilon_2$$

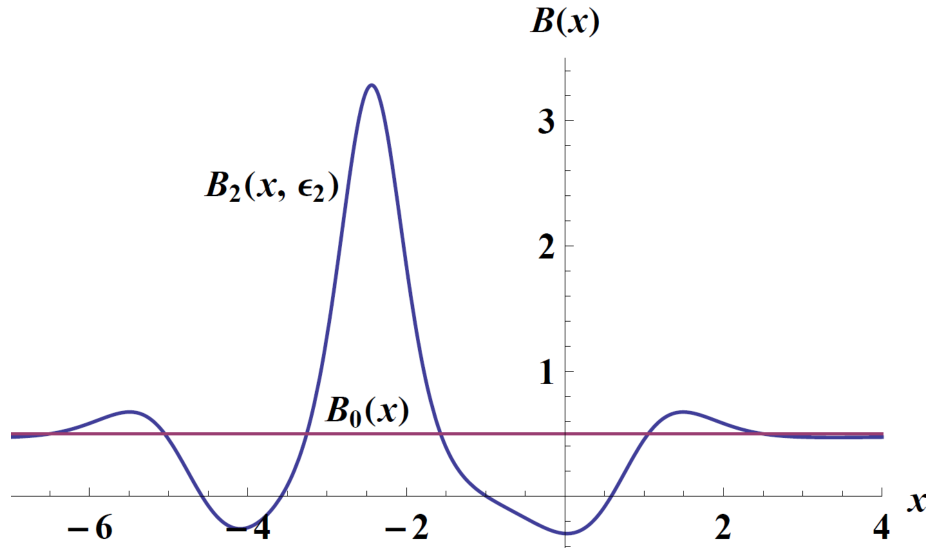
$$B_2(x,\epsilon_2) = -B_1(x,\epsilon_1) + \frac{c\hbar}{e}\{\ln[\mathrm{W}(u_1^{(0)},u_2^{(0)})]\}''$$

$u_2^{(0)}$ arises from $u_1^{(0)}$ by changing $\epsilon_1 \to \epsilon_1 + \epsilon_2$

- For $\epsilon_1 = -\omega/5$, $\epsilon_2 = -3\omega$, $\omega = 1$, $\nu_1 = 0$, $\nu_2 = \frac{3}{2}$:

Iterative DT





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Singular DT

- Real seed solutions with nodes can be used in the first step $\Rightarrow$ singularities arise in $W_1(x, \epsilon_1)$, $V_1(x, \epsilon_1)$ and $B_1(x, \epsilon_1)$
- An appropriate transformed seed solution of H^- with nodes is used in the second step to cancel some singularities induced in the first step. At the end singularities still appear in $W_2(x, \epsilon_2)$ and in $B_2(x, \epsilon_2)$ but disappear from $V_2(x, \epsilon_2)$
- A third non-singular Darboux transformation is required to cancel completely the singularities remaining of the second step

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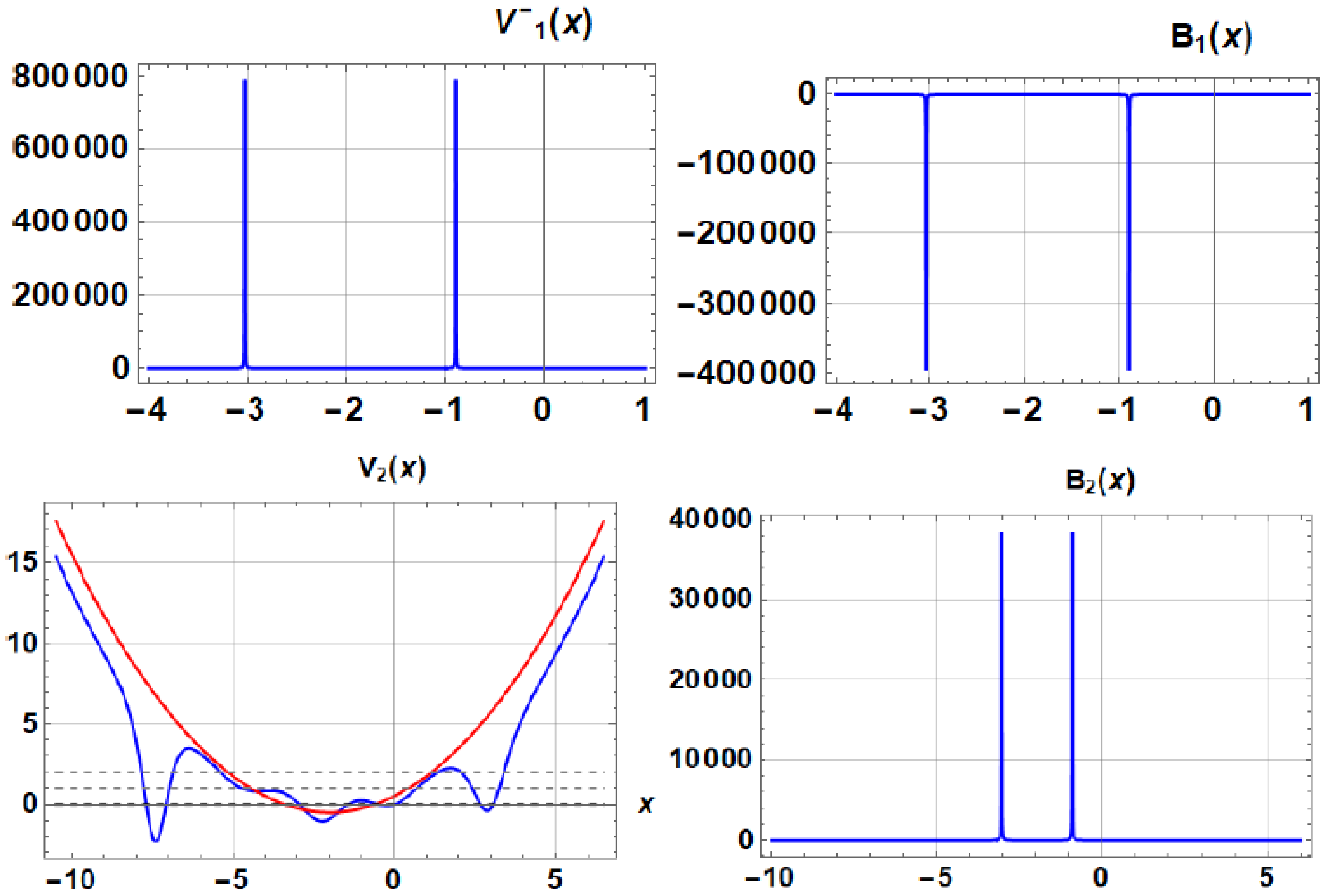
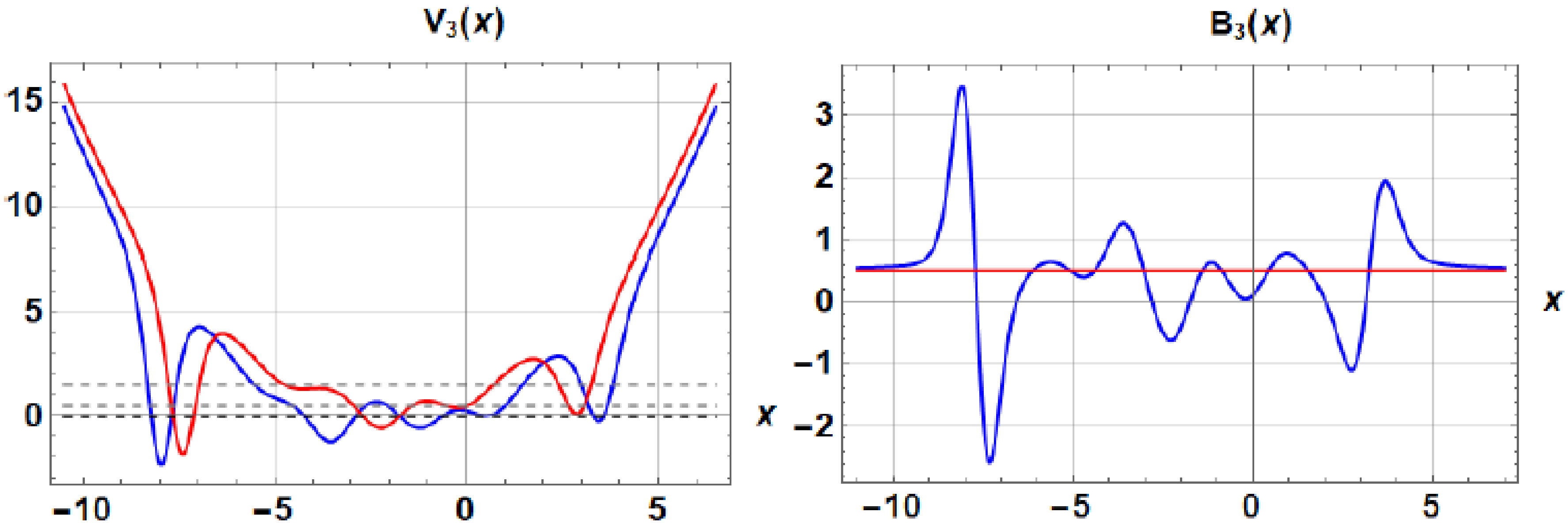
Singular DT



$$\epsilon_1 = 1.7, \quad \nu_1 = 0.1, \quad \epsilon_2 = 1.5, \quad \nu_1 = 1.1$$

Singular DT



$$\epsilon_1 = 1.7, \quad \nu_1 = 0.1$$
$$\epsilon_2 = 1.5, \quad \nu_2 = 1.1$$
$$\epsilon_3 = -0.5, \quad \nu_3 = 0.5$$

Complex DT

Seed solutions for $\epsilon \in \mathbb{C}$ can be used, such that after 3 steps a Hermitian graphene Hamiltonian is obtained:

– A complex seed solution $u^{(0)}$ for $\epsilon \in \mathbb{C}$, vanishing at one of the ends of the x-domain, is chosen

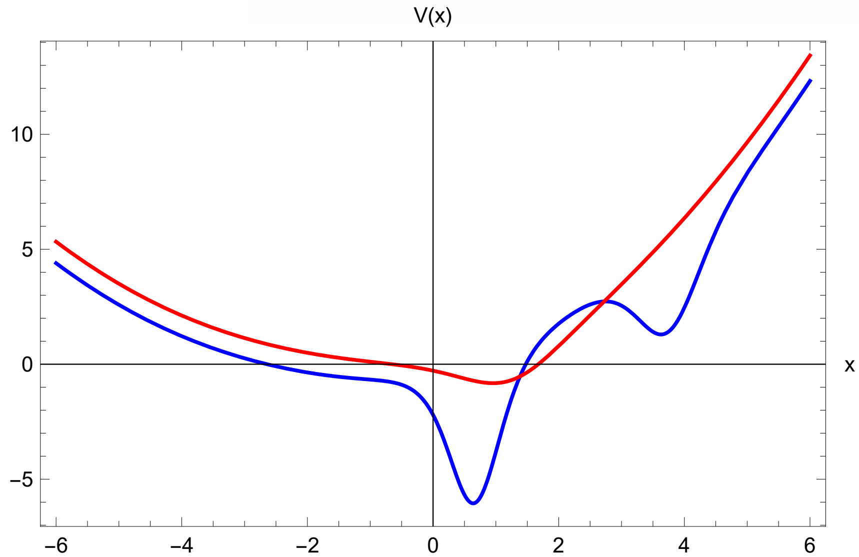
– Then, the (transformed) seed solution $-iu^*(x)$ associated to ϵ^* is used, such that $-i\mathsf{W}(u^{(0)}, u^{*(0)})$ is real nodeless

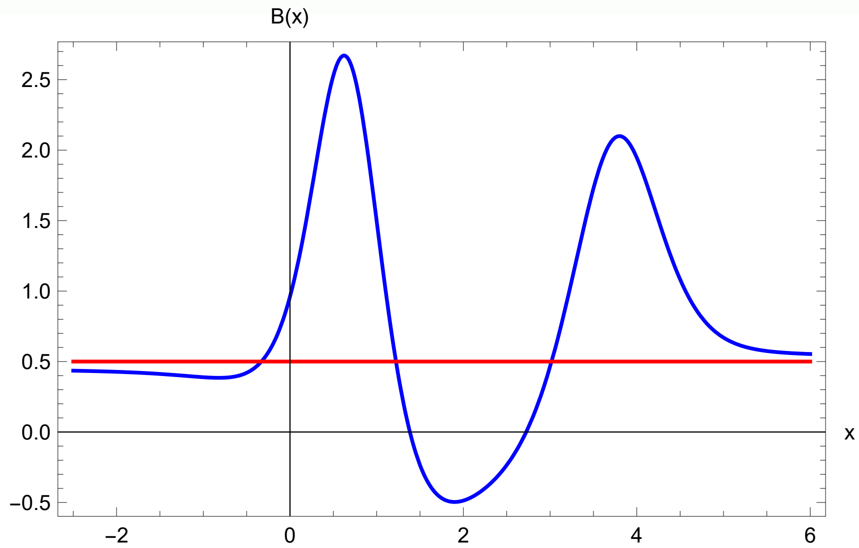
– A real nodeless (transformed) seed solution $u_3^{(0)}$ for $\epsilon_3 \leq \mathcal{E}_0^- = 0$ is used. At the end a real potential and magnetic field are obtained,

$$V_3(x, \epsilon_3) = V^-(x) - 2\left\{\ln\left[\mathsf{W}\left(u^{(0)}, -iu^{*(0)}, u_3^{(0)}\right)\right]\right\}'' - \epsilon_3$$

$$B_3(x, \epsilon_3) = \frac{c\hbar}{e}\left\{\ln\left[\frac{\mathsf{W}(u^{(0)}, -iu^{*(0)}, u_3^{(0)})}{\mathsf{W}(u^{(0)}, -iu^{*(0)})}\right]\right\}''$$

Complex DT





$$\epsilon_1 = -\frac{1}{2} + \frac{i}{10}$$

$$\epsilon_2 = -\frac{1}{2} - \frac{i}{10}$$

$$\epsilon_3 = -\frac{1}{2}, \quad \Lambda = \frac{1}{10}$$

Periodic DT

- Finally, let us consider a periodic magnetic field,

$$\mathcal{B}(x) = B_0 \frac{m-1+\mathrm{dn}(x|m)^4}{\mathrm{dn}(x|m)^2}$$

- The superpotential is,

$$W_0(x) = m \frac{\mathrm{sn}(x|m)\mathrm{cn}(x|m)}{\mathrm{dn}(x|m)}$$

- The two self-isospectral periodic Lamé potentials are:

$$V^{-}(x) = 2\,m\,\mathrm{sn}(x|m)^2 - m$$
$$V^{+}(x) = 2\,m\,\mathrm{sn}(x+K|m)^2 - m$$

$\mathrm{sn}(x|m),\ \mathrm{cn}(x|m),\ \mathrm{dn}(x|m)$ are Jacobi elliptic functions, $0 \leq m \leq 1$ is the modulus, $2K(m)$ is the period of $V^{\pm}$, $K(m) = \int_0^{\pi/2} d\theta/\sqrt{1 - m\sin^2(\theta)}$

Periodic DT

- The bande edge eigenfunctions and eigenvalues are

$$
\begin{array}{lll}
\mathcal{E}_0^- = 0 & \mathcal{E}_1^- = 1 - m & \mathcal{E}_{1'}^- = 1 \\
\psi_0^- = \mathrm{dn}(x|m) & \psi_1^- = \mathrm{cn}(x|m) & \psi_{1'}^- = \mathrm{sn}(x|m) \\
\psi_0^+ = \mathrm{dn}(x+K|m) & \psi_1^+ = \mathrm{cn}(x+K|m) & \psi_{1'}^+ = \mathrm{sn}(x+K|m)
\end{array}
$$

The band edge solutions for electrons in the graphene periodic magnetic superlattice:

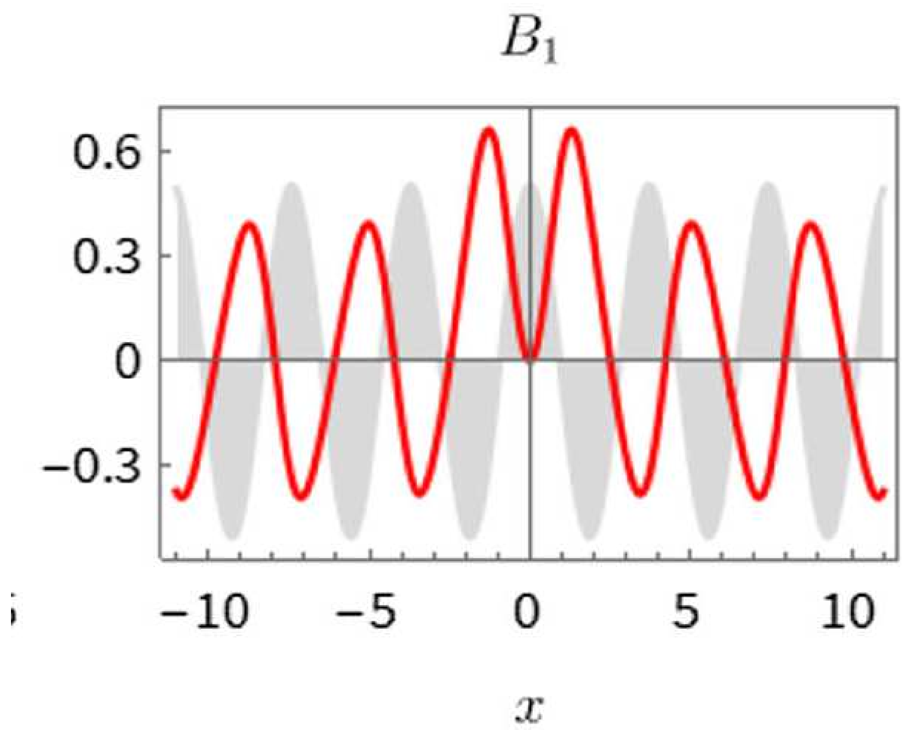
$$
E_0^- = 0 \qquad \Psi_0 = e^{iky} \begin{bmatrix} A_1\,\mathrm{dn}(x+K|m) \\ i\,A_2\,\mathrm{dn}(x|m) \end{bmatrix}
$$

$$
E_1^- = \sqrt{1-m} \qquad \Psi_1 = e^{iky} \begin{bmatrix} \mathrm{cn}(x+K|m) \\ i\,\mathrm{cn}(x|m) \end{bmatrix}
$$

$$
E_{1'}^- = 1 \qquad \Psi_{1'} = e^{iky} \begin{bmatrix} \mathrm{sn}(x+K|m) \\ i\,\mathrm{sn}(x|m) \end{bmatrix}
$$

Quasi-periodic DT

- By applying the general DT, it is possible to create a bound state at zero energy departing from $\tilde{V}_0 = V^- - \epsilon_1$. Thus, a new potential $V_1(x, \epsilon_1)$ is generated, which is not longer periodic but it is quasiperiodic. The same property is acquired by the new magnetic field



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Conclusions

- We have analyzed the behavior of graphene in external magnetic fields orthogonal to the layer through DT. The seed solutions employed can produce either shape invariant or more general potentials
- Non-singular DT and their iterations have been proposed as an algorithm to generate magnetic field for which the graphene Hamiltonian admits exact solutions
- Singular DT and their iterations producing non-singular potentials and associated magnetic field (after 3 steps) have been explored
- Complex DT and their iterations generating real SUSY partner potentials and associated magnetic field (after 3 steps) have been studied

Conclusions

- Complex DT involving complex magnetic fields and leading to non-Hermitian graphene Hamiltonians start to be explored
- The second-order DT emerges as the natural tool to address a similar study for bilayer graphene in the so-called Bernal stacking
- We would like to explore if for twisted bilayer graphene it is possible to implement a similar study
- The coherent states approach to graphene is a feasible study, that we started in 2017 and it is still in progress

References

DT, SUSY QM, Factorization Method

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