XIII SCHOOL ON GEOMETRY AND PHYSICS

Białystok, Poland

8 July - 12 July 2024

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LIST OF COURSES

1. **Alexey BOLSINOV** — *Loughborough University*, *United Kingdom*

   Nijenhuis Geometry and its applications

   This course is an introduction to Nijenhuis Geometry, a new challenging area in Differential Geometry that studies local and global properties of geometric structures given by a field of endomorphisms with vanishing Nijenhuis torsion. This topic is located on the crossroad of Geometry, Mathematical Physics and Algebra as Nijenhuis structures naturally appear in many seemingly unrelated research areas such as bi-Hamiltonian integrable systems (both finite and infinite-dimensional), projective geometry, theory of left-symmetric algebras and others.

   Programme includes the following topics:

   - Fields of endomorphisms. Nijenhuis torsion and Nijenhuis operators.
   - Basic properties of Nijenhuis operators. Splitting theorem.
   - Diagonalisable and differentially non-degenerate Nijenhuis operators.
   - Nijenhuis operators with complex eiganvalues.
   - Nilpotent Nijenhuis operators and Jordan blocks.
   - Singular points of Nijenhuis operators and linearisation.
   - Left-symmetric algebras. Linearisability and non-degenaracy.
   - gl-regular Nijenhuis operators and their canonical forms.
   - Nijenhuis perturbations of a Jordan block.
   - Normal forms for gl-regular Nijenhuis operators in dimension 2.
   - Global properties of Nijenhuis operators on closed manifolds.
   - Nijenhuis operators and geodesically equivalent metrics.
   - Open problems in Nijenhuis Geometry.

   The course will be based on a series of recent papers/preprints:

   - A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev,Nijenhuis geometry. *Adv. in Math.*, 394: 108001, (2022).
   - A. Konyaev, Nijenhuis geometry II: Left-symmetric algebras and linearization problem for Nijenhuis operators. *Diff. Geom. and Appl.*, 74: 101706, 2021.
   - A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Nijenhuis Geometry III: gl-regular Nijenhuis operators. *Rev. Mat. Iberoamericana*, 2023, DOI 10.4171/RMI/1416.
   - A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Nijenhuis Geometry IV: conservation laws, symmetries and integration of certain non-diagonalisable systems of hydrodynamic type in quadratures, *arXiv:2304.10626.*
   - A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Applications of Nijenhuis geometry: nondegenerate singular points of Poisson–Nijenhuis structures, *Europ. Jour. of Math.*, 8: 1355–1376, 2022.
   - A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Applications of Nijenhuis geometry II: maximal pencils of multi- Hamiltonian structures of hydrodynamic type. *Nonlinearity*, 34(8): 5136–5162, 2021.

- A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Applications of Nijenhuis geometry III: Frobenius pencils and compatible non-homogeneous Poisson structures. *Jour. of Geom. Anal.*, 33:193, 2023.
- A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Applications of Nijenhuis Geometry IV: multicomponent KdV and Camassa–Holm equations. *Dynamics of PDEs*, 20(1): 73–98, 2023.
- A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Applications of Nijenhuis Geometry V: geodesic equivalence and finite-dimensional reductions of integrable quasilinear systems, Jour. of Nonlinear Science, 34:33, 2024.

2. **Jerzy KIJOWSKI** – *Center for Theoretical Physics, Polish Academy of Sciences, Poland*

On Quantization

The laws of classical physics are considered approximate. They result from the laws of quantum physics (considered to be fundamentally rigorous) applied to "large" objects. This means that classical physics can be formally obtained from quantum physics as the limit: $\hbar \to 0$. It is therefore difficult to expect that it will be possible to reason in the opposite direction, i.e. to build a quantum theory based on knowledge of a classical theory. And yet, paradoxically, all important quantum theories (quantum mechanics, quantum electrodynamics, even quantum chromodynamics) have been formulated in this way: by "quantizing" classical theory. The "quantization procedure", initiated by W. Heisenberg's hypothesis, which consists in replacing the classical observables (position and momentum) with their non-commutative counterparts in such a way that the Poisson bracket is represented by their commutator, hides many puzzles and surprises that have been overlooked by both physicists and mathematicians. Some of these surprises are very painful (not everything can be "quantized", and even if something can be quantized, the result may be ambiguous), but some are unexpectedly positive (in some situations the result is unique and unambiguous). The course will analyze the physical and mathematical aspects of various "quantization schemes" proposed in the literature. Mathematically, I will not focus on the technical side of the proofs, but on showing the essence of various mathematical structures that appear automatically as a result of the adopted physical assumptions. The following issues will be discussed (in more or less detail):

1. Quantization according to Heisenberg: successes and limitations.
2. Quantization according to Schrödinger.
3. Equivalence analysis of both approaches.
4. Beams of classical free particles and the quantum mechanical momentum operator.
5. Wave function as a "half-density".
6. Un-ambiguous quantization of "vector fields".
7. Quantization of the Galilean transformations.
8. "Fractional Fourier transform" and the harmonic oscillator.
9. Weyl rule. Moyal bracket.
10. Spin quantization.
11. Hermitian and self-adjoint operatorse.
12. What is a "continuous spectrum"?
13. Paradoxes. Example: "Arrival time" in quantum mechanics.
14. Newton–Wigner position operator.

I assume knowledge of physics and mathematics resulting from completing first two years of bachelor's studies of physics.

3. **Gabriel LAROTONDA** — *University of Buenos Aires / CONICET*, *Argentina*

   Geometry of infinite dimensional Grassmannians: exponential map and conjugate points

   For a C^*-algebra $\mathcal{A}$, we consider $Gr(P)$ the unitary orbit of an orthogonal projection $P \in \mathcal{A}$, which is one of the connected components of the Grassmanian manifold of $\mathcal{A}$. In this mini-course we will first present its differentiable structure, and then we will give a unified presentation of the canonical linear connection that appears in several sources of the literature in $Gr(P)$. We will describe in detail its geodesics, exponential map and curvature tensor. We will then give a concise description of the conjugate points along geodesics. These come in two flavours: conjugate points in the metric sense (cut locus for the Finsler metric induced by the norm of $\mathcal{A}$), and conjugate points in the tangent sense (the differential of the exponential map along γ is not invertible). Along the course, several tools from operator theory will be introduced and applied to this specific framework, which generalizes the classical Grassmann manifolds over $\mathbb{R}$ or $\mathbb{C}$.

4. **Vladimir SALNIKOV** — *La Rochelle University*, *France*

   Generalized geometry in relations to physics and mechanics

   Concents: Poisson and Dirac geometry, some related Q-manifolds constructions for sigma models, structure preserving numerical methods in mechanics, Poisson and Dirac integrators.

   In this minicourse I will make an overview of some recent advances in applying differential and generalized geometry to exhibiting intrinsic properties of physical and mechanical systems. I will start by recalling the standard constructions from Poisson geometry and Dirac structures and fit them to the description using differential graded geometry. I will then define the so-called equivariant Q-cohomology and explain the link of it to the gauging procedure. The two main examples will be the Poisson sigma model (the most general AKSZ one in space-time dimension 2) and the Dirac sigma model (the most general one obtained from the gauging procedure of a 3d Wess–Zumino term). As for mechanics, I will explain how Dirac structures appear naturally in dynamics of systems with constraints or coupled systems. We will also address the question of defining dynamics on Dirac structures via some variational principles. Time permitting, I will conclude by mentioning some consequences of these results for construction of geometric integrators — numerical methods preserving the geometric structure of the equations and qualitative properties of respective mechanical systems.

   References

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[3] A. Kotov, V. Salnikov, The category of $\mathbb{Z}$-graded manifolds: what happens if you do not stay positive. Preprint: arXiv:2108.13496

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5. **Anna ZAMOJSKA-DZIENIO** — *Warsaw University of Technology, Poland*

Barycentric algebras — convexity and order

Originating from the use of barycentric coordinates in geometry by Möbius (1827), barycentric algebras were introduced in the nineteen-forties for the axiomatization of real convex sets, presented algebraically with binary operations given by weighted means, the weights taken from the (open) unit interval in the real numbers.

Barycentric algebras unify ideas of convexity and order. They provide intrinsic descriptions of convex sets (cancellative b.a.), semilattices (commutative and associative b.a. with all operations being equal), and more general semilattice-ordered systems of convex sets, independently of any ambient affine or vector space. Natural applications can be found in physics, biology, and social sciences for the modeling of systems that function on (potentially incomparable) multiple levels, and also in computational geometry to analyze systems of barycentric coordinates.

In this minicourse, we first examine the algebraic aspects of barycentric algebras. Then, we focus on various examples and applications, reviewing the pertinence of the barycentric algebra structure.

References

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