XLI WORKSHOP ON GEOMETRIC METHODS IN PHYSICS

Białystok, Poland

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The Workshop is organized under the auspices of the Rector of the University of Białystok, Professor Robert W. Ciborowski.

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XLI Workshop on Geometric Methods in Physics is organized by:
Faculty of Mathematics, University of Białystok
Ciołkowskiego 1M, 15-245 Białystok, Poland

e-mail: wgmp@uwb.edu.pl

LIST OF ABSTRACTS

Plenary lectures

1. **Edwin BEGGS** — *Swansea University, Wales*

   Noncommutative geometry, parallel transport and quantum mechanics

   I shall consider parallel transport of states on noncommutative algebras, and differentiating positive maps more generally using Hilbert C^*-bimodules. This will be applied to the Heisenberg algebra of position and momentum operators. From this we can form a geometric picture of quantum mechanics using parallel transport on the Heisenberg algebra.

2. **Alexey BOLSINOV** — *Loughborough University, United Kingdom*

   Multicomponent generalisations of KdV and CH equations

   We construct a new series of multi-component integrable PDE systems that contains as particular examples (with appropriately chosen parameters) and generalises many famous integrable systems including KdV, Harry Dym, Camassa–Holm, Dullin–Gottwald–Holm and Kaup–Boussinesq systems. We discuss finite-dimensional reduction of these systems and their relations with the theory of geodesically equivalent metrics.

3. **David J. FERNÁNDEZ C.** — *CINVESTAV, Mexico*

   Darboux transformations applied to graphene in external magnetic fields

   The Dirac electron in graphene with magnetic fields which are orthogonal to the graphene surface is studied. The eigenvalue problem is reduced to two one-dimensional Schrödinger Hamiltonians which are Darboux-transformed to each other. The magnetic field is initially chosen such that the associated Schrödinger potentials turn out to be shape invariant. Then, more general first-order Darboux transformations are used to generate magnetic fields leading to new analytic solutions for the graphene problem. The iterations of the method are discussed in the real, singular, and complex cases, looking for new graphene Hermitian solvable Hamiltonians. The case when the magnetic fields are complex, leading to non-Hermitian graphene Hamiltonians with complex eigenvalues, is addressed. Finally, the magnetic periodic superlattices are explored.

4. **Noriaki IKEDA** — *Ritsumeikan University, Japan*

   Q-manifolds and sigma models

   We discuss relations of Q-manifolds (differential graded manifolds) to physical theories. A Q-manifold is called a QP-manifold if it has a compatible symplectic structure. One of main applications is the BRST-BV formalism of gauge theories. We discuss sigma models. There are two types of sigma models, topological ones and non-topological ones. In topological versions, ASKZ sigma models are directly constructed from Q-manifolds, which give a large class of topological sigma models. We discuss a generalization of AKSZ sigma models by deforming a Q-structure on the mapping space. Next, we discuss a non-topological version, a gauged nonlinear sigma model (GNLSM). Though physicists have constructed GNLSMs by hand, a Q-manifold structure naturally appears in this construction and we obtain a theoretical construction of GNLSMs.

5. **Marek KUŚ** — *Center for Theoretical Physics, Polish Academy of Sciences, Poland*

   Geometry of quantum entanglement

   I will review a geometric approach to classification and examination of quantum correlations (entanglement) in composite quantum systems. Using momentum map geometry, in particular the convexity of the momentum map, the Kirwan–Ness stratification, Morse theory and geometric invariant theory I will show how to classify states from the point of view of their application in quantum information processing.

6. **Jacek MIĘKISZ** — *University of Warsaw, Poland*

   Ergodic theory and topology of nonperiodic tilings and Gibbs states of interacting particles

   We will present lattice models of interacting particles based on nonperiodic tilings. Basic ergodic and topological properties of tiling spaces will be discussed. The main open problem is stability of ground states (minimizers of certain energy functionals) corresponding to tilings. The important property here seems to be rapid convergence to equilibrium of local patterns – the so called strict boundary property of uniquely ergodic tiling (ground-state) measures. No tiling system with such property is known.

   The interested reader of this abstract may look at slides of my recent presentation `https://www.cirm-math.fr/RepOrga/3002/Slides/Miekisz-CIRM2024.pdf`.

7. **Nicolai RESHETIKHIN** — *Yau Mathematical Sciences Center, Tsinghua University, China*

   Hybrid integrable systems

   The talk will focus on integrable systems where a quantum system is driven by an integrable quantum background.

8. **Karen STRUNG** — *Czech Academy of Sciences, Czech Republic*

   Realising Quantum Flag Manifolds as Graph C^*-algebras

   In this talk I will show how the C^*-completions of the so-called quantum flag manifolds—noncommutative spaces arising as homogeneous spaces of quantum groups—can be realised as graph C^*-algebras. After recalling the definition of a quantum flag manifold and its C^*-algebra, I will describe how to compute the primitive ideal space using Dijkhuizen and Stokmann's description of a complete set of irreducible $*$-representations. This allows one to to construct a graph directly from the Weyl group of the associated Lie algebra, and appeal to classification results of Eilers, Ruiz and Sorensen to show that this graph C^*-algebra is isomorphic to the C^*-algebra of the relevant quantum flag manifold. This recovers some known isomorphisms between the C^*-algebras of quantum flag manifolds, as well as determining surprising new ones.

   Joint work with Tomasz Brzeziński, Ulrich Krähmer, and Réamonn Ó Buachalla.

9. **Daniele VALERI** — *University of Rome La Sapienza, Italy*

   Integrability of classical affine W-algebras

   Classical affine W-algebras $W(\mathfrak{g}, \mathcal{O})$ are algebraic structures associated to a simple Lie algebra $\mathfrak{g}$ and a nilpotent orbit $\mathcal{O}$. In this talk we will describe how to associate to $W(\mathfrak{g}, \mathcal{O})$ an integrable hierarchy of PDEs. When $\mathcal{O}$ is the principal nilpotent orbit one gets the Drinfeld–Sokolov hierarchy, which gives the famous Korteweg–de Vries hierarchy for $\mathfrak{g} = sl_2$. The talk is based on joint works with Alberto De Sole, Mamuka Jibladze and Victor G. Kac.

10. **Tilmann WURZBACHER** — *Université de Lorraine, France*

    Multisymplectic geometry and dynamics — a case study

    I will first discuss the notion of "(dynamical) Hamilton–DeDonder–Weyl equations" generalizing Hamilton's equations on symplectic manifolds and Volterra's equations in classical field theory at the same time. Then I will specialise to a surprising example: two-plectic geometry and dynamics on the six-sphere. It is well-known that the octonionic structure on an 8-dimensional real space yields a closed three-form and a non-integrable almost-complex structure on the six-sphere. We present joint work with Maxime Wagner, notably concerning its non-flatness, automorphisms and dynamics.

Contributed lectures

1. **Krzysztof BARDADYN** — *University of Białystok, Poland*

   Inverse semigroup Banach algebras

   In 2008 Ruy Exel introduced tight C^*-algebras associated to inverse semigroups, that generalize group C^*-algebras but also work well with Cuntz algebras, graph C^*-algebras, self-similar actions, etc. I will present how to generalize these constructions and results to Banach algebras based on the theory of groupoid Banach algebras that we developed recently together with Bartosz Kwaśniewski and Andrew McKee.

2. **Goce CHADZITASKOS** — *Czech Technical University in Prague, Czech Republic*

   The Schwarzschild's solution and Mach's principle

   We derived a relation for Mach's principle by applying Schwarzschild's procedure to solve Einstein's equations within a homogeneous mass sphere. This results in the proportionality relation $c^2 \sim \frac{MG}{R} = \Phi$. We then use this conclusion to extend the solution to a three-dimensional sphere as a model of the spatial component of space-time. For a stationary model of the universe as a three-dimensional sphere in four-dimensional space, the possible proportionality coefficients are calculated.

3. **Johan Michel CHAVEZ TOVAR** — *CINVESTAV, Mexico*

   Geometric flows, entropy, and nonlinear electrodynamics

   When applied to a gravitational theory like general relativity, the Raychaudhuri equation is a fundamental result that describes the motion of nearby particles in curved spacetime. In this work, we present a method for deriving it by assuming a geometric flow in spacetime. Finally, an interesting relation between geometrical entropy and mean geodesic deviation can be established, a property that we study for black hole geometries coupled to nonlinear electrodynamics.

4. **Olga CHEKERES** — *La Rochelle University, France*

   Odd and generalized Wilson surfaces

   In this talk we discuss various extensions and generalizations of Wilson surface observables in gauge theories. Previously, Wilson surface observables were interpreted as a class of Poisson sigma models. We profit from this construction to define and study the super version of Wilson surfaces. We provide some 'proof of concept' examples to illustrate modifications resulting from appearance of odd degrees of freedom in the target. We also explain some natural directions for defining the analogues of Wilson surface observables in higher dimensions.

   The talk is mostly based on https://arxiv.org/abs/2403.09820. This is a joint work with Vladimir Salnikov.

5. **Alina DOBROGOWSKA** — *University of Białystok, Poland*

   A new look at Lie algebras

   We present a new look at description of real finite-dimensional Lie algebras. The basic ingredient is a pair (F, v) consisting of a linear mapping $F \in End(V)$ with an eigenvector v. This pair allows to build a Lie bracket on a dual space to a linear space V. The Lie algebra obtained in this way is solvable. In particular, when F is nilpotent, the Lie algebra is actually nilpotent. We show that these solvable algebras are the basic bricks of the construction of all other Lie algebras. Using relations between the Lie algebra, the Lie–Poisson structure and the Nambu bracket, we show that the algebra invariants (Casimir functions) are solutions of an equation which has an interesting geometric significance. Several examples illustrate the importance of these constructions.

6. **Gerald GOLDIN** — *Rutgers University, USA*

Quantum Mechanics without Quantization

I shall discuss foundational reasons why a certain infinite-dimensional semidirect product group serves as a universal kinematical group for quantum mechanics. This provides a unifying account of all possible quantum kinematics for systems with mass in an arbitrary physical space, and clarifies the role of topology. Our development does not require quantization of any classical phase space; rather, the classical limit follows from the quantum mechanics. My talk is based on joint work with David H. Sharp, Los Alamos National Laboratory.

7. **Piotr P. GOLDSTEIN** — *Theoretical Physics Division, National Centre for Nuclear Research, Poland*

Variants of asymptotic behaviour in the BKL scenario as described with a quadric of kinetic-energy

The Belinski–Khalatnikov–Lifshitz (BKL) scenario [1], which describes asymptotic behaviour of an anisotropic universe near the cosmological singularity, is found to manifest several interesting variants of behaviour. These variants are classified and analysed in terms of the model's diagonal Hamitonian variables. The geometric picture of the dynamics may be seen as advancing of the system in the 3-dimensional momentum space. This motion takes place within a quadric of the "kinetic energy". As the latter is an indefinite quadratic form, the quadric is a cone. Since the governing equations are time-reversible, the description may be used for a collapse towards or expansion from the singularity.

We find that the possible "subscenarios" of the collapse are:

(a) unstable squeeze of the universe in all directions to a point;

(b) stable approaching a limit in which the universe collapses in one or two directions, while infinitely stretching in the remaining two or one, like in Kasner's models [2];

(c) an oscillatory approach to such a limit;

(d) finally, an approach to a limit in two directions with unstable oscillations of increasing amplitude in the third.

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8. **Tomasz GOLIŃSKI** — *University of Białystok, Poland*

Integrable system related to restricted Grassmannian on partial isometries

This talk is about a certain hierarchy of integrable bihamiltonian systems constructed on Banach Lie–Poisson spaces related to the restricted Grassmannian $\mathrm{Gr}_{\mathrm{res}}(\mathcal{H})$. This hierarchy was introduced in the paper with A. Odzijewicz by introducing a family of Casimir functions for a pencil of Poisson brackets on the predual space to the central extension of the unitary restricted algebra $\mathfrak{u}_{\mathrm{res}}(\mathcal{H})$. Recently, or more precisely during the previous Workshop, we realized that under an additional condition this system defines a system of equations in involution on the Banach Lie groupoid of partial isometries. A particular solution for partial isometries of rank one will also be presented.

This is a joint work with A.B. Tumpach.

9. **Jerzy KIJOWSKI** — *Center for Theoretical Physics, Polish Academy of Sciences, Poland*

Variational Principles in Physics

It is shown that the standard derivation of field equations (Euler–Lagrange equations), based on fixing *a priori* the field boundary conditions, cannot be applied in the case of hyperbolic field theory, because leads to a contradiction. This contradiction has been overlooked by theoretical physicists. A remedy for this contradiction is proposed. It consists in describing Lagrangian field theory as a classical symplectic control system. Implications for various branches of physics are discussed.

10. **Bartosz KWAŚNIEWSKI** — *University of Białystok, Poland*

Topological freeness and ideals in twisted Banach algebra crossed products

We generalize some fundamental C*-algebraic result for crossed products of discrete transformation groups to the realm of Banach algebras and twisted actions. We show that topological freeness is equivalent to the intersection property for all reduced twisted Banach algebra crossed products coming from subgroups, and in the untwisted case to a generalized intersection property for a full Lp-operator algebra crossed product. This gives efficient simplicity criteria for various Banach algebra crossed products.

This talk is based on a joint work with Krzysztof Bardadyn.

11. **Gabriel LAROTONDA** — *University of Buenos Aires / CONICET, Argentina*

Conjugate points in the Grassmann manifold of a C^*-algebra

For a (real or complex) C^*-algebra $\mathcal{A}$, let $P^* = P^2 = P \in \mathcal{A}$. Consider $Gr(P)$ the unitary orbit of P in $\mathcal{A}$, which is one of the connected components of the Grassmannian manifold of $\mathcal{A}$. For a natural connection (which appears in different disguises) in $Gr(P)$, its geodesics are described by $\gamma(t) = e^{tZ}Pe^{-tZ}$ for $Z^* = -Z$ and $Z = ZP + PZ$. The exponential map of this connection is then $\mathrm{Exp}_P(Z) = e^Z Pe^{-Z}$. In this talk we will describe conjugate points along geodesics in the Grassmannian, both in the metric sense (cut locus for the Finsler metric induced by the norm of $\mathcal{A}$) and in the tangent sense (the differential of the exponential map along γ is not invertible). In the infinite dimensional setting, the distinction between monoconjugate and epiconjugate points will be relevant.

This is part of recent research in collaboration with Esteban Andruchow and Lázaro Recht (CONICET, Argentina).

12. **Anatolij PRYKARPATSKI** — *Cracow University of Technology, Poland*

On the conformal Lie superalgebras $K(1; N = 1; 2; 3)$, their super-pseudodifferential generalizations and related semi-supersymmetric integrable systems

As one of most important legacy of modern mathematical physics is deemed nowadays a new fruitful conception of supersymmetry, whose main idea [9, 11] is to treat elementary bosonic and fermionic particles equally, what mathematically amounts to incorporating [2, 7] anticommuting variables of Grassmann type together with the usual commuting variables. In such way a number of well known mathematical physical equations have been generalized into the supersymmetric analogues, amongst which we find [5, 8, 12] supersymmetric versions of sine-Gordon, Korteweg–de Vries, Burgers, Kadomtsev–Petviashvili hierarchy, Boussinesq, nonlinear Schrödinger equation and many others. The investigation of the problem of particle-like behavior in supersymmetric field theories naturally leads to a theory of super-integrable systems and studying their properties, which can be helpful in part in analysis of modern super-string mathematical physics problems. It is nowadays well known [3]) that there exist integrable fermionic extensions [12] of the completely integrable field theory systems on functional supermanifolds, related with conformal superalgebra symmetries, and which are not supersymmetric. There was also observed [6] that in most cases the supersymmetric integrable extensions were associated [8, 12] to evolution superflows, generated by means of the super- Lax type representations, yet as it was clearly demonstrated in the work [3], there exist also supersymmetric integrable extensions, related with semi-supersymmetric systems, generated by the centrally extended $N = 2$-superconformal loop Lie superalgebra symmetry and allowing reduction to the supersymmetric flows. Here is worth also to mention that, in fact, almost all of so called "new" nonlinear $N = 1$-superintegrable dynamical systems, published [3] during past decades, are related to coadjoint flows of the affine conformal $N = 1$-symmetry Lie superalgebra $\mathcal{K}(1|1)$, described in detail first in [4], and in general, are related [3] to the affine Sturm–Liouville type superconformal spectral problem $(-D_\theta^3 + (\sum_{j=-m}^{m+p-1}(u_j\theta + v_j)\lambda^j) + \theta\lambda^{m+p})f(x,\theta) = 0, m,p \in \mathbb{N},\ \lambda \in \mathbb{C}$, on the supercircle $\mathbb{S}^{1|1} \simeq \{(x,\theta) \in \mathbb{S}^1 \times \Lambda_1^{(1)}\}$ for a smooth function $f \in C^\infty(\mathbb{S}^{1|1}; \Lambda_0^{(1)})$, where $\Lambda_0^{(1)} \oplus \Lambda_1^{(1)} := \Lambda^{(1)}$ — the corresponding $\mathbb{Z}_2$-graded superalgebra. Meanwhile, as the description of $N=1$-supersymmetric Lax type flows was elaborated [8, 12] are known widely enough, the $N \geq 2$-supersymmetric integrable flows, analyzed in [6, 8], still should be paid more attention.

Owing to the interesting observation in the work [3], based on the affine Sturm–Liouville type superconformal spectral problem $(D_{\theta_1}D_{\theta_2}+\sum_{j=-m}^{m+p-1}u_j(x,\theta)\lambda^j+\lambda^{m+p})f(x,\theta)=0,\ m,p\in\mathbb{N},\ \lambda\in\mathbb{C}$, on the supercircle $\mathbb{S}^{1|2}\simeq\{(x,\theta)\in\mathbb{S}^1\times\Lambda_1^{(2)}\}$ for $f\in C^\infty(\mathbb{S}^{1|2};\Lambda_0^{(2)})$ the special reductions of the related nonlinear integrable superconformal evolution flows prove to be supersymmetric dynamical systems on functional supermanifolds. Another interesting Backlund type construction of nonlinear $N=2$-superconformal semi-supersymmetric dynamical systems was suggested in [3], generalizing in part those obtained before in [6].

Within our Report we successively analyze modern Lie algebraic approaches, lying in the background of effective constructions of integrable in general semi-supersymmetric Hamiltonian systems on functional $N\geq 2$-supermanifolds, possessing rich yet hidden super-symmetries and endowed with suitably related super-Poisson structures. As an application, we describe countable hierarchies of new Lax type integrable nonlinear $N=3$-semi-supersymmetric dynamical systems. In particular, we analyze the suitably central extended super-conformal affine Lie superalgebra $\hat{\mathcal{K}}(1|3)$ and its finite-dimensional coadjoint orbits, generated by the related Casimir functionals on the super-coalgebra $\hat{\mathcal{K}}(1|3)^*$, and construct an infinite hierarchy of completely integrable super-Hamiltonian systems on smooth functional supermanifolds, which also prove to be supersymmetric. Moreover, we generalized these results subject to the suitably factorized super-pseudodifferential Lax type representations, taking into account the devised before algebro-analytic constructions both in mentioned above works [3, 6] and in devoted to Lie algebraic properties of factorized Lax type representations [10] and the respectively factorized Hamiltonian systems. As a new interesting result, we succeed in algorithmic construction of integrable super-Hamiltonian factorized systems, generated by Casimir invariants of centrally extended pseudo-differential operator superalgerbras.

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13. **Praful RAHANGDALE** — *Paderborn University, Germany*

Poisson Geometry and related Geometric and Algebraic Structures

A Poisson structure is a foliation on a manifold, which partitions it into leaves. These leaves have a symplectic structure, and transverse to the leaves, there is a Lie algebra structure. Thus, Poisson geometry amalgamates foliation theory, symplectic geometry, and Lie theory.

The original motivation for studying Poisson structures comes from classical mechanics and integrable systems. Phase spaces of classical mechanical systems are modeled on Poisson manifolds. The time evolution of the system is given by the flow of the vector field corresponding to a distinguished function called the Hamiltonian of the system. The system is constrained to submanifolds, which are the level sets of functions that Poisson commute with the Hamiltonian. The set of these commuting functions are conserved quantities of the system.

In this talk, we shall first discuss the motivation and important examples of Poisson structures. Then, we will explore the local description of a Poisson manifold according to Weinstein splitting theorem, as a product of a symplectic manifold with another manifold transverse to it, whose Poisson structure vanishes at the point. After introducing symplectic realizations of Poisson manifolds, we will see that any Poisson manifold can be viewed as the quotient of a symplectic manifold. Finally, we will discuss Lie bialgebras, the algebraic counterpart of Poisson Lie groups, and the one-to-one correspondence between connected, simply connected Poisson Lie groups and finite-dimensional Lie bialgebras.

14. **Stefan RAUCH** — *Linkoping University, Sweden*

Understanding reversals of a Rattleback

The rattleback is a rigid body having a boat like shape (modelled as a bottom half of an 3-axial ellipsoid) having asymetric (chiral) distribution of mass. When the rattleback is spun on its bottom in the “wrong” direction then it starts to rattle, it slows down and acquires rotation in the opposite, preferred sense of spinning. This behaviour defies our intuition about conservation of angular momentum as the force and the torque responsible for changing the angular momentum (and the direction of spinning) is not easily discernible. Overwhelming majority of papers on the rattleback motion study the dependence of stability for spinning solutions: on the sense of rotation, on the shape of rattleback’s surface and on the distribution of mass. There has been no simple explanation of the rattleback behaviour in terms of physical forces and torques.

This question has been the subject of our paper with M. Przybylska that have appeared in Regular and Chaotic Dynamics, a journal of Russian Academy of Sciences. In this paper we study the motion of a toy rattleback by using frictionless Newton equations of motion for a rigid body rolling without sliding in a plane. In these equations it is the reaction force of the supporting surface that is the source of the torque turning the rattleback in the preferred sense of rotation. The picture is, however, more subtle as it appears that the direction of the torque depends on the initial conditions and a frictionless, low energy rattleback admits reversals in both directions (!).

I will present a simple, intuitive understanding of how the rattleback’s motion depends on initial conditions and will discuss how it is consistent with numerical simulations of rattlebacks equations for tapping and for spinning initial conditions. Simulations show also that long time behaviour of such rattleback is, for low energy initial conditions, quasi-periodic and there are inifinitly many reversals in both directions.

15. **Alice Barbara TUMPACH** — *Wolfgang Pauli Institut / Lille University, Austria*

Subtleties of Poisson Geometry in infinite dimensions

In this short talk, I will present different possible definitions of Poisson structures in the infinite-dimensional context. Examples will be given in order to highlight the differences and the contexts of applications.

16. **Anna ZAMOJSKA-DZIENIO** — *Warsaw University of Technology, Poland*

The algebraic approach to barycentric coordinates

Barycentric coordinates provide solutions to the problem of expressing an element of a compact convex set as a convex combination of a finite number of extreme points of the set. They have been studied widely within the geometric literature, typically in response to the demands of numerical analysis and computer graphics. In this talk we bring an algebraic perspective to the problem, based on barycentric algebras. We present some recent results obtained together with A. Romanowska (Warsaw University of Technology) and J.D.H Smith (Iowa State University).

Virtual contributed lectures

1. **Luca CAMPOBASSO** — *Łódź University of Technology, Poland*

   Approximations to the Wigner function

   The Green's function for the eigenvalue equation for the Wigner distribution was found and applied, and three different potentials were analysed as examples. A recursive formula approximating the eigenvalue equation is presented alongside.

2. **Eduardo CHIUMIENTO** — *University of La Plata / CONICET, Argentina*

   Geometric approach to the Moore–Penrose inverse and polar decomposition in operator ideals

   In this talk we consider three fundamental maps in matrix analysis and operator theory: the Moore–Penrose inverse, the operator modulus, and the polar factor arising in the polar decomposition. We work in an infinite-dimensional setting, where we use norms given by symmetrically-normed ideals, and the notion of index of a pair of projections. We will show that the Moore–Penrose inverse is a real binalytic map between homogeneous spaces of Lie–Banach groups associated with symmetrically-normed ideals. Furthermore, the maps given by the operator modulus and polar factor are real analytic fiber bundles. This is based on a recent joint work with Pedro Massey (IAM-UNLP).

3. **Mahouton Norbert HOUNKONNOU** — *University of Abomey-Calavi, Benin*

   Invariance of the scalar curvature for the noncommutative 2-torus equipped with a perturbed complex structure

   This talk addresses a noncommutative 2-torus $\mathcal{A}_\theta = C(\mathbb{T}^2_\theta)$ equipped with a perturbed complex structure by inner derivation. The corresponding Laplacian and the perturbed pseudo-differential symbols are obtained. The local expression of the scalar curvature and its invariance are investigated.

4. **Taika OKUDA** — *Tokyo University of Science, Japan*

   Explicit Formulae for Deformation Quantization with Separation of Variables of $G_{2,4}(\mathbb{C})$

   This is the joint work with Akifumi Sako (Tokyo University of Science). We will talk about the explicit formula that gives the deformation quantization with separation of variables of $G_{2,4}(\mathbb{C})$. For arbitrary locally symmetric Kähler manifolds, the construction methods of deformation quantization with separation of variables of it were proposed by Sako–Suzuki–Umetsu and Hara–Sako. These constructions make it possible to give the star product on a locally symmetric Kähler manifold such that its coefficients satisfy some recurrence relations. The explicit star product for $\mathbb{C}^N$, $\mathbb{C}P^N$, $\mathbb{C}H^N$ and arbitrary one- and two-dimensional ones were obtained from this construction method so far. In this talk, we denote the explicit formula of the star product on $G_{2,4}(\mathbb{C})$ by using the construction method proposed by Hara–Sako. We show that for $G_{2,4}(\mathbb{C})$, the explicit form of general terms of the recurrence relations given by Hara–Sako can be determined explicitly via the Fock representations. From the obtained general terms, we also give the star product on $G_{2,4}(\mathbb{C})$ explicitly.

5. **Dzianis ZHALUKEVICH** — *University of Białystok, Poland*

   Group classification of second-order quasi-linear equations with two independent variables in homogeneous isotropic media

   In this paper, a group classification of second-order quasi-linear equations with two independent variables in homogeneous isotropic media is carried out, and analytical solutions of these equations are sought.

Poster presentations

1. **Jarah FLUXMAN** — *University of Edinburgh, United Kingdom*

   UIRs of Lifschitz groups

   Classifying the Unitary Irreducible Representations of certain Lie groups of the Lifschitz type using the method developed by Mackey.

2. **Jiří HRIVNÁK** — *Czech Technical University in Prague, Czech Republic*

   Honeycomb Weyl orbit functions of G_2

   Motivated by current extensive research of triangular graphene quantum dots, the two-dimensional honeycomb lattice is constructed via subtraction of the weight and dual weight lattices of the crystallographic root system G_2. Four types of Weyl orbit functions, that correspond to four sign homomorphisms of the associated Weyl group, are labelled by the dominant weights from the weight lattice. The fundamental and dual fundamental domains of the induced affine and dual affine Weyl groups determine both finite point and weight sets utilized for simultaneous weight and dual weight discretizations of Weyl orbit functions. Modified weight sets of the dual weight discretization represent cornerstones for introduction of G_2 honeycomb Weyl orbit functions. Subtracting the weight and dual weight point sets produces honeycomb point sets shaped as 30-60-90 triangles over which the discrete orthogonality of honeycomb Weyl orbit functions is discussed. This is a joint work with Tomasz Czyżycki.

3. **Severin POŠTA** — *Czech Technical University in Prague, Czech Republic*

   Construction and application of quasicrystals

   We discuss three methods that generate n-dimensional quasicryatals and propose two applications of quasicrystals to data processing. The first application is to use the mapping between the physical and internal spaces of a quasi-crystal to evenly distribute data that is lost in the process of transmitting or storing information. At the same time, it is possible to unambiguously restore the rest of the data. The second application consists in the construction of special quasi-crystals that satisfy the requirements of keys of any length for the classical Vernam cipher method. Several examples of construction of quasicrystals with predetermined properties and examples of image processing that makes the loss of its part uniformly distributed are given.

4. **Elwira WAWRENIUK** — *University of Białystok, Poland*

   Title to be announced

5. **Karolina WOJCIECHOWICZ** — *University of Białystok, Poland*

   Linear bundle of Lie algebras applied to the classification of real Lie algebras

   We present a new look at the classification of real low-dimensional Lie algebras based on the notion of linear bundle of Lie algebras. Belonging to suitable family of Lie bundles entails compatibility of Lie–Poisson structures on dual space to those algebras. It gives us thus compatibility of bi-Hamiltonian structure on space of upper triangular matrices and with a bundle at the algebra level.

6. **Akira YOSHIOKA** — *Department of Mathematics, Tokyo University of Science, Japan*

   Star product deformation of the zeta function

   One parameter deformation of function is given by means of convergent star product, where the deformation parameter varies in some domain of $\mathbb{C}$. The deformed functions are called *star functions*. In this presentation, we give a short survey of star product, especially as a simple case, a star product of functions of one variable. We discuss a deformation of the Riemann zeta function by the star product.

LIST OF PARTICIPANTS

1. **BARDADYN, Krzysztof**
University of Białystok
Białystok, Poland
kbardadyn@math.uwb.edu.pl

2. **BEGGS, Edwin**
Swansea University
Swansea, Wales
e.j.beggs@swansea.ac.uk

3. **BELTIŢĂ, Daniel**
Institute of Mathematics “Simion Stoilow” of the Romanian Academy
Bucharest, Romania
beltita@gmail.com, Daniel.Beltita@imar.ro

4. **BOLSINOV, Alexey**
Loughborough University
Loughborough, United Kingdom
A.Bolsinov@lboro.ac.uk

5. **BRZEZIŃSKI, Tomasz**
University in Białystok / Swansea University
Białystok / Swansea, Poland / Wales
t.brzezinski@swansea.ac.uk

6. **CAMPOBASSO, Luca**
Łódź University of Technology
Łódź, Poland
luca.campobasso@dokt.p.lodz.pl

7. **CEWKOWICZ, Szymon**
University of Białystok
Białystok, Poland
sc87251@student.uwb.edu.pl

8. **CHADZITASKOS, Goce**
Czech Technical University in Prague
Prague, Czech Republic
chadzgoc@cvut.cz

9. **CHAVEZ TOVAR, Johan Michel**
CINVESTAV
Mexico City, Mexico
johan.chavez@cinvestav.mx

10. **CHEKERES, Olga**
La Rochelle University
La Rochelle, France
olya.chekeres@gmail.com

11. **CHIUMIENTO, Eduardo**
University of La Plata / CONICET
Buenos Aires, Argentina
eduardo@mate.unlp.edu.ar

12. **CZYŻYCKI, Tomasz**
University of Białystok
Białystok, Poland
tomczyz@math.uwb.edu.pl

13. **DOBROGOWSKA, Alina**
University of Białystok
Białystok, Poland
alina.dobrogowska@uwb.edu.pl

14. **FERNÁNDEZ C., David J.**
CINVESTAV
Mexico City, Mexico
david@fis.cinvestav.mx

15. **FLUXMAN, Jarah**
University of Edinburgh
Edinburgh, United Kingdom
s2340356@ed.ac.uk

16. **GOLDIN, Gerald**
Rutgers University
New Brunswick, NJ, USA
geraldgoldin@dimacs.rutgers.edu

17. **GOLDSTEIN, Piotr P.**
Theoretical Physics Division, National Centre for Nuclear Research
Warsaw, Poland
piotr.goldstein@ncbj.gov.pl

18. **GOLIŃSKI, Tomasz**
University of Białystok
Białystok, Poland
tomaszg@math.uwb.edu.pl

19. **HOROWSKI, Maciej**
University of Białystok
Białystok, Poland
horowski@math.uwb.edu.pl

20. **HOUNKONNOU, Mahouton Norbert**
University of Abomey-Calavi
Cotonou, Benin
hounkonnou@yahoo.fr

21. **HRIVNÁK, Jiří**
Czech Technical University in Prague
Prague, Czech Republic
jiri.hrivnak@fjfi.cvut.cz

22. **IKEDA, Noriaki**
Ritsumeikan University
Kusatsu, Japan
nikeda@se.ritsumei.ac.jp

23. **JAKIMOWICZ, Grzegorz**
University of Białystok
Białystok, Poland
g.jakimowicz@uwb.edu.pl

24. **KIELANOWSKI, Piotr**
CINVESTAV
Mexico City, Mexico
kiel@fis.cinvestav.mx

25. **KIJOWSKI, Jerzy**
Center for Theoretical Physics, Polish Academy of Sciences
Warsaw, Poland
kijowski@cft.edu.pl

26. **KUŚ, Marek**
Center for Theoretical Physics, Polish Academy of Sciences
Warsaw, Poland
marek.kus@cft.edu.pl

27. **KWAŚNIEWSKI, Bartosz**
University of Białystok
Białystok, Poland
b.kwasniewski@uwb.edu.pl

28. **LAROTONDA, Gabriel**
University of Buenos Aires / CONICET
Buenos Aires, Argentina
glaroton@dm.uba.ar

29. **MIĘKISZ, Jacek**
University of Warsaw
Warszawa, Poland
miekisz@mimuw.edu.pl

30. **NOVOTNÝ, Petr**
Czech Technical University in Prague
Prague, Czech Republic
petr.novotny@fjfi.cvut.cz

31. **OKUDA, Taika**
Tokyo University of Science
Tokyo, Japan
warriors.aboot@gmail.com

32. **POŠTA, Severin**
Czech Technical University in Prague
Prague, Czech Republic
severin.posta@fjfi.cvut.cz

33. **PRYKARPATSKI, Anatolij**
Cracow University of Technology
Kraków, Poland
pryk.anat@cybergal.com

34. **RADZISZEWSKI, Krzysztof**
University of Białystok
Białystok, Poland
k.radziszewski@uwb.edu.pl

35. **RAHANGDALE, Praful**
Paderborn University
Paderborn, Germany
praful@math.uni-paderborn.de

36. **RAUCH, Stefan**
Linkoping University
Linkoping, Sweden
Stefan.Rauch@liu.se

37. **RESHETIKHIN, Nicolai**
Yau Mathematical Sciences Center, Tsinghua University
Beijing, China
reshetik@math.berkeley.edu

38. **SALNIKOV, Vladimir**
La Rochelle University
La Rochelle, France
vladimir.salnikov@univ-lr.fr

39. **SLIŻEWSKA, Aneta**
University of Białystok
Białystok, Poland
anetasl@uwb.edu.pl

40. **SZAJEWSKA, Marzena**
University of Białystok
Białystok, Poland
m.szajewska@math.uwb.edu.pl

41. **STRUNG, Karen**
Czech Academy of Sciences
Prague, Czech Republic
strung@math.cas.cz

42. **TUMPACH, Alice Barbara**
Wolfgang Pauli Institut / Lille University
Vienna, Austria
alice-barbora.tumpach@univ-lille.fr

43. **VALERI, Daniele**
University of Rome La Sapienza
Rome, Italy
daniele.valeri@uniroma1.it

44. **WAWRENIUK, Elwira**
University of Białystok
Białystok, Poland
e.wawreniuk@uwb.edu.pl

45. **WOJCIECHOWICZ, Karolina**
University of Białystok
Białystok, Poland
kwojciechowicz@math.uwb.edu.pl

46. **WURZBACHER, Tilmann**
Université de Lorraine
Metz, France
tilmann.wurzbacher@univ-lorraine.fr

47. **YOSHIOKA, Akira**
Department of Mathematics, Tokyo University of Science
Tokyo, Japan
yoshioka_a@rs.tus.ac.jp

48. **ZAMOJSKA-DZIENIO, Anna**
Warsaw University of Technology
Warsaw, Poland
anna.zamojska@pw.edu.pl

49. **ZHALUKEVICH, Dzianis**
University of Białystok
Białystok, Poland
den.zhal@yandex.by