On Quantization

The laws of classical physics are considered approximate. They result from the laws of quantum physics (considered to be fundamentally rigorous) applied to "large" objects. This means that classical physics can be formally obtained from quantum physics as the limit: $\hbar \to 0$. It is therefore difficult to expect that it will be possible to reason in the opposite direction, i.e. to build a quantum theory based on knowledge of a classical theory. And yet, paradoxically, all important quantum theories (quantum mechanics, quantum electrodynamics, even quantum chromodynamics) have been formulated in this way: by "quantizing" classical theory. The "quantization procedure", initiated by W. Heisenberg's hypothesis, which consists in replacing the classical observables (position and momentum) with their non-commutative counterparts in such a way that the Poisson bracket is represented by their commutator, hides many puzzles and surprises that have been overlooked by both physicists and mathematicians. Some of these surprises are very painful (not everything can be "quantized", and even if something can be quantized, the result may be ambiguous), but some are unexpectedly positive (in some situations the result is unique and unambiguous).

The course will analyze the physical and mathematical aspects of various "quantization schemes" proposed in the literature. Mathematically, I will not focus on the technical side of the proofs, but on showing the essence of various mathematical structures that appear automatically as a result of the adopted physical assumptions. The following issues will be discussed (in more or less detail):

1. Quantization according to Heisenberg: successes and limitations.
2. Quantization according to Schrödinger.
3. Equivalence analysis of both approaches.
4. Beames of classical free particles and the quantum mechanical momentum operator.
5. Wave function as a "half-density".
6. Un-ambiguous quantization of "vector fields".
7. Quantization of the Galilean transformations.
8. "Fractional Fourier transform" and the harmonic oscillator.
9. Weyl rule. Moyal bracket.
10. Spin quantization.
11. Hermitian and self-adjoint operatorse.
12. What is a "continuous spectrum"?
13. Paradoxes. Example: "Arrival time" in quantum mechanics.
14. Newton-Wigner position operator.

I assume knowledge of physics and mathematics resulting from completing first two years of bachelor's studies of physics.

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