Alexey Bolsinov (A.Bolsinov@lboro.ac.uk)
Minicourse on Nijenhuis Geometry

This course is an introduction to Nijenhuis Geometry, a new challenging area in Differential Geometry that studies local and global properties of geometric structures given by a field of endomorphisms with vanishing Nijenhuis torsion. This topic is located on the crossroad of Geometry, Mathematical Physics and Algebra as Nijenhuis structures naturally appear in many seemingly unrelated research areas such as bi-Hamiltonian integrable systems (both finite and infinite-dimensional), projective geometry, theory of left-symmetric algebras and others.

Programme includes the following topics
- Fields of endomorphisms. Nijenhuis torsion and Nijenhuis operators.
- Basic properties of Nijenhuis operators. Splitting theorem.
- Diagonalisable and differentially non-degenerate Nijenhuis operators.
- Nijenhuis operators with complex eiganvalues.
- Nilpotent Nijenhuis operators and Jordan blocks.
- Singular points of Nijenhuis operators and linearisation.
- Left-symmetric algebras. Linearisability and non-degenaracy.
- gl-regular Nijenhuis operators and their canonical forms.
- Nijenhuis perturbations of a Jordan block.
- Normal forms for gl-regular Nijenhuis operators in dimension 2.
- Global properties of Nijenhuis operators on closed manifolds.
- Nijenhuis operators and geodesically equivalent metrics.
- Open problems in Nijenhuis Geometry.

The course will be based on a series of recent papers/preprints:
- A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Nijenhuis geometry. *Adv. in Math.*, 394: 108001, (2022).
- A. Konyaev, Nijenhuis geometry II: Left-symmetric algebras and linearization problem for Nijenhuis operators. *Diff. Geom. and Appl.*, 74: 101706, 2021.
- A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Nijenhuis Geometry III: gl-regular Nijenhuis operators. *Rev. Mat. Iberoamericana*, 2023, DOI 10.4171/RMI/1416.
- A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Nijenhuis Geometry IV: conservation laws, symmetries and integration of certain non-diagonalisable systems of hydrodynamic type in quadratures, *arXiv:2304.10626*.
- A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Applications of Nijenhuis geometry: non-degenerate singular points of Poisson–Nijenhuis structures, *Europ. Jour. of Math.*, 8: 1355–1376, 2022.
- A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Applications of Nijenhuis geometry II: maximal pencils of multi- Hamiltonian structures of hydrodynamic type. *Nonlinearity*, 34(8): 5136–5162, 2021.
- A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Applications of Nijenhuis geometry III: Frobenius pencils and compatible non-homogeneous Poisson structures. *Jour. of Geom. Anal.*, 33:193, 2023.
- A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Applications of Nijenhuis Geometry IV: multicomponent KdV and Camassa-Holm equations. *Dynamics of PDEs*, 20(1): 73–98, 2023.
- A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, Applications of Nijenhuis Geometry V: geodesic equivalence and finite-dimensional reductions of integrable quasilinear systems, Jour. of Nonlinear Science, 34:33, 2024.