XII SCHOOL ON GEOMETRY AND PHYSICS

Białystok, Poland

June 26 - June 30, 2023

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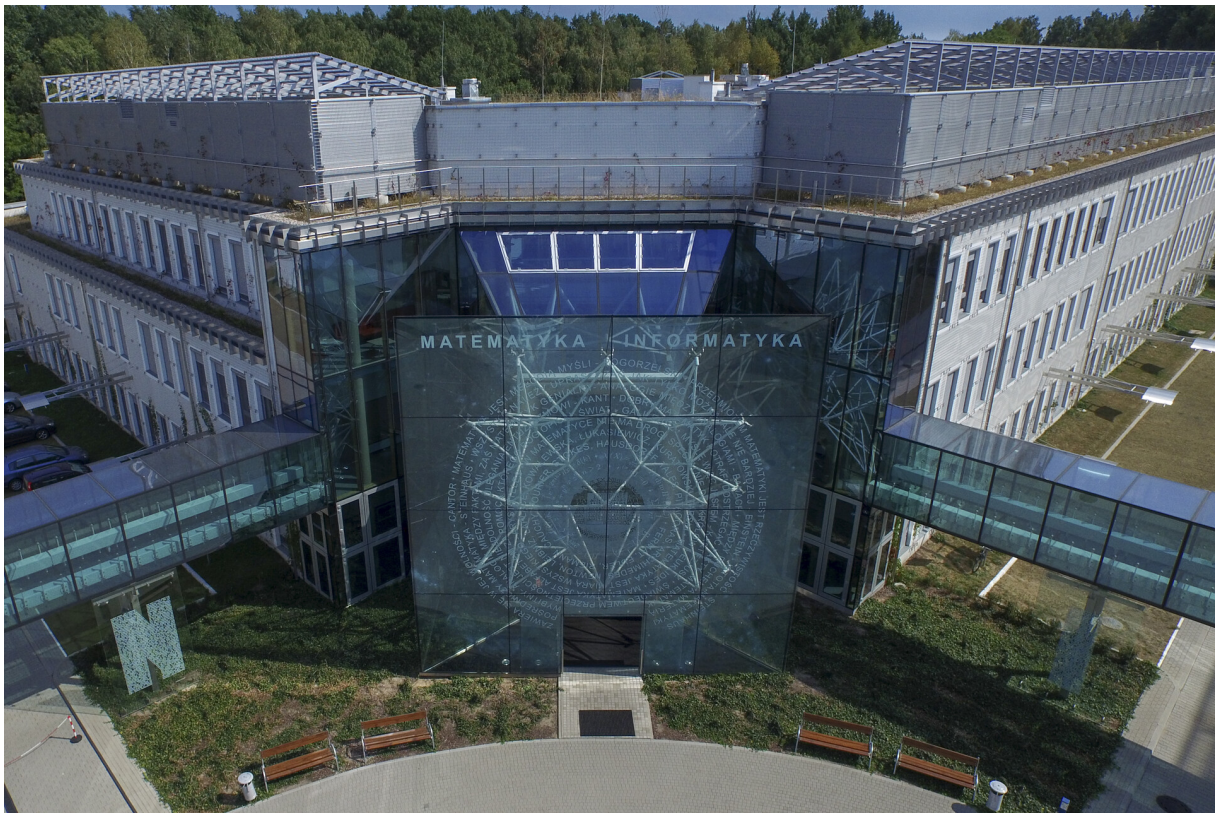
The Workshop and the School are co-funded by Ministry of Education and Science, Poland programme “Doskonała Nauka” under the project ”XL Konferencja *Workshop on Geometric Methods in Physics*” nr DNK/SP/548722/2022. Funding amount 101,970.00 zł, total project value 131,270.00 zł.

Campus of the University of Białystok



School: map of Białystok

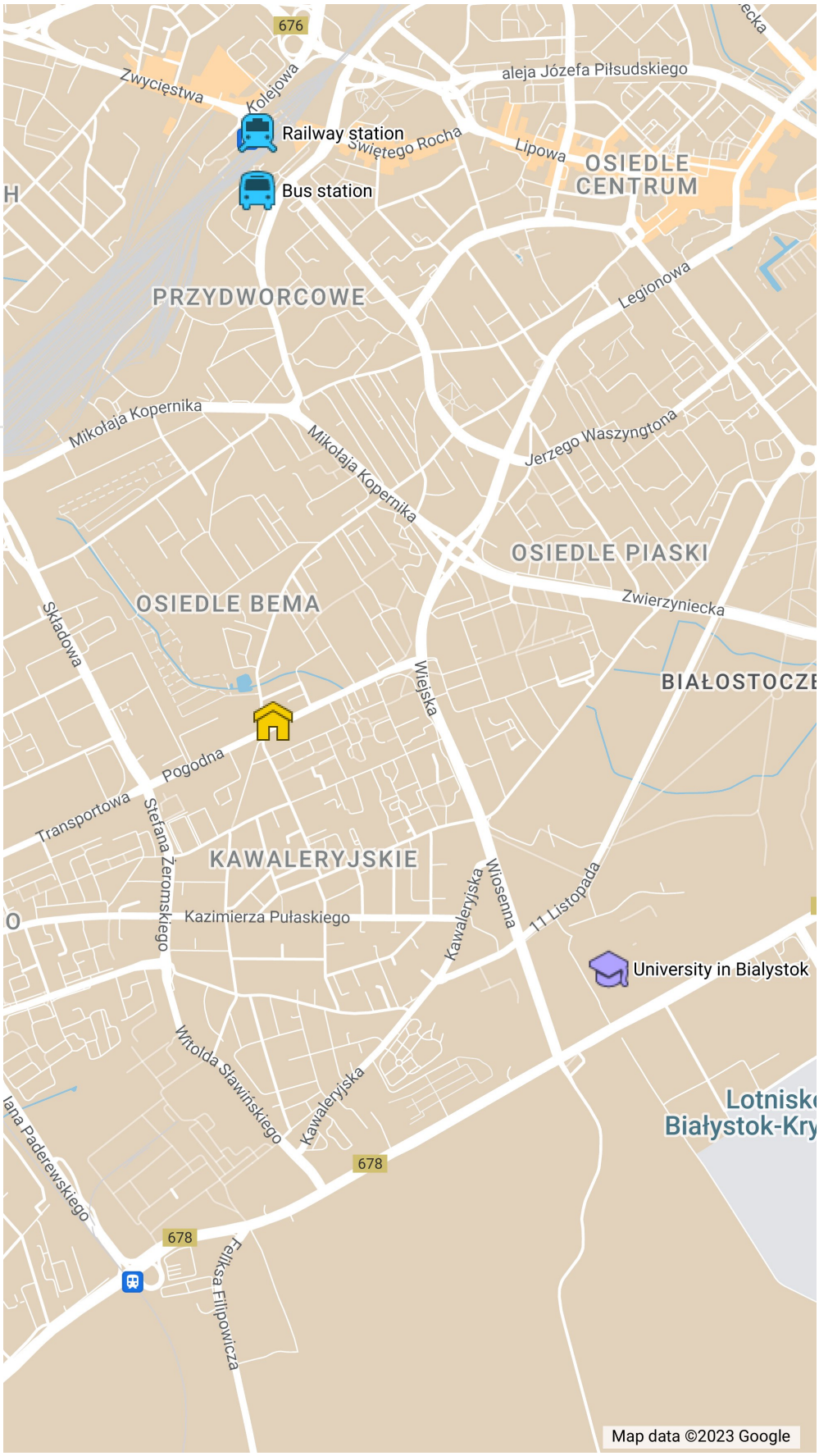
Points of interest

University in Bialystok

Railway station

Bus station

Centrum Titanic



LIST OF COURSES

1. **Marco BERTOLA** — *Concordia University, Canada*

   Orthogonal polynomials, some of their applications and asymptotic analysis

   The course provides an overview to the theory and applications of orthogonal polynomials (OPs) The undergraduate student most likely encounters OPs when discussing separation of variables in solutions of important PDEs, notably the harmonic oscillator in quantum mechanics (i.e. Hermite polynomials). However, their applications cover a much wider range of topics, whose list includes (but is not limited to): elements of combinatorics, number theory (e.g. the proof that the Euler constant is transcendental); integrable systems (e.g. the Toda lattice equations); stochastic models (random matrices); special equations (Painlevé equations).

   The topics covered in the course will be:

   - Origins, definitions and fundamental properties.
   - Asymptotic analysis for large degrees; elements of nonlinear steepest descent analysis
   - Some applications to spectral theory of large random matrices (hopefully with mention of Fredholm determinants and Tracy–Widom distribution, time permitting).

   The course is aimed at graduate students (or advanced undergraduate) with a solid grasp of complex analysis (contour integration, conformal properties of holomorphic functions, Cauchy theorem(s)), linear algebra and elementary measure theory.

2. **Helge GLÖCKNER** — *Universität Paderborn, Germany*

   Introduction to Infinite-Dimensional Lie Groups

   The talks shall provide an introduction to infinite-dimensional Lie groups and the underlying differential calculus in locally convex spaces.

   We shall encounter the main classes of infinite-dimensional Lie groups (like linear Lie groups, mapping groups, diffeomorphism groups, and direct limits of finite-dimensional Lie groups).

   We shall also distinguish certain classes of infinite-dimensional Lie groups which are well behaved, like Lie groups possessing an exponential function, locally exponential Lie groups (whose exponential function is a local diffeomorphism at 0), BCH-Lie groups, and regular Lie groups.

3. **Katarzyna GRABOWSKA** — *University of Warsaw, Poland*

   Contact geometry with applications

   Contact manifold is a manifold equipped with codimension one distribution which is maximally nonintegrable. The last condition implies that contact manifolds are always odd-dimensional. In the literature they are regarded as odd-dimensional generalizations of symplectic manifolds. During the mini-course we shall examine the traditional definition of a contact manifold and explore the alternative approach via homogeneous symplectic structures. Using the symplectic approach we shall discuss contact Hamiltonian dynamics as well as a version of contact Lagrangian mechanics, Hamilton Jacobi theory and contact reductions. We shall learn by closely examining several examples. Each lecture will be accompanied by a few homework problems for willing participants.

4. **Andriy PANASYUK** — *Cardinal Wyszyński University, Poland*

Kronecker webs and nonlinear PDEs

A web on a manifold is a collection of foliations of constant dimension in a general position. Theory of classical webs, i.e. finite collection of foliations, was developed in the first half of XX century by the school of W. Blaschke. In the end of that century I. Gelfand and I. Zakharevich introduced Kronecker webs [GZ91], collections of foliations parametrized by one-dimensional projective space with a particular dependence on the parameter, as objects encoding the local geometry of finite-dimensional bi-hamiltonian structures, i.e. pairs of compatible Poisson structures. Later Kronecker webs appeared as independent object of investigations in relation with integrable nonlinear systems of PDEs [Zak00], [DK14], [KP17], [Pan19].

In this series of lectures I will give an outline of this relation based on the last two references and also explain recent ideas putting preceding results into the context of the so-called heavenly PDEs describing vacuum self-dual Einstein metrics in neutral signature [PS22].

References

[DK14] M. Dunajski, W. Kryński, *Einstein–Weyl geometry, dispersionless Hirota equation and Veronese webs*, Math. Proc. Camb. Phil. Soc. **157** (2014), 139–150, doi:10.1017/S0305004114000164.

[GZ91] I. Gelfand, I. Zakharevich, *Webs, Veronese curves, and bihamiltonian systems*, J. Funct. Anal. **99** (1991), 150–178.

[KP17] B. Kruglikov, A. Panasyuk, *Veronese webs and nonlinear PDEs*, J. Geom. Phys. **115** (2017), 45–60.

[Pan19] A. Panasyuk, *Kronecker webs, Nijenhuis operators, and nonlinear PDEs*, Banach Center Publications **117** (2019), 177–210.

[PS22] A. Panasyuk, A. Szereszewski, *Webs, Nijenhuis operators, and heavenly PDEs*, arXiv:2211.03197.

[Zak00] I. Zakharevich, *Nonlinear wave equation, nonlinear Riemann problem, and the twistor transform of Veronese webs*, math-ph/00006001.

5. **Lyudmila TUROWSKA** — *Chalmers University of Technology / University of Gothenburg, Sweden*

Quantum correlations, non-local games and operator algebras

There are several mathematical models to describe the conditional probability densities/correlations that can occur when two labs in entangled state conduct quantum measurements. It has been fundamental research done to study whether these models give rise to the same sets of correlations. The celebrated Bell theorem demonstrates that the set of classical correlations are strictly smaller than the quantum ones, while the Tsirelsson problems are related to differences between "physically realizable" bipartite probability distributions. Operator systems, operator algebras and their tensor products have been an important tool to study such distributions. One of the Tsirelson problems is related to finite approximability in operator algebras and is equivalent to the Connes' embedding problem in von Neumann algebras, the solution of which was recently announced. Many results on correlations have come from the study of non-local games and their winning strategies. They witness the differences between classes of correlations and provide ways of constructing new interesting classes of operator algebras.

In these lectures I will give an introduction of non-local games. After going over the basic theory of operator systems I will highlight the role C^*-algebras and operator systems play in mathematical understanding of quantum correlations and perfect strategies of non-local games. Synchronous games as for example graph homomorphism/isomorphism games are of particular interests as their perfect strategies can be described through traces of affiliated C^*-algebras. I will discuss differences between classes of quantum correlations.

The last lecture will be devoted to some recent results on quantum no-signaling correlations which appear as strategies of non-local games with quantum inputs and outputs. If time allows I will also talk about quantum graphs and quantum homomorphism/isomorphism games.

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Monday, June 26

LECTURES 09:30–12:40

09:30–10:20 *Kronecker webs and nonlinear PDEs*
Andriy PANASYUK, Cardinal Wyszyński University, Poland

10:20–10:50 Coffee break

10:50–11:40 *Introduction to Infinite-Dimensional Lie Groups*
Helge GLÖCKNER, Universität Paderborn, Germany

11:50–12:40 *Orthogonal polynomials, some of their applications and asymptotic analysis*
Marco BERTOLA, Concordia University, Canada

18:00 School dinner

Tuesday, June 27

LECTURES 09:30–12:40

09:30–10:20 *Quantum correlations, non-local games and operator algebras*
Lyudmila TUROWSKA, Chalmers University of Technology / University of Gothenburg, Sweden

10:20–10:50 Coffee break

10:50–11:40 *Kronecker webs and nonlinear PDEs*
Andriy PANASYUK, Cardinal Wyszyński University, Poland

11:50–12:40 *Orthogonal polynomials, some of their applications and asymptotic analysis*
Marco BERTOLA, Concordia University, Canada

Wednesday, June 28

LECTURES 09:30–12:40

09:30–10:20 *Contact geometry with applications*
Katarzyna GRABOWSKA, University of Warsaw, Poland

10:20–10:50 Coffee break

10:50–11:40 *Introduction to Infinite-Dimensional Lie Groups*
Helge GLÖCKNER, Universität Paderborn, Germany

11:50–12:40 *Kronecker webs and nonlinear PDEs*
Andriy PANASYUK, Cardinal Wyszyński University, Poland

13:30 Excursion

Thursday, June 29

LECTURES 09:30–12:40

09:30–10:20 *Quantum correlations, non-local games and operator algebras*
Lyudmila TUROWSKA, Chalmers University of Technology / University of Gothenburg, Sweden

10:20–10:50 Coffee break

10:50–11:40 *Contact geometry with applications*
Katarzyna GRABOWSKA, University of Warsaw, Poland

11:50–12:40 *Introduction to Infinite-Dimensional Lie Groups*
Helge GLÖCKNER, Universität Paderborn, Germany

Friday, June 30

LECTURES 09:30–12:40

09:30–10:20 *Orthogonal polynomials, some of their applications and asymptotic analysis*
Marco BERTOLA, Concordia University, Canada

10:20–10:50 Coffee break

10:50–11:40 *Contact geometry with applications*
Katarzyna GRABOWSKA, University of Warsaw, Poland

11:50–12:40 *Quantum correlations, non-local games and operator algebras*
Lyudmila TUROWSKA, Chalmers University of Technology / University of Gothenburg, Sweden