XL WORKSHOP ON GEOMETRIC METHODS IN PHYSICS

Białowieża, Poland

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LIST OF ABSTRACTS

Plenary lectures

1. **Anastasia DOIKOU** — *Heriot–Watt University, UK*

   Quantum groups, discrete Magnus expansion, pre-Lie & tridendriform algebras

   We review the discrete evolution problem and the corresponding solution as a discrete Dyson series in order to rigorously derive a discrete version of Magnus expansion. We also systematically derive the discrete analogue of the pre-Lie Magnus expansion and show that the elements of the discrete Dyson series are expressed in terms of a tridendriform algebra action. Key links between quantum algebras, tridendriform and pre-Lie algebras are then established. This is achieved by examining tensor realizations of quantum groups, such as the Yangian. We show that these realizations can be expressed in terms of tridendriform and pre-Lie algebras actions.

2. **Adam DOLIWA** — *University of Warmia and Mazury, Poland*

   Rational approximation, multiple orthogonal polynomials, and integrability

   I plan to discuss the connections between the three issues above. At the level of two independent variables, the structural relationships between the theory of orthogonal polynomials and the Padé approximants can be reduced to the study of the Hankel type determinants. The Frobenius identities they satisfy are a special case of the integrable discrete-time Toda equations introduced by Ryogo Hirota.

   It is difficult to overestimate the role of orthogonal polynomials in theoretical physics and applied mathematics. They appear in the separation of variables of the fundamental partial differential equations of theoretical physics, representations of Lie groups and algebras and their quantum versions, random walks, numerical integration, to name a few notable examples. The spectral approach to orthogonal polynomials can be treated as a bridge between them and the theory of integrable systems, and the classical three-term recurrence relation is the spectral problem for the Toda system with continuous or discrete time.

   The Hermite–Padé approximants, introduced by Hermite in his proof of the transcendence of Euler's number e, are closely related to the theory of multiple orthogonal polynomials, which has been intensively studied in recent years. It turns out that the recurrence relations, studied by Mahler and Paszkowski within the theory of multiple rational approximation, provide integrable extension of the Frobenius identities to arbitrary number of discrete variables. Their quasi-determinant generalizations can be interpreted as new non-commutative integrable discrete systems.

3. **Sorin DRAGOMIR** — *University of Basilicata, Italy*

   Kostant–Souriau–Odzijewicz quantization of a mechanical system whose classical phase space is a complex manifold

   We adopt the Kostant–Souriau–Odzijewicz quantization scheme for quantizing both the quantizable observables and the classical states of a mechanical system whose classical phase space is the Siegel domain $\Omega_n = \left\{\zeta \in \mathbb{C}^n \, : \, \rho(\zeta) > 0\right\}$, $\rho(\zeta) \equiv \mathrm{Im}(\zeta_1) - |\zeta'|^2$. We compute the transition probability amplitude $a_{0\overline{0}}(\zeta,\, z)$ from the state $z \in \Omega_n$ to the state $\zeta \in \Omega_n$. When the system interacts with weak external fields $\epsilon\, B$, $B \in L^\infty(\Omega_n)$, $0 < \epsilon \ll 1$, we show that the corresponding transition probability amplitudes are

$a_{0\overline{0}}(\zeta,\, z) + \mathrm{O}(\epsilon)$. We discuss A. Odzijewicz's assumption that the measure on phase space [associated to the reproducing kernel of $L^2H(\Omega_n\,,\,\gamma_\alpha)$, $\gamma_\alpha = \rho^\alpha$, $\alpha > -1$] should coincide, up to a multiplicative constant, with the Liouville measure. If the classical phase space M is a compact complex manifold without boundary, we show that the quantization map $\mathscr{K} : M \to \mathbb{CP}\big(L^2H\big(\Lambda^{n,0}(M) \otimes E\big)\big)$ is a harmonic map and an absolute minimum in the homotopy class of $\mathscr{K}$ (cf. A. Lichnerowicz, [7], for finite dimensional case). We study the boundary behavior of the quantization map $\mathscr{K} : \overline{\Omega} \to \mathbb{CP}\big(L^2H(\Omega,\,\gamma)\big)$, where $\Omega \subset \mathbb{C}^n$ is a smoothly bounded strictly pseudoconvex domain, and $\gamma \in AW(\Omega)$ an admissible weight.

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4. Vladimir DRAGOVIĆ — *The University of Texas at Dallas, USA*

Bridging Statistics with Geometry and Mechanics

We emphasize the importance of bridges between statistics, mechanics, and geometry. We develop and employ links between pencils of quadrics, moments of inertia, and linear and orthogonal regressions. For a given system of points in $\mathbb{R}^k$ representing a sample of a full rank, we construct a pencil of confocal quadrics which appears to be a useful geometric tool to study the data. Some of the obtained results can be seen as generalizations of classical results of Pearson on orthogonal regression. Applications include statistics of errors-in-variables models (EIV) and restricted regressions, both ordinary and orthogonal ones. For the latter, a new formula for test statistic is derived. The developed methods and results are illustrated in natural statistics examples. The talk is based on a joint work with Borislav Gajić.

5. **László FEHÉR** — *Wigner RCP / University of Szeged, Hungary*

Poisson reductions of master integrable systems on doubles of compact Lie groups

We consider three 'classical doubles' of any semisimple, connected and simply connected compact Lie group G: the cotangent bundle, the Heisenberg double and the internally fused quasi-Poisson double. On each double we identify a pair of 'master integrable systems' and investigate their Poisson reductions. In the simplest cotangent bundle case, the reduction is defined by taking quotient by the cotangent lift of the conjugation action of G on itself, and this naturally generalizes to the other two doubles. In each case, we derive explicit formulas for the reduced Poisson structure and equations of motion, and find that they are associated with well known classical dynamical r-matrices. This yields a unified treatment of a large family of reduced systems, which contains new models as well as well familiar spin Sutherland and Ruijsenaars–Schneider models. It is proved that the reduced systems restricted on a dense open subset of the Poisson quotients are integrable in the degenerate sense. The talk is based on arXiv:2208.03728 complemented by more recent results.

6. **Janusz GRABOWSKI** — *Institute of Mathematics, Polish Academy of Sciences, Poland*

Contact geometry as a chapter in symplectic geometry

I will present an approach to the contact geometry which, in contrast to the one dominating in the physics literature, serves also for non-trivial contact structures. In this approach contact geometry is not an 'odd-dimensional cousin' of symplectic geometry, but rather a part of the latter, namely 'homogeneous symplectic geometry'. This understanding of contact structures has a long tradition and taken seriously is very effective in applications. Instead of ad hoc definitions in the contact case, we have therefore obvious concepts coming directly from the standard symplectic picture. In particular, the contact Hamiltonian Mechanics, extensively studied nowadays, can be fully expressed in terms of the traditional symplectic one, and Legendre submanifolds turn out to be Lagrangian. Also a natural understanding of contact reductions with respect to group actions comes easily from the Marsden–Weinstein–Meyer symplectic reduction.

7. **John HARNAD** — *Université de Montréal / Concordia University, Canada*

Hamiltonian structure of rational isomonodromic deformation systems

The Hamiltonian approach to isomonodromic deformation systems is extended to generic rational covariant derivative operators on the Riemann sphere with irregular singularities of arbitrary Poincaré rank. The space of rational connections with given pole degrees carries a natural Poisson structure corresponding to the standard classical rational R-matrix structure on the dual space $L^*gl(r)$ of the loop algebra $Lgl(r)$. Nonautonomous isomonodromic counterparts of the isospectral systems generated by spectral invariants are obtained by identifying the deformation parameters as Casimir elements on the phase space. These are shown to coincide with the higher Birkhoff invariants determining the local asymptotics near to irregular singular points, together with the pole loci. Pairs consisting of Birkhoff invariants, together with the corresponding dual spectral invariant Hamiltonians, appear as "mirror images", matching, at each pole, the negative power coefficients in the principal part of the Laurent expansion of the fundamental meromorphic differential on the associated spectral curve with the corresponding positive power terms in the analytic part.

Infinitesimal isomonodromic deformations are shown to be generated by the sum of the Hamiltonian vector field and an explicit derivative vector field that is transversal to the symplectic foliation. The Casimir elements serve as coordinates complementing those along the symplectic leaves, extended by the exponents of formal monodromy, defining a local symplectomorphism between them. The explicit derivative vector fields preserve the Poisson structure and define a flat transversal connection, spanning an integrable distribution whose leaves, locally, may be identified as the orbits of a free abelian group action. The projection of the infinitesimal isomonodromic deformation vector fields to the quotient manifold under this action gives the commuting Hamiltonian vector fields corresponding to the spectral invariants dual to the Birkhoff invariants and the pole loci.

8. **Jerzy KIJOWSKI** — *Center for Theoretical Physics, Poland*

Quantization (55 years later)

9. **Gandalf LECHNER** — *FAU Erlangen–Nürnberg, Germany*

Integrable quantum field theories between modular theory and the Yang–Baxter equation

Integrable quantum field theories on two-dimensional Minkowski space can be realized in terms of a solution to the Yang–Baxter equation (two-particle scattering operator) and a localization structure (defined in terms of Tomita–Takesaki modular theory). In this talk I will review this construction programme from a mathematical perspective, outlining existing results and open problems.

10. **Douglas LUNDHOLM** — *Uppsala University, Sweden*

Twisted perspectives on quantum mechanics

Our understanding of the influence of geometry and topology in quantum systems has seen some rapid developments recently, with experimental confirmations of intermediate notions of quantum statistics and "anyons", as well as with the recent Nobel prize in physics, in which the deeper aspects of entanglement and non-locality are beginning to be fully appreciated. In this talk I aim to discuss some bridges between these developments using the general concepts of perspectives, twisting and vorticity. Namely, while "anyons" (intermediate exchange quantum statistics) are a consequence of the twisting of the phase of a wave function under particle exchange, and their corresponding exclusion statistics may be viewed as a resulting vorticity in the probabilities around the diagonals of the configuration space, also contextual systems and non-local entanglement quantum games may be understood from a more general twisting of probability distributions. As has been also emphasized by e.g. Mermin and Mansfield, such twisting may be illustrated geometrically using impossible figures such as in the artistic works of Escher, Penrose and Reutersvärd.

11. **Jean-Pierre MAGNOT** — *LAREMA – University of Angers, France*

On the interplay between gauge theory and decision theory

This talk is centered on a rigorous interplay between topics in decision theory and Yang–Mills theory from the viewpoint of quantum gravity. Addressed to specialists on physics and mathematics, I recall basics on pairwise comparisons, inconistency indicators and priority vectors and develop an almost fully symmetric picture between pairwise comparisons and an Abelian Yang–Mills theory in the spirit of quantum gravity. The latest results in this direction are discussed, and open directions, such as pairwise comparisons in a non abelian group, considerations on the meaning of a quantization procedure for pairwise comparisons, as well as questions on the meaning in physics of the transposition of methods from information theory are ending the talk.

12. **John SIPE** — *University of Toronto, Canada*

    Microscopic polarization and magnetization fields: Towards a “post-modern” theory

    The response of solids to incident electromagnetic fields is often heuristically described in terms of macroscopic polarization and magnetization fields. In condensed matter physics, the “modern theory of polarization”, and its extension to magnetization, gives this a new level of rigour for time independent and uniform applied fields. We review the philosophy and main results of that strategy, and report on a new approach based on introducing microscopic polarization and magnetization fields. This “post-modern” approach can be used to address the response of crystals to electromagnetic fields varying arbitrarily in space and time, and connects that response to aspects of the underlying topology of the band structure. We compare it to earlier work on atoms and molecules, identifying important similarities and differences.

13. **Jacek SZMIGIELSKI** — *University of Saskatchewan, Canada*

    The Camassa–Holm equation—trente ans après: on the interplay between Approximation Theory, Inverse Problems, and non-smooth Solitons

    In Memoriam Anatol Odzijewicz

    It has been 30 years since the derivation of the shallow water equation with peaked solitons by R. Camassa and D. Holm. The Camassa–Holm equation has become one of the most studied non-linear equations in recent years. This talk reviews the interplay between the mathematics of peakons (non-smooth solitons) and Approximation Theory. I will survey some decisive developments shaping my understanding of peakons and my motivation to study peakon-bearing equations. I will highlight the role of the Padé and Hermite-Padé approximations in the solution of inverse problems intrinsically related to these non-linear wave equations. This talk is partly based on my recent joint work on the beam problem with Richard Beals and peakons with Hans Lundmark and Xiang-ke Chang.

14. **Leandro VENDRAMIN** — *Vrije Universiteit Brussel, Belgium*

    Groups, rings and the Yang–Baxter equation

    Skew braces are ring-theoretical algebraic structures to study non-degenerate set-theoretic solutions of the Yang–Baxter equation. A typical example of a skew brace is a Jacobson radical ring. In this talk, we will discuss the basic properties of skew braces and how these structures are related to a discrete version of the celebrated Yang–Baxter equation. We will also discuss connections with groups and non-commutative rings.

Contributed lectures

1. **Krzysztof BARDADYN** — *University of Białystok, Poland*

    Groupoid Banach algebras

2. **Daniel BELTIŢĂ** — *Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Romania*

   Groupoid techniques in Hilbert space operator theory

   We study the action of the groupoid of partial isometries on the set of normal operators with respect to the moment map given by the range projection.The corresponding groupoid orbits turn out to have Banach manifold structures for which the target map is a smooth submersion. We also describe the norm closure of the groupoid orbit of any normal operator and we investigate the way its topological or differentiable structures are encoded by the spectral properties of the operator under consideration. The presentation is based on joint work with Gabriel Larotonda.

3. **Petr BLASCHKE** — *Silesian University in Opava, Czech Republic*

   Spherical pedal coordinates

   It has been shown that "pedal coordinates" are well suited for describing solutions of force problems in the plane. In particular, central force problems can be solved algebraically in pedal coordinates. Their 3-D analogue seems to be not unique, however. A generalization of pedal coordinates for space curves restricted to the surface of the unit sphere will be presented. We will showcase their usefulness on a number of examples, including: Spherical Kepler problem, movement of a non-planar pendulum, Spherical analogues of Brachistochrone, Catenary and more.

4. **Tomasz BRZEZIŃSKI** — *Swansea University / University of Białystok, UK / Poland*

   Lie affgebras revisited

5. **Goce CHADZITASKOS** — *Czech Technical University in Prague, Czech Republic*

   Coherent states of the asymmetric harmonic oscillator

   We have constructed formal coherent states for an asymmetric harmonic oscillator, where the asymmetry parameter is the ratio of spring constants. The contribution deals with the study of their properties. These states generally do not satisfy all the required properties for coherent states. During the time development, coherent states introduced in this way become decoherent. For specific asymmetry parameters, coherent states can be constructed on the subspace of states that preserve coherence during time evolution.

6. **Johan Michel CHAVEZ TOVAR** — *CINVESTAV, Mexico*

   Higher order curvature terms corrections in the Raychaudhuri equation

   In general relativity, gravity is attractive. This feature is present in the Raychaudhuri equation provided the strong or weak energy conditions are fulfilled. In this scenario the expansion of a congruence of geodesics is consistently decreasing. As it is well known Raychaudhuri equation is an important component of singularity theorem which shows that spacetime is geodesically incomplete implying that general relativity is itself incomplete. It is possible that modified versions of general relativity in particular the inclusion of higher order curvature terms can resolve or attenuate this kind of singularities. It is important then to study the behavior of geodesic congruences in these theories.

In this work, we present a brief review of the Raychaudhuri equation, we begin with a summary of the essential features of this equation, and we move on to a discussion of the equation in the context of the alternative gravitation theory known as Critical Gravity. Finally, we discuss some effects of Critical Gravity theory in the Raychaudhuri equation and the geodesic congruences in an AdS type black hole solution.

7. **Alina DOBROGOWSKA** — *University of Białystok, Poland*

Second order q-difference equations solvable by factorization method

In memory of Prof. Anatol Odzijewicz

By solving an infinite nonlinear system of q-difference equations one constructs a chain of q-difference operators. The eigenproblems for the chain are solved and some applications, including the one related to q-Hahn orthogonal polynomials, are discussed. It is shown that in the limit $q \to 1$ the present method corresponds to the one developed by Infeld and Hull.

8. **Ziemowit DOMAŃSKI** — *Poznań University of Technology, Poland*

Quantization in arbitrary coordinate system and transformations of coordinates in quantum mechanics

The quantization is the process of constructing a quantum system from a classical one. The basic ingredient in the usual quantization procedure is a way to construct operators on a certain Hilbert space from functions defined on the phase space of the system. A standard way to do this is to write a function in Cartesian coordinates and replace position and momentum variables with appropriate operators. Since the position and momentum operators do not commute it is also necessary to appropriately order them. The problem arises when one would like to repeat this procedure in different coordinates, because the received operators corresponding to the same phase space function will not be unitarily equivalent. The usual way of resolving this inconsistency is to quantize only in Cartesian coordinates or in a coordinate independent way. However, it should be possible to quantize in any coordinate system in a consistent way.

We resolve this problem by adding to the phase space function $\hbar$-dependent correction terms and then constructing an operator from such modified function in a usual way. We find a systematic way of constructing such corrections by employing the phase space formalism of quantum mechanics. This formalism also allows for introduction of transformation of coordinates in a natural way. In particular we define quantum canonical transformations of coordinates.

9. **Jesús FUENTES** — *University of Luxembourg, Luxembourg*

Unraveling Soft Squeezing Transformations

Quantum squeezing, a captivating phenomenon that increases one variable's uncertainty while decreasing its conjugate's, can be examined as a time-dependent process, with precise solutions often found using adiabatic invariants-based frameworks. Impressively, Mielnik demonstrated that exact solutions can be identified with time-varying elastic forces, bypassing reliance on invariant formalism. Analyzing these solutions as an inverse problem reveals their direct association with designing elastic fields that generate squeezing transformations on canonical variables. The dynamic transformations explored belong to a gentle quantum operations class, characterized by refined particle manipulation, which avoids the sudden energy spikes typically found in conventional control protocols.

10. **Gerald GOLDIN** — *Rutgers University, USA*

Predicting Nonabelian Anyons

Nonabelian anyons are conjectured particles or excitations in two-dimensional space, whose "exchange statistics" satisfy unitary representations of the braid group of dimension greater than one. Their first prediction came in 1985 from Menikoff, Sharp, and myself, in published research whose citation researchers in the field still systematically omit. Recently nonabelian anyons have attracted considerable attention in the field of quantum computing, and experimental results have ensued. In this talk, I will outline how their prediction followed from classifying the unitary representations of a certain local symmetry group: the natural semi-direct product of a group of scalar functions with a group of diffeomorphisms of physical space. For fundamental reasons, this group serves as a universal kinematical group for quantum mechanics. Our development made clear how the topology of configuration space enters inevitably into quantum mechanics, resolving some earlier controversies about "multivalued wave functions". If time permits, I will compare and contrast this approach with conventional Schrödinger and Heisenberg quantization schemes, and mention some of my ongoing research directions.

11. **Piotr P. GOLDSTEIN** — *National Centre for Nuclear Research, Poland*

A quadric of kinetic energy in the role of phase diagrams — application to the BKL scenario

The phase diagrams turn into intractable objects if a Hamiltonian system is more than one dimensional. However, it is convenient to describe the evolution of the bound states, whose potential part $E_p < 0$, by the sole kinetic part, especially if it is quadratic in the momenta. In the space of momenta the evolution takes place inside or outside of a quadric $E_k - H < 0$ for each value of H. Moreover, it provides complete information on the dynamics of the system like phase diagrams do in 1 dimension.

We apply this tool to the Belinski–Khalatnikov–Lifshitz (BKL) scenario, which describes behaviour of an anisotropic Universe near the cosmic singularity. It consists of 3 equations and a constraint (which arise as asymptotics of the Einstein equations in the synchronous frame of reference)

$$\frac{d^2 \ln a}{dt^2} = \frac{b}{a} - a^2, \qquad \frac{d^2 \ln b}{dt^2} = a^2 - \frac{b}{a} + \frac{c}{b}, \qquad \frac{d^2 \ln c}{dt^2} = a^2 - \frac{c}{b},$$

subject to the constraint

$$\frac{d \ln a}{dt}\,\frac{d \ln b}{dt} + \frac{d \ln a}{dt}\,\frac{d \ln c}{dt} + \frac{d \ln b}{dt}\,\frac{d \ln c}{dt} = a^2 + \frac{b}{a} + \frac{c}{b}\,,$$

where t is the time parameter while $a = a(t)$, $b = b(t)$ and $c = c(t)$ are the directional scale factors, whose evolution defines the dynamics of the characteristic lengths in three principal directions. The limit $t \to \infty$ corresponds to the cosmological singularity.

A transformation of variables leads to a Hamiltonian model with the constraint $H = 0$, while $E_p < 0$. The boundary of the quadric, $E_k = 0$, which is a cone, corresponds to the limit $t \to \infty$. A comprehensive description of the asymptotics for large t is provided by this method. For systems like BKL, it seems to be more natural and simpler than the diagrams of Misner, Thorne and Wheeler [2].

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12. Tomasz GOLIŃSKI — *University of Białystok, Poland*

Banach Lie groupoids, algebroids, von Neumann algebras and restricted Grassmannian

13. Daniel HODGSON — *University of Leeds, UK*

Resolving the Abraham–Minkowski controversy with symmetry arguments

In the presence of dielectric media, the speed of light, as well as the direction of propagation of incoming wave packets, can change. The Abraham–Minkowski controversy examines the question of which form of the electromagnetic momentum is conserved when transitioning between media of different refractive indices. By adopting a recent quantisation of the EM field in the position representation, in this talk I shall derive a locally-acting mirror Hamiltonian for the reflection and transmission of light near the boundary between two different media. This shall be achieved by applying symmetry arguments, as well as by ensuring that photon number is conserved. As the expectation values of all observables commuting with this Hamiltonian must be conserved, different expressions for momentum can be more fully understood in the context of local unitary interactions.

14. Pavel HOLBA — *Silesian University in Opava, Czech Republic*

Complete classification of local conservation laws for generalized Cahn–Hilliard–Kuramoto–Sivashinsky equation

In this talk we consider nonlinear multidimensional Cahn–Hilliard and Kuramoto–Sivashinsky equations that have many important applications in physics, chemistry, and biology, and a certain natural generalization of these equations.

Namely, for an arbitrary number n of spatial independent variables we present a complete list of cases when the following PDE in $n+1$ independent variables $t, x_1 \ldots, x_n$ and one dependent variable u

$$u_t = a\Delta^2 u + b(u)\Delta u + f(u)|\nabla u|^2 + g(u),$$

to which we refer as to the generalized Cahn–Hilliard–Kuramoto–Sivashinsky equation, admits nontrivial local conservation laws of any order, and for each of those cases we give an explicit form of all the local conservation laws of all orders modulo trivial ones admitted by the equation under study. Here b, f, g are arbitrary smooth functions of the dependent variable u, a is a nonzero constant, $\Delta = \sum_{i=1}^n \partial^2/\partial x_i^2$ is the Laplace operator and $|\nabla u|^2 = \sum_{i=1}^n (\partial u/\partial x_i)^2$.

In particular, we show that the original Kuramoto–Sivashinsky equation admit no nontrivial local conservation laws and list all of the nontrivial local conservation laws for original Cahn–Hilliard equation.

For more details see P. Holba, Complete classification of local conservation laws for generalized Cahn–Hilliard–Kuramoto–Sivashinsky equation. *Stud Appl Math.*, to appear, `https://doi.org/10.1111/sapm.12576`.

15. **Mahouton Norbert HOUNKONNOU** — *University of Abomey-Calavi, Benin*

Discrete mechanics in non-uniform time α-lattices

In this talk, we consider special types of non-uniform time lattices by introducing the α-addition and α-subtraction from the point of view of pseudo-analysis, a generalization of the classical analysis, where instead of the field of real numbers, a semiring is taken on a real interval $[a, b] \subset [-\infty, +\infty]$. In this framework, we deduce related algebraic operations and discuss their properties. We define the discrete conformable fractional derivative and integral by introducing the fractional falling factorial. We derive the Langevin equation and describe the two-dimensional discrete mechanics. Besides, we introduce the polar coordinate analysis to describe some relevant physical phenomena.

16. **Yasushi IKEDA** — *Sapporo, Japan*

Quasiderivations and Quantum Mishchenko–Fomenko Construction

In this talk I speak about the quantum analogue of the theorem of A. Mishchenko and A. Fomenko, which allows one to construct commutative subalgebras in the universal enveloping algebras. We replace the derivation of the symmetric algebra $Sgl(d, \mathbb{C})$ (used in the original theorem) with the quasiderivation of the universal enveloping algebra $Ugl(d, \mathbb{C})$, proposed by Gurevich, Pyatov, and Saponov. In my paper I derived an explicit formula for such quasiderivations and used it to prove the quantum analogue of the theorem at the first order. In this talk I proceed with the calculations for the second order: we show that complex combinatorial formulas play an important role here. We believe that understanding of the representation theory underlying these formulas can lead to a general proof of the quantum analogue.

17. **Giorgi KHIMSHIASHVILI** — *Ilia State University, Georgia*

Maxwell's conjecture for three aligned point charges

We will present several results concerned with the J.C. Maxwell's conjecture on the number of equilibrium points of a system of point charges. Specifically, we will show that this conjecture holds true for three aligned point charges of arbitrary signs and magnitudes. To this end, we use an analytic approach developed in our two joint papers with G. Giorgadze published in J. Math. Phys. (vol. 62, no. 5, 2021) and in Georgian Math. J. (vol. 29, no. 4, 2022). More concretely, for a triple of non-aligned points, we compute the so-called canonical stationary charges and describe their behavior under the deformations of the reference triangle, which is the crucial point in the proof. We also establish functorial properties of the nonlinear system of coordinates in the ambient plane given by the canonical charges and present some geometric applications of such coordinates.

18. **Bartosz KWAŚNIEWSKI** — *University of Białystok, Poland*

Noncommutative Cartan C^*-subalgebras

Celebrated Renault's result states that commutative Cartan C^*-subalgebras, i.e. regular maximal abelian C^*-subalgebras $A \subseteq B$ with faithful conditional expectation, are in bijective corresponence with topologically free twisted groupoids. Recently, Exel generalized the notion of a Cartan inclusion $A \subseteq B$ to the case where A is an arbitrary (noncommutative) C^*-algebra, and proved a part of a Renault's theorem. In this talk I present a number of characterisations of noncommutative Cartan C^*-inclusions that fully generalize Renault's Theorem, and give tools to study properties of the ambient algebra. (Based on a number of joint works with Ralf Meyer.)

19. **Dušan NAVRÁTIL** — *Brno University of Technology, Czech Republic*

Lie Symmetry Analysis of the Charney–Hasegawa–Mima equation

The Charney-Hasegawa-Mima equation (CHM) is a partial differential equation that governs the behavior of large-scale waves in rotating fluids, such as Earth's atmosphere and oceans. Specifically, we will consider the CHM equation in the β-plane model, which takes the form:

$$\frac{\partial}{\partial t}(\Delta u - Fu) + \beta \frac{\partial u}{\partial x} + [u, \Delta u] = 0,$$

where t is the temporal coordinate, x and y are spatial coordinates, u is the stream function, and β/F are constants that depend on the specific situation.

We will focus on the derivation of Lie-point symmetries of the CHM equation and their geometrical interpretation. Lie-point symmetries are important in the study of differential equations, as they provide insight into the underlying structure of the equations and allow us to find exact solutions. By understanding the symmetries of the CHM equation, we can gain a deeper understanding of the behavior of large-scale waves in rotating fluids.

20. **Filip PETRÁK** — *VUT Brno (University of Technology Brno), Czech Republic*

Higher-order and Weil Grassmannian as orbit spaces

We define a foliation on $regJ_0^r(\mathbb{R}^k, \mathbb{R}^m)$ with G_m^r-orbits as its leaves and the left action given by the jet composition. We construct an atlas on $Gr(r, k, m)$ from the finite system of local sections of $p\#$ identified with local maps with supports dense in $V = Gr(r, k, m)$. In the present work, we investigate the case of $m < k$. The symbol reg indicates submersions, the jet group acts from the left and in the most general situation corresponding to a Weil functor T^A we do not work with the group $AutA$ but still with G_m^r.

21. **Anatolij PRYKARPATSKI** — *Cracow University of Technology / Lviv Polytechnic National University, Poland / Ukraine*

Affine Courant algebroid, its coadjoint orbits and related integrable systems

The Poisson structures related with the affine Courant algebroid are analyzed. The coadjoint action orbits are studied, infinite hierarchies of the Casimir functionals are described. A wide class of integrable flows on functional manifolds is considered.

The Lie algebroid as a mathematical object is an unrecognized part of the folklore of differential geometry. They have been introduced repeatedly [3, 5] into differential geometry since the early 1950's, and also into physics and algebra, under a wide variety of names, chiefly as infinitesimal invariants associated to geometric structures: in connection theory, as a means of treating de Rham cohomology by algebraic methods, as invariants of foliations and pseudogroups of various types, in symplectic and Poisson geometry and, in a more algebraic setting, as algebras of differential operators associated with vector bundles and with infinitesimal actions of Lie groups. These and related differential-geometric structures [1, 2, 4] make it possible to construct new types of geometric objects on fibered spaces, study their algebro-analytical properties and apply to many problems of mechanics, physics and other natural sciences.

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22. Maciej PRZANOWSKI — *Łódź University of Technology, Poland*

Quantum mechanics of a single linear graviton

Wave function of a single linear graviton is proposed. We find the evolution equation of this function. A simple interpretation of the linear graviton wave function in momentum representation is given. Finally, we find a Hermitian operator consisting of three mutually commuting components which fulfill the canonical commutation relations with the momentum operator of the linear graviton. That operator is a close counterpart of the Hawton position operator of the photon.

23. Tudor RATIU — *Shanghai Jiao Tong University, China*

Clebsch optimal control and its link to variational principles in mechanics

I will introduce a class of control problems that possess symmetry and will show that their solution is obtained via an Euler–Poincaré equation. Examples will be provided.

24. Stefan RAUCH — *Linköping University, Sweden*

Driven cofactor systems of Newton equations and separability of time dependent potentials

By Newton equation we mean equation of the form $d^2q_r/dt^2 = F_r(q_1, \dots q_m; q_{m+1} \dots q_n)$, $r = 1, \dots, m+n$, that is **acceleration = force** not depending on time or velocities. It is driven when it has the form $d^2y/dt^2 = N(y)$, $d^2x/dt^2 = M(y, x)$ with $(x, y) = (q_1, \dots q_m; q_{m+1} \dots q_n)$ in which a solution $y(t)$ of the first y-subsystem "drives" the force of the second x-subsystem, called "driven" equation.

Theory of such equations becomes interesting when there are known one or two energy-like integrals of motion (**which are quadratic in velocities**) of **cofactor** type. If Newton equation admits two such quadratic cofactor integrals then this Newton equation is completely integrable and is separable in a non-standard sense. If the whole Newton system has **only one quadratic cofactor integral** then it has further $n+1$ quadratic integrals, due to special structure of equations. In particular the driving subsystem $d^2y/dt^2 = N(y)$ inherits a quadratic cofactor integral of motion depending only on y-variables.

When a solution $y(t)$ of the driving subsystem is known and the driven subsystem force has a potential, then for $x(t)$ one obtains a system generated by time dependent potential $d^2x/dt^2 = M(y, x) = -grad_x V(y(t), x)$. It appears that such system has separable time-dependent Hamilton–Jacobi equation and is solvable by natural extension of the Stäckel separability procedure. It gives rise to time-dependent separation variables. This procedure is illustrated by a simple example.

25. **Bernard RYBOŁOWICZ** — *Heriot–Watt University, UK*

From skew braces to the set-theoretic Yang–Baxter equation from affine viewpoint

In 2007 W. Rump discovered an interesting connection between algebraic structures, which he named braces, and the set-theoretic Yang–Baxter equation. During the talk, I will introduce the audience to skew braces, the generalisation of braces, and discuss their properties and links with the set-theoretic Yang–Baxter equation. Next, I will explain how by looking at them from a somehow affine point of view, we can notice new solutions. Then, I will reverse the process and start with a particular family of solutions of the set-theoretic equation and a group. I will construct a structure which resembles a skew brace, which will be called a near brace.

26. **Alice Barbara TUMPACH** — *Wolfgang Pauli Institut / Lille University, Austria*

The Universal Teichmüller space and the restricted Grassmannian

We will describe the universal Teichmüller space and its connexion to some infinite-dimensional hermitian-symmetric spaces like the restricted Grassmannian and infinite-dimensional Siegel domains. Some new and old results on the geometry of these spaces will be presented.

27. **Iryna YEHORCHENKO** — *Institute of Mathematics of the NAS of Ukraine, Ukraine*

Using contact transformations for construction of exact solutions of the Schrödinger equation with power, logarithm and derivative nonlinearities

We will use a reduction procedure to the Schrödinger equation with power, logarithm and derivative nonlinearities. The resulting reduction conditions (overdetermined systems of first- and second-order PDEs) may be solved by means of contact and hodograph transformations. Solutions produce ansatzes leading to exact solutions of the original equations. In some cases we can obtain only solutions that may be obtained also by Lie symmetry algorithm, but there are special cases when it is possible to obtain new families of exact solutions.

28. **Slimane ZAIEM** — *University of Batna 1, Algeria*

Evaporation of black hole in the non-commutative gauge theory of gravity

In this work, we used the non-commutative (NC) gauge theory of gravity to analysis the thermodynamic properties of a deformed Schwarzschild black hole (SBH). As a first step, the Hawking temperature with all order in theta, and the entropy of NC SBH. The non-commutativity removes the divergence behavior of temperature and these corrections reveal an estimation of the NC parameter theta which is founded at the Planck scale order. Then, the description of the heat capacity of the deformed black hole shows the effect of the NC geometry on the thermodynamic stability and the phase transitions. Finally, we investigate the influence of the non-commutativity on the Luminosity of SBH, where this geometry removes the divergence behavior of SBH luminosity. In this study, we found that the NC geometry show the same role of the quantization of the black body radiation in the classical theory.

Virtual contributed lectures

1. **Michael FRANCIS** — *University of Western Ontario, Canada*

   On automorphisms of complex b^k-manifolds

   Foundations of symplectic b^k-geometry were laid by Guillemin et al and has its roots in the b-calculus of Melrose. In a similar spirit, complex b-calculus was worked out by Mendoza. In this talk, we discuss the local and global automorphisms of complex b^k upper half planes. We consider analogues of weighted Bergman spaces and associated Toeplitz operators as well as coherent states. This is joint work with Tatyana Barron.

2. **Dimitry GUREVICH** — *Université Polytechnique Hauts-de-France, France*

   Reflection Equation Algebra and related combinatorics

   Reflection Equation Algebras constitute a subclass of the so-called Quantum Matrix algebras. Each of the REA is associated with a quantum R-matrix. In a sense the REA corresponding to quantum R-matrix of Hecke type can be considered as q-counterparts of the commutative algebra $Sym(gl(N))$ or the enveloping algebras $U(gl(N))$. On any such RE algebra there exist analogs of some symmetric polynomials, namely the power sums and the Schur functions. In my talk I plan to exhibit q-versions of the Capelli formula, the Frobenius formula, related to this combinatorics. Also I plan to introduce analogs of the Casimir operators and partially perform their spectral analysis.

3. **Igor KANATCHIKOV** — *National Center of Quantum Information, Poland*

   The cosmological constant and the minimal acceleration of MOND from precanonical quantum gravity

   We outline the approach of precanonical quantization (which is based on a space-time symmetric generalization of the Hamiltonian formalism), explain its relation to the functional Schrödinger representation, and apply it to quantization of tetrad general relativity and pure gauge theory. The simplest solutions of the precanonical Schrödinger equation for quantum gravity, which correspond to a quantum (wave) analogue of the Minkowski space-time, naturally lead to the acceleration threshold $a_0 = 8\pi G\hbar\varkappa$ which is related to the range of the Yukawa modes propagating in the spaces of spin connection coefficients. The cosmological constant $\Lambda \sim (8\pi G\hbar\varkappa)^2$ emerges from the re-ordering of operators in the precanonical Schrödinger equation consistently with the scalar product with an operator-valued Misner-like measure. The observable values of Λ and a_0 of the Milgromian MOND correspond to the subnuclear scale of the parameter $\varkappa$, which is introduced by the precanonical quantization. This is consistent with its appearance in the estimation of the gap in the mass spectrum of quantum SU(2) Yang–Mills theory $\Delta m \sim (g^2\hbar^4\varkappa)^{1/3}$, which we have obtained earlier. Thus, both the cosmological constant (as the simplest form of dark energy) and the minimal acceleration of MOND (which is an alternative description of the dynamics of galaxies and their clusters without dark matter), and the mysterious Milgrom's relation between them: $a_0 \approx \sqrt{\Lambda}$, appear as elementary consequences of precanonical quantum gravity and the "spin connection foam" picture of quantum geometry of space-time it leads to.

4. **Fatemeh NIKZAD PASIKHANI** — *Institute for Advanced Studies in Basic Sciences, Iran*

   Stability Theorem for $\mathbb{Z}_2^n$-Lie supergroups

   We show that every pre-representation of a $\mathbb{Z}_2^n$-Lie supergroup has a unique extension to a unitary representation. By a $\mathbb{Z}_2^n$-Lie supergroup we mean a Harish–Chandra pair

$(G_0, \mathfrak{g}_{\mathbb{C}})$ where G_0 is a common Lie Group and $\mathfrak{g}_{\mathbb{C}}$ is a $\mathbb{Z}_2^n$-graded Lie superalgebra such that there exists an action $Ad : G_0 \times \mathfrak{g}_{\mathbb{C}} \to \mathfrak{g}_{\mathbb{C}}$ preserves the $\mathbb{Z}_2^n$-grading and $\mathrm{Ad}|_{\mathfrak{g}_o} : G_0 \times \mathfrak{g}_o \to \mathfrak{g}_o$ is the adjoint action of G_0 on $\mathfrak{g}_o \cong Lie(G_0)$.

5. **Fernand PELLETIER** — *Université Savoie Mont Blanc, France*

On partial Dirac structure on a Banach manifolds and constraint dynamical systems

In finite dimension, the idea of interconnections plays an essential role in modeling physical systems interacting with various energy fields, such as electro-mechanical systems, bio-chemical reaction systems. For example, the interconnection of L-C circuits was shown to be represented by interconnexion of Dirac structures by Van der Schaft and Maschke (1995) and Bloch and Crouch (1997). Thus Dirac structure became the basic tool for modeling such complex systems not only in finite dimension but also in Hilbert and reflexive Banach setting. In the same way, Dirac structure gives rise to application in Banach sub-Riemannian geometry. This will be the purpose of this conference.

6. **Artur SERGYEYEV** — *Silesian University in Opava, Czech Republic*

Multidimensional integrable systems and Jacobi structures

In this talk, motivated by our earlier contact-geometric construction (AS, New integrable $(3+1)$-dimensional systems and contact geometry, Lett. Math. Phys. 108 (2018), no. 2, 359-376) for a large new class of nonlinear partial differential systems in four independent variables integrable in the sense of soliton theory, we explore the possibility of generalizing this construction by using Jacobi structure instead of the contact one.

Poster presentations

1. **Alfonso Salomón ACEVEDO JUÁREZ** — *CINVESTAV, Mexico*

How birefringence arise from nonlinear electrodynamics

We analyze the propagation of light signals in the context of nonlinear electrodynamics. As a general feature of the non linear theories the superposition principle is no longer satisfied, on electromagnetic theory this is because light propagation is influenced by the electromagnetic background. We present a simple derivation of the two light cones that arise if the Lagrangian depends nonlinearly on the electromagnetic invariants. The analysis is based on the algebraic properties of the electromagnetic tensor $f_{\mu\nu}$. We show that the problem can be treated as a Sturm–Liouville problem where the eigenvalues are related to the principal null directions. It turns out that, in the presence of the background field, the propagation can be slower or faster than that of light.

2. **Tomasz CZYŻYCKI** — *University of Białystok, Poland*

Evolution difference equation with random coefficients

In the theory and applications of stochastic difference equations an important role play difference equations with random coefficients, which describe a broad class of real phenomena in physics, biology, financial mathematics and many others. We study moment equations for evolution difference equation (EDE) of the first order with random coefficients dependent on Markov chain to analyse asymptotic be- haviour and stability of its solutions. We derive moment equations of the first and second order and solve them analytically to compare the trajectories for solutions of EDE and solution of moment equations.

This is joint work with prof. I. Dzhalladova.

3. **Jiří HRIVNÁK** — *Czech Technical University in Prague, Czech Republic*

   Discrete E-transforms of $A_1 \times A_1$

   The reducible crystallographic root system $A_1 \times A_1$, together with the even subgroup of the associated Weyl group, determines two-variable even Weyl orbit functions. These E-functions form the kernels of the developed discrete Fourier–Weyl E-transforms with the finite point and label sets realized by rectangular fragments of the admissibly shifted weight lattices. General form of the sixteen types of the point and label sets along with related discrete orthogonality relations of the E-functions are presented. The forward and backward transforms as well as the linked interpolation formulas and orthogonal transform matrices are exemplified. This is a joint work with Goce Chadzitaskos and Jan Thiele.

4. **Petr NOVOTNÝ** — *Czech Technical University in Prague, Czech Republic*

   Quantum Particle on G_2 Dual Weight Lattice in Even Weyl Alcove

   Model of a free non-relativistic quantum particle propagating on the dual weight lattice inside the scaled fundamental domain is described and solved using Weyl orbit functions. Example concerning G_2 root system is presented.

5. **Akira YOSHIOKA** — *Tokyo University of Science, Japan*

   Star Product and Several Star Functions

   One parameter deformation of function is given by means of convergent star product, where the deformation parameter varies in some domain of $\mathbb{C}$. The deformed functions are called *star functions*. In this presentation, we discuss, as a simple case, a star product of functions of one variable, and then the product becomes commutative and associative. Several concrete examples, e.g., star Gamma function, are given together with their basic identities.

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