KOSTANT-SOURIAU-ODZIJEWICZ QUANTIZATION OF A MECHANICAL SYSTEM WHOSE CLASSICAL PHASE SPACE IS A COMPLEX MANIFOLD

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Abstract. We adopt the Kostant-Souriau-Odzijewicz quantization scheme for quantizing both the quantizable observables and the classical states of a mechanical system whose classical phase space is the Siegel domain $\Omega_n = \{\zeta \in \mathbb{C}^n : \rho(\zeta) > 0\}$, $\rho(\zeta) \equiv \mathrm{Im}(\zeta_1) - |\zeta'|^2$. We compute the transition probability amplitude $a_{0\overline{0}}(\zeta, z)$ from the state $z \in \Omega_n$ to the state $\zeta \in \Omega_n$. When the system interacts with weak external fields ϵB, $B \in L^\infty(\Omega_n)$, $0 < \epsilon << 1$, we show that the corresponding transition probability amplitudes are $a_{0\overline{0}}(\zeta, z) + \mathrm{O}(\epsilon)$. We discuss A. Odzijewicz's assumption that the measure on phase space [associated to the reproducing kernel of $L^2H(\Omega_n, \gamma_\alpha)$, $\gamma_\alpha = \rho^\alpha$, $\alpha > -1$] should coincide, up to a multiplicative constant, with the Liouville measure. If the classical phase space M is a compact complex manifold without boundary, we show that the quantization map $\mathscr{K} : M \to \mathbb{CP}\big(L^2H\big(\Lambda^{n,0}(M) \otimes E\big)\big)$ is a harmonic map and an absolute minimum in the homotopy class of $\mathscr{K}$ (cf. A. Lichnerowicz, [Lic], for finite dimensional case). We study the boundary behavior of the quantization map $\mathscr{K} : \overline{\Omega} \to \mathbb{CP}\big(L^2H(\Omega, \gamma)\big)$, where $\Omega \subset \mathbb{C}^n$ is a smoothly bounded strictly pseudoconvex domain, and $\gamma \in AW(\Omega)$ an admissible weight.

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Talk delivered at XL Workshop on Geometric Methods in Physics (`https://wgmp.uwb.edu.pl/`), Białowieza (July 2 – July 8, 2023).

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