Unifying Classical & Quantum Physics + Quantum Fields & Gravity

· *Unusual classical & quantum physics*

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· *Coherent states & two* **Bridges**

· *A marvelous* **unification**

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· *Affine quantization for fields*

· *Affine quantization for gravity*

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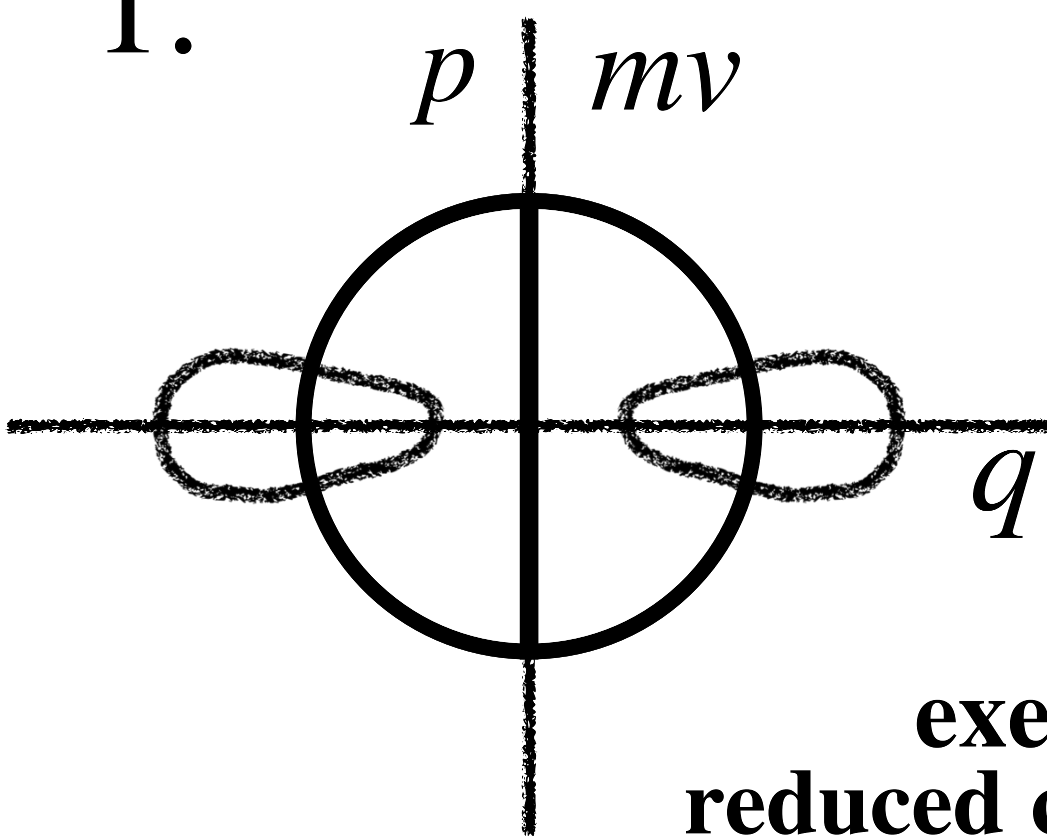
Unusual Classical Physics

1. $H = (p^2 + q^2)/2 + g/q^2\ ,\quad g \to 0\ ,\quad q \neq 0\ ,\ q > 0 \longleftarrow$

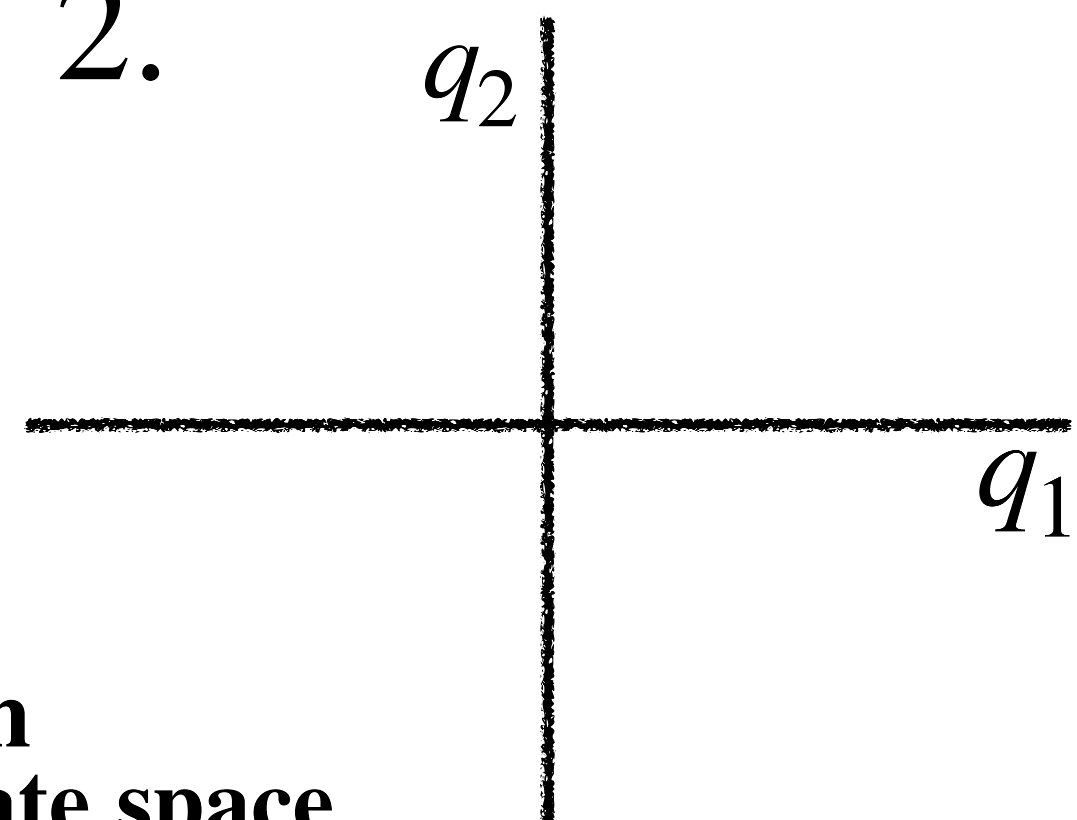
2. $H = (\overrightarrow{p}^2 + \overrightarrow{q}^2)/2 + g/(\overrightarrow{q}^2 - d^2)^2 + g'/(\overrightarrow{q}^2 - c^2)^2,\ \ g, g' \to 0$

$d > c\ ,\quad (\overrightarrow{q}^2 - d^2) < 0\ ,\quad (\overrightarrow{q}^2 - c^2) < 0$

1.



2.



exercises in reduced coordinate space

CANONICAL QUANTIZATION

tool ↑

Coherent states $-\infty < p, q < \infty$, $\{q, p\} = 1$

$p \to P$ & $q \to Q$, $[Q, P] = i\hbar 1$

$|p, q\rangle = e^{-iqP/\hbar}\, e^{ipQ/\hbar}\, |\omega\rangle$

🙂 **Dirac :** **Having** $H(p, q) = \mathscr{H}(p, q)$ **requires Cartesian coordinates in order to obtain physically correct quantizations** .

$$\langle\omega|\,[aQ + bP + c\,(Q^2 + P^2)]\,|\omega\rangle = c\ \mathscr{O}(\hbar)$$

$$H(p, q) = \langle p, q\,|\,\mathscr{H}(P, Q)\,|\,p, q\rangle = \mathscr{H}(p, q) + \mathscr{O}(\hbar; p, q)$$ ← **1st key**

$$d\sigma^2 = 2\hbar\,[\,\|\,d\,|p, q\rangle\,\|^2 - |\langle p, q\,|\,d\,|\,p, q\rangle|^2]\ ,\quad \mathscr{F} - \mathscr{S}$$

🙂 $$d\sigma^2 = \omega^{-1}\,dp^2 + \omega\,dq^2$$ **(created, not found!)**

CQ → flat surface = constant zero curvature := 0

A SIMPLE TRUTH

(*variables & functions*)

$$AB = C \quad \leftrightarrow \quad A = C/B$$

If two are known, the third should be known

$$IF\ B\ \&\ C = 0\ ,\ \ WHAT\ IS\ A?$$

$$IF\ B\ \&\ C = \infty\ ,\ \ WHAT\ IS\ A?$$

🙂 B & C **should NOT block** A $\begin{cases} math! \\ phys? \end{cases}$

$mv = p \ \ (m \neq 0)\ , \quad \varphi(x)\,\pi(x) = \kappa(x) \ \ (\varphi(x) \neq 0)$ $\longleftarrow$

We will draw attention to ABC – terms

AFFINE QUANTIZATION $_{tool}$

Coherent states $\quad d = pq \xrightarrow{ABC} q \neq 0\,,\ \text{accept}\ \ q > 0$

$d \rightarrow D = (P^{\dagger}Q + QP)/2\ \ \textit{and}\ \ q \rightarrow Q > 0 \quad \longleftarrow$ '*dimensionless*'

$[Q, D] = i\hbar\, Q \quad , \quad \{q, d\} = q$

$$|p;q\rangle = e^{ipQ/\hbar}\, e^{-i\ln(q)D/\hbar}\,|\beta\rangle$$

$$\langle\beta|[aQ + bD + c\,((Q - 1\!\!1)^2 + D^2)]|\beta\rangle = a + c\ \mathcal{O}(\hbar)$$

$$H'(pq, q) = \langle p;q|\mathscr{H}'(D, Q)|p;q\rangle = \mathscr{H}'(pq, q) + \mathcal{O}(\hbar; pq, q)$$ $\longleftarrow$ **2nd key**

$$d\sigma^2 = 2\hbar\,[\,\|\,d|p;q\rangle\|^2 - |\langle p;q|\,d|p;q\rangle|^2]\,,\ \ \mathscr{F} - \mathscr{S}$$

$$d\sigma^2 = (\beta\hbar)^{-1}q^2dp^2 + (\beta\hbar)\,q^{-2}dq^2$$ (**created, not found**!)

AQ → constant negative curvature : $= -\,2/\beta\hbar$ 🙂

negative curvature 2d : Scholarpedia

(SEMI) CLASSICAL HAMILTONIANS (keys available)

Canonical story $(\hbar \sim 10^{-33}\mathrm{m}^2\mathrm{kg/s})$

the 1st key

$$H(p,q) = \langle p,q|\mathscr{H}(P,Q)|p,q\rangle = \langle\omega|\mathscr{H}(P+p,Q+q)|\omega\rangle$$

$$= \mathscr{H}(p,q) + \mathscr{O}(\hbar;p,q) \longrightarrow H(p,q) \simeq \mathscr{H}(p,q)$$

Affine story $(d \leftarrow \text{dilation})$

the 2nd key ←

$$H'(pq,q) = \langle p;q|\mathscr{H}'(D,Q)|p;q\rangle = \langle\beta|\mathscr{H}'(D+pqQ,qQ)|\beta\rangle$$

$$= \mathscr{H}'(pq,q) + \mathscr{O}(\hbar;pq,q) \longrightarrow H(pq,q) \simeq \mathscr{H}(pq,q)$$

10^{33} *kilometers is further than the* furtherist known galaxy

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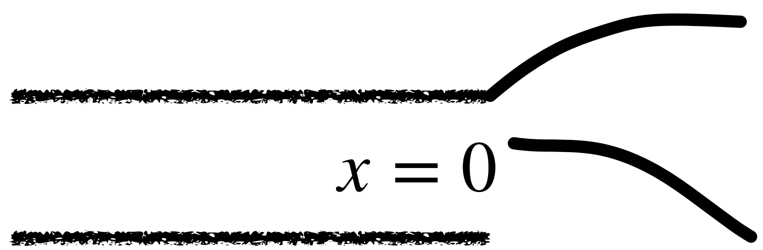
The half-harmonic oscillator

By CQ : $\text{‘}\mathscr{H} = (P^{\dagger}P + Q^2)/2\text{’}$, $0 < q \;\&\; Q$ $\quad (|s| < \infty)$

1 $q = s^2\,,\quad p = r/2\,q^{1/2},\ \{s, r\} = 1 \rightarrow \mathscr{H}' = [(R\,S^{-2}\,R/4) + S^4]/2$

2 $\psi(x) =$ $\quad -\infty < x < \infty$

$\psi'(x) =$ $\quad$ ('*infinite wall*' $x \leq 0$) $\leftarrow$



$\psi''(x) = c\,\delta(x) + ..\ \rightarrow \int |\psi''(x)|^2\,dx = \infty \rightarrow \ \psi''(x)$ *is NOT in* $\mathfrak{H}$

By AQ : $\mathscr{H} = (DQ^{-2}D + Q^2)/2$, $0 < Q$, $\quad 0 < x < \infty$

$H = (d^2/q^2 + q^2)/2$ $\quad = [P^2 + (3/4)\hbar^2/Q^2 + Q^2]/2 \qquad (3/4)$

the 2nd key

🙂 🙂

$$\psi_n(x) = x^{3/2}\, polynomial_n\, e^{-x^2/2\hbar}$$

$$E_n = 2\hbar(n+1)\ ,\ \ n = 0,1,2,3,... \quad (= \quad) \leftarrow$$

COHERENT STATES-1

tool

Canonical quantization $\quad [Q, P] = i\hbar \mathbb{1}$

$$|p,q\rangle = e^{-iqP/\hbar}\, e^{ipQ/\hbar}\, |\omega\rangle \quad , \quad (Q + iP/\omega)\,|\omega\rangle = 0 \quad \longleftarrow$$

$$\langle p', q' | p, q\rangle = \exp\{i(p'+p)(q'-q)/2\hbar - [\omega^{-1}(p'-p)^2 + \omega\,(q'-q)^2]/2\hbar\}$$

🙂 $$\mathbb{1} = \int |p,q\rangle\langle p,q|\; dp\, dq/2\pi\hbar \quad , \qquad ('p'c\,,'q'/c)$$

$\Uparrow\!\!\!\!\times\Uparrow$

The CQ metric (*a reminder*)

$$d\sigma^2 = \omega^{-1}\, dp^2 + \omega\, dq^2$$ 🙂 **Cartesian**

COHERENT STATES-2

tool

Affine quantization $\quad [Q, D] = i\hbar\, Q$

$$|p;q\rangle = e^{ipQ/\hbar}\, e^{-i\ln(q)\, D/\hbar}\,|\beta\rangle, \quad [(Q - 1\!\!1) + iD/\beta\hbar]\,|\beta\rangle = 0 \leftarrow$$

$$\langle p';q'|p;q\rangle = \left[[(q'/q)^{1/2} + (q/q')^{1/2}]/2 + i(q'q)^{1/2}(p'-p)/2\beta\hbar\right]^{-2\beta}$$

🙂 $\quad 1\!\!1 = \int |p;q\rangle\langle p;q|\,[1 - 1/2\beta]\;\, dp\,dq/2\pi\hbar\;, \;\; \beta > 1/2$

The *AQ* metric $\;$ ($q > 0$, *a reminder*)

$$d\sigma^2 = (\beta\hbar)^{-1}\, q^2\, dp^2 + (\beta\hbar)\, q^{-2}\, dq^2$$ 🙂 $\quad ('p'\,c\,,'q'/c)$

A constant negative curvature = – 2/$\beta\hbar$ $\quad \Uparrow$

Building two BRIDGES

construction

The building material : 🙂 🙂 🙂

$i,\ \hbar,\ \partial/\partial t,\ \langle p',q'|,\langle p';q'|,|p,q\rangle,\ |p;q\rangle, \mathscr{H}(P,Q),\ \&\ \mathscr{H}'(D,Q)$

A Canonical **BRIDGE** 🙂

$\longrightarrow\ B(p',q'\ |p,q) \equiv \langle p',q'|[\ i\hbar(\partial/\partial t) - \mathscr{H}(P,Q)\ ]\,|p,q\rangle$

An Affine **BRIDGE** 🙂

$\longrightarrow\ B'(p';q'|p,;q) \equiv \langle p';q'|[\ i\hbar(\partial/\partial t) - \mathscr{H}'(D,Q)\ ]\,|p;q\rangle$

Using a (CQ/AQ)-BRIDGE

$CR \leftrightarrow QR$

🙂 *The (semi) classical realm (CQ)*

$$A_{cl} = \int_0^T c\{ \iint \langle p(t), q(t) | p', q' \rangle \, B(p', q' | p, q) \, \langle p, q | p(t), q(t) \rangle \, dp' \, dq' \, dp \, dq \} \, dt$$
$$= \int_0^T \{ \langle p(t), q(t) | [i\hbar(\partial/\partial t) - \mathscr{H}(P, Q)] | p(t), q(t) \rangle \} \, dt$$
$$= \int_0^T \{ p(t) \, \dot{q}(t) - \mathscr{H}(p(t), q(t)) - \mathscr{O}(\hbar; p(t), q(t)) \} \, dt$$ 🙂

🙂 *The quantum realm (CQ)*

$$A_{qu} = \int_0^T c\{ \iint \langle \Psi(t) | p', q' \rangle \, B(p', q' | p, q) \, \langle p, q | \Psi(t) \rangle \, dp' \, dq' \, dp \, dq \} \, dt$$
$$= \int_0^T \{ \langle \Psi(t) [i\hbar(\partial/\partial t) - \mathscr{H}(P, Q)] | \Psi(t) \rangle \} \, dt$$ 🙂

$\longrightarrow$ **For AQ** : $c \to c'$, **all** $p, q \to p; q > 0$, $P \to D$, **and** $B \to B'$

HOW MATHEMATICS CAN FAIL PHYSICS

Mathematics : *when* $\varphi(x)$ *has* $\varphi(0) = \infty$

$$\varphi(x)^2 = \frac{1}{|x|^{2/3}} \;, \quad \varphi(x)^4 = \frac{1}{|x|^{4/3}}$$

integrable infinity →

$$\int_{-1}^{1} \frac{dx}{|x|^{2/3}} < \infty \;, \quad \int_{-1}^{1} \frac{dx}{|x|^{4/3}} = \infty$$

Physics : $\varphi(x)$ *can represent nature*!

Nature's wonders do not need infinity!

🙂 Can Math be Made to Save Physics? **YES**! 🙂

CLASSICAL AND QUANTUM

Usual classical and quantum issues

$$\infty > H(\pi, \varphi) = \int \{ [\pi(x)^2 + (\overrightarrow{\nabla} \varphi(x))^2 + m^2 \varphi(x)^2]/2 + g\, \varphi(x)^p \} \; d^s x$$

$p = power\ of\ interaction\ term$

$n = s + 1,\ number\ of\ spacetime\ dimensions$

$p \geq 2n/(n-2)$ **will lead to NON – renormalizability**

$$\mathscr{H}\, \hat{\pi}, \varphi)_{CQ} = \int \{ [\hat{\pi}(x)^2 + (\overrightarrow{\nabla} \varphi(x))^2 + m^2 \varphi(x)^2]/2 + g\, \varphi(x)^p \} \; d^s x$$

Monte Carlo studies for both $\varphi_n^p = \varphi_3^{12}$ *and* $\varphi_n^p = \varphi_4^4$ *are*

UNACCEPTABLE using canonical quantization, but

→ 🙂 *ACCEPTABLE using affine quantization* . 🙂

QUANTUM FIELD THEORY

$\downarrow$ **AQ** $\kappa(x) = \pi(x)\,\varphi(x)^{ABC}$, $\varphi(x) \neq 0$, $|\varphi|, |\pi|$ & $|\kappa| < \infty^{ABC}$

$$\infty > H'(x) = [\kappa(x)^2\, \varphi(x)^{-2} + (\vec{\nabla}\varphi(x))^2 + m^2\varphi(x)^2]/2 < \infty$$ 🙂

$$\infty > H'(\kappa, \varphi) = \int \{[\kappa(x)^2\, \varphi(x)^{-2} + (\vec{\nabla}\varphi(x))^2 + m^2\varphi(x)^2]/2 + g\,\varphi(x)^p\}\, d^sx$$

To summerize $\varphi(x)^2 > 0$, $\varphi(x)^{-2} > 0$

$$0 < |\varphi(x)| < \infty \quad \rightarrow \quad 0 < |\varphi(x)|^p < \infty$$ 🙂

To quantize, use $\hat{\varphi}(x) \rightarrow \varphi(x) \neq 0$ $\downarrow$

$$\hat{\kappa}(x) = [\hat{\pi}(x)^\dagger \varphi(x) + \varphi(x)\,\hat{\pi}(x)]/2$$ $(MC: \ \varphi_3^{12}, \varphi_4^4)$

$$\mathscr{H}'(\hat{\kappa}, \varphi)_{AQ} = \int \{ [\hat{\kappa}(x)\, \varphi(x)^{-2}\, \hat{\kappa}(x) + (\vec{\nabla}\varphi(x))^2 + m^2\, \varphi(x)^2]/2 + g\,\varphi(x)^p \}\, d^sx$$ 🙂

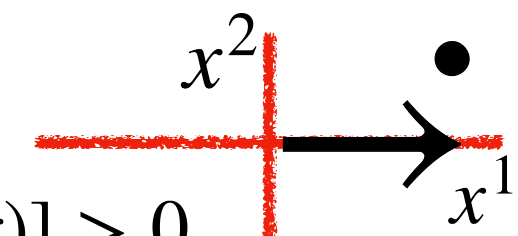
🙂 RENORMALIZABLE 🙂

QUANTUM GRAVITY

A *math* **view** $\pi^a_b(x) = \pi^{ac}(x)\, g_{bc}(x)$, ‘$g_{de}(x) \neq 0$’, SO(3)

$\sim ABC$

A *physics* **view** $x^1 = \pm\,\epsilon\,,\ \ x^2 = 0\ \ (= x^3)$



$$\infty > ds(x)^2 = \ g_{ab}(x)\ dx^a\, dx^b > 0 \ \rightarrow \ g(x) \equiv det[g_{ab}(x)] > 0$$

ADM

$$\infty > H(\pi, g) = \int \{\, g(x)^{-1/2}\, [\pi^a_b(x)\, \pi^b_a(x) - (1/2)\, \pi^a_a(c)\, \pi^b_b(x)\,] + g(x)^{1/2}\, {}^{(3)}R(x)\ \}\ d^3x > \overset{ABC}{-\infty}$$

To quantize, use $\hat{g}_{ab}(x) \rightarrow g_{ab}(x) > 0$

$$\hat{\pi}^a_b(x) = [\hat{\pi}^{ac}(x)^\dagger g_{bc}(x) + g_{bc}(x)\, \hat{\pi}^{ac}(x)]/2$$

$$\hat{\pi}^a_b(x)\, g(x)^{-1/2} = 0$$ 🙂

$$\mathscr{H}(\hat{\pi}, g)_{AQ} = \int \{\, [\hat{\pi}^a_b(x)\, g(x)^{-1/2}\, \hat{\pi}^b_a(x) - (1/2)\, \hat{\pi}^a_a(x)\, g^{-1/2}(x)\, \hat{\pi}^b_b(x)\,] + g(x)^{1/2}\, {}^{(3)}R(x)\ \}\ d^3x$$ 🙂

A **valid Hamitonian** *is the* **key** *to quantum gravity* 🙂

Constraints and other items can be added

Suggested Publications

1 – *intro to affine quantization* arXiv : 1912.08047 — $Q > 0$ $(R \to D)$

2 – *quantizing a special field* arXiv : 2109.13447 — φ_4^4

3 – *path integration of gravity* arXiv : 2203.15141 — $g_{ab}(x) > 0$

THANK YOU