Predicting “Anyons”:
Implications of History for Science

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Thank you to all the local organizers of WGMP XXXIX in Bialystok for their work in achieving this excellent meeting under extraordinarily challenging conditions.

In memory today of two dear colleagues and friends – Bogdan Mielnik, whose creative spirit and original ideas inspired us across the years of WGMP in Białowieża, and Anatol Odziewicz, without whose dedicated leadership we would not be here for WGMP at all.

We miss them both greatly.

Overview of my talk

1. "Anyons" and "nonabelian anyons" in two-space: The easiest of ideas
2. Why so long? Profound epistemological and cognitive obstacles
3. Antecedent ideas: (Ehrenberg-Siday) Aharonov-Bohm effect, Feynman paths, topology in quantum mechanics
4. Further antecedent ideas: Group representations, local current algebras, ∞-dimensional groups; parastatistics; quantum field theory, braid group
5. Three independent predictions of intermediate statistics in two-space
6. Implications for educating future mathematicians and physicists: creative activity and learning
7. Issues of acknowledgment, and their wider consequences: academic integrity, risk-taking, fairness and equity

1. “Anyons” and “nonabelian anyons" in two-space

The basic idea behind “anyons” is extremely easy.

Let two indistinguishable particles in the plane exchange positions (without passing “through” each other). Then we can consistently keep track of the number of (clockwise or counterclockwise) windings of one about the other.

This is not true in three or more space dimensions.

So instead of being limited to a factor of +1 or -1, the wave function could pick up a phase $\exp i\theta$.

Two clockwise exchanges in succession are inequivalent to making no exchange. So we need not set $\exp 2i\theta = 1$.

When indistinguishable particles exchange position, predictions of observables should be invariant. This will still hold.

For three or more particles path-dependent exchanges do not commute, and nonabelian representations exist.

So why did this take over half a century – from understanding bosons and fermions (1924-25), to intermediate statistics (1977-82)?

What then led to three predictions independent of each other, in close succession?

2. Why so long?

The concept of "epistemological obstacle"

These pertain to historical paths of discovery, evolving conceptual schemes, and the meanings ascribed to representations.

Examples in mathematics: negative numbers, "imaginary" numbers, non-Euclidean geometries widely believed not to really exist.

Examples in physics: relativity of spacetime, wave-particle duality contradicted universal categories of everyday experience.

In physics, we depend on the accessibility of empirical observations.

The concept of “cognitive obstacle”:

The psychology of individual thinkers, reinforced by widely shared belief structures, limits the consideration of possibilities.

"Ontogeny recapitulates phylogeny"

The stages of learning by individuals, as they break through cognitive obstacles, recapitulate historical processes of overcoming epistemological obstacles.

Several obstacles to “anyons”

The meaning of particle exchange for indistinguishable particles

Labels and the exchange of labels, rather than particles. Index permutations vs. value permutations

Configuration space (exchange not meaningful) vs. coordinate space (exchange of indices) vs. physical space (exchange of positions)

Established empirical knowledge of fundamental particles: only two kinds (bosons and fermions), falling within powerful, all-embracing theories (Pauli exclusion principle, Bose-Einstein condensation, etc.)

Single-valuedness of wave functions

The Bohr atom and quantization

The Schrödinger equation: wave functions continuous on a physical space, satisfying boundary conditions

Axiomatic quantum mechanics: unintended consequences

Wanting to prove well-established beliefs as theorems (e.g., totally symmetric or totally antisymmetric wave functions), one accomplishes this by assuming stronger than warranted axioms.

3. Antecedent ideas

These refer to developments that enable the overcoming of epistemological and cognitive obstacles:

Aharonov-Bohm effect (1959), earlier by Ehrenberg and Siday (1949)

Charged particles circling but not entering regions of magnetic flux

Role of topology in quantum mechanics

Multivalued wave functions achieve legitimacy

Possible physical meaning of the electromagnetic potential A_μ (open to question)

Alternative self-adjoint extensions of differential operators

Feynman path integral formulation and topology

Paths from initial to final state: particle locations traverse paths (tacit implication: indices do not)

Feynman path models for spin, statistics: Lawrence Schulman; Michael Laidlaw and Cécile Morette DeWitt; J. S. Dowker

General finite occupation number statistics

G. Gentile (1940, 1942) “Si studiano le statistiche quantistiche, intermedie tra quella di Bose-Einstein e l’altra di Fermi-Dirac ...”

4. Further antecedent ideas

Group representations

Unitary representations of Lie groups in quantum mechanics; Eugene Wigner, George Mackey, theory of induced representations

High-energy particle physics, quarks: Murray Gell-Mann; Yuval Ne'eman; George Zweig

Local current algebras and ∞-dimensional groups: Dashen and Adler

Nonrelativistic current algebra and diffeomorphism group representations: Dashen, David Sharp, GG, Ralph Menikoff

Rotation generators in two-space: Christiane Martin

Further antecedent ideas (continued)

Parastatistics

Quantum fields (trilinear brackets), nonabelian representations of S_N: H. S. Green; A. Messiah & O. W. Greenberg: opened consideration of exotic statistics, presaged nonabelian anyons

Possible application to quarks superseded by their description via additional quantum numbers

Quantum field theory, braid group (Yang-Baxter relations)

Analogous concept in quantum field-theoretic models with soliton fields in two dimensions: Ray Streater and Ivan Wilde; Jürg Fröhlich

5. Three independent predictions

Leinaas and Myrheim (1977-78)

Schrödinger quantization of particle dynamics in two-space

Singular points where particles coincide: Why can't dynamical paths cross? Possible need for a hardcore potential ...

Connection with electromagnetism: charged-particle/monopole composite model

Three independent predictions (continued)

Goldin, Menikoff, and Sharp (1980-81)

Mathematically rigorous prediction, the culmination of 15 years' work.

Quantum kinematics from unitary representations of $\mathit{Diff}_0(\mathbb{R}^2)$, rather than prediction from dynamics. Shift in the kinetic angular momentum spectrum.

Wave functions single-valued on configuration space (exchange statistics encoded in the operators describing local currents).

No arbitrary exlcusion of coincidence points; no hard-core repulsive potential required.

(continued)

Goldin, Menikoff, and Sharp (continued)

First explicit identification of the braid group as the homotopy group governing N-anyon statistics (1983)

Rigorous kinematical framework for fractional spin of anyons (1983)

First prediction of "nonabelian anyons" equivariant under higher-dimensional unitary braid group representations (1985)

Intermediate phases of topological origin for distinguishable particles: the group of colored braids (1985)

Three independent predictions (continued)

Wilczek (1982a,b)

Investigating fractional quantum numbers suggests fractional spin in two dimensions

Model based on charged particles bound to units of magnetic flux (“particle-flux tube composites”)

Coinage of catchy name “anyons” for“any” intermediate statistics; suggestion of fractional spin-statistics connection

Subsequent powerful advocacy for theoretical importance of anyons

6. Educational implications

What can we learn from this history of ideas about educating future (or current) mathematicians and physicists? Or educating ourselves?

What does it mean for a student to "really understand" a newly-studied concept in mathematics or physics?

What *should* it mean, to be abe to overcome cogntiive obstacles?

As learning occurs, how can we foster ...
... questioning of established constructs?
... generation of new ones?

Teaching physics and mathematics: What is optimal?

How to state, apply, calculate with a concept ... emphasized strongly

Knowing the accepted rationale ... usually well explained

Understanding the "why" behind the concept ... sometimes developed

Questioning its foundations ... done much less frequently

How and why the concept was first invented rarely explored in depth

How the student herself or himself might have invented the concept, without having seen it ... almost never considered

Encouraging creativity in mathematics and physics

Creativity usually involves breaking conventions, crossing boundaries, introducting new forms or variations.

Art and literature: constrained only by imagination

Architecture and design: constraints of functionality, safety, etc.

Mathematics: constraints of precision and logic
(historical constraints of meaning and existence)

Physics: constraints of experimental agreement
(sometimes interpreted in too-restrictive ways)

Israell Gelfand's perspective (from a talk in 2003)

Mathematics shares four features with classical music and poetry:

(1) Beauty
(2) Simplicity
(3) Exactness
(4) Crazy ideas (most important)

Predicting anyons involved courageous ideas that seemed "crazy" then, but less crazy than before the antecedents.

STEM education: Emphasize exploration early, with open-ended problems and experiments without predetermined outcomes. Hold high aspirations for girls and boys of all races and nations.

7. Issues of truthful acknowledgment

A difficult, highly personal subject

The most wonderful aspiration of a young mathematical physicist is to predict a new phenomenon, and then see it confirmed.

In 2020 anyonic excitations were achieved experimentally!

This should be a source of deep satisfaction and fulfillment. Developing the general theory that led to the prediction took 15 years, during difficult times.

But prediction of anyons is now publicly attributed only to Frank Wilczek, repeating a pattern that began forty years ago. Professor Wilczek shared the Nobel Prize for his renowned earlier work on quark confinement.

Some scientists – and all journalists – fail to cite our prior prediction of intermediate statistics, or any of the theoretical observations we were first to publish.

To be “written out of history” before one’s eyes, across 40 years – when there is no actual dispute – is indeed a painful experience.

Issues of acknowledgment (continued)

Acknowledging the physicists who first predicted “anyons”:

Long-term failures of proper citation and acknowledgment are also part of the history of anyons, and have affected it.

1980-89

Early omissions of fair acknowledgment occurred, some inadvertent, some quite deliberate, with long-enduring effects.

Correspondence by letter then was slow, difficult, and ineffective.

Risk-taking in science has a price. Power relationships influence acknowledgment, with psychological and career consequences.

1980-89 (continued)

Consequences for scientific investigation during this period: some methods are pursued, others disregarded. Missed opportunities.

1989-1991

Physics Today (1989, 1990): A major battle for correct citation is required, at heavy personal cost. It is favorably resolved, but ...

Scientific American (1991), *Science* magazine, and elsewhere: The acknowledgment failures continue on, though the truth is undisputed. Some citations in the research literature, as thirty years pass ...

Recent events (2020-21)

Anyonic excitations are confirmed experimentally. Fame takes over, dictating attribution. Omissions are no longer inadvertent.

Discover Magazine chooses intentionally dishonest journalism. Crediting only Nobelist Frank Wilczek, its editors ignore detailed communications and refuse to update on-line posts with corrections.

Quanta Magazine credits Wilczek only, as its editors disregard repeated efforts to communicate.

Wikipedia (“anyon” entry): An anonymous editor with screen name “HouseofChange” takes out every correct citation put in by independent physicists, and makes false and malicious accusations.

Why does it matter? A wider perspective

Proper acknowledgment benefits the whole scientific community.

The loss when correct citation is withheld is not only to individuals. Denying information slows the progress of research and its benefits.

Long-term, original projects whose success is not assured, and predictions of new, unsuspected phenomena, involve investment of time, possible loss of career opportunities, and reputational risk.

When even dramatic success is unrecognized, disillusionment and alienation result.

Why it matters (continued)

Accepting dishonesty is dangerous, because the value of truth and respect for integrity in science is damaged.

Many others have had experiences similar to ours. Failure to attribute scientific achievements undermines fairness and equity.

Non-recognition and career obstacles hurt women, minorities, scientists in developing countries, and those without "connections."

It may one day hurt some young scientists here, or even be costing you opportunities now.

Do we recognize excellent science, or do we value fame and promotional ability instead?

Conclusion

David Sharp and I are among the most fortunate. We had fellowships, fine educations at the best universities, successful careers, honors and awards in physics.

We took risks in our early careers, and endured some difficult consequences. But we emerged as continuing, active scientists.

Unfortunately, it was only our status and connections that let us achieve the earlier corrections published in *Physics Today* and *Scientific American* in 1989-91. But the damage was done. Today we still encounter impasse after impasse.

Others without such resources have little or no recourse.

We both feel it is our obligation to the field to speak up. The outcome will not affect us materially. The process, though difficult, is necessary to address a serious and widespread problem.

Several pages of references form the balance of my slides.

Thank you for your kind attention.

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