Malignant and non-malignant nonlinearity in QM

or

Russian roulette with a cheating player and EPR correlations in nonlinear QM

or

Is there any no-go theorem about nonlinear QM?

Marek Czachor

POLITECHNIKA GDAŃSKA

Białystok, 20.06.2022

prof. Bogdan Mielnik memorial session

In memory of prof. Bogdan Mielnik, a man to whom I owe exceptionally much

Commun. math. Phys. 60, 1—6 (1978)

Communications in Mathematical Physics

Prehistory of the problem (1978)

Comments on Mielnik's Generalized (Non Linear) Quantum Mechanics*

Rudolf Haag and Ulrich Bannier

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Abstract. We discuss a model of non linear quantum mechanics in which the wave equation satisfies the homogeneity condition (2.1). It is argued that in this model the set of (mixed) states is a simplex.

Thus the fact that we can influence the motion by the non linear A-term in this model makes mixtures of different sets of pure states distinguishable. As far as only single particle systems are considered one might claim that $|\varphi(\boldsymbol{x})|$ and $\nabla S(\boldsymbol{x})$ describe the "objective state" of an individual particle. However it is not our intention to maintain that thereby one achieves a classical theory (where $|\varphi|$ and ∇S are hidden variables of the particle) because problems with such an interpretation would immediately arise as soon as one considers 2-particle systems.

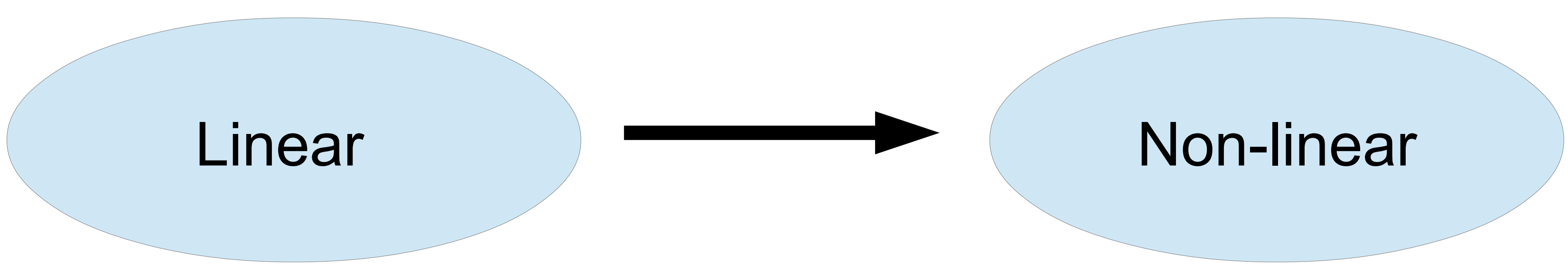
More recent prehistory

- N. Gisin - 1989 (in fact the first version of the paper was rejected in PRL several years earlier)

Similar idea as in Haag-Bannier (distinguishability of local non-linearly evolving mixtures via their entanglement with a linearly evolving system)

N. Gisin, Helv.Phys.Acta **62**, 363 (1989)

Faster than light telegraph acting by creation at-a-distance of initial conditions for a nonlinear evolution



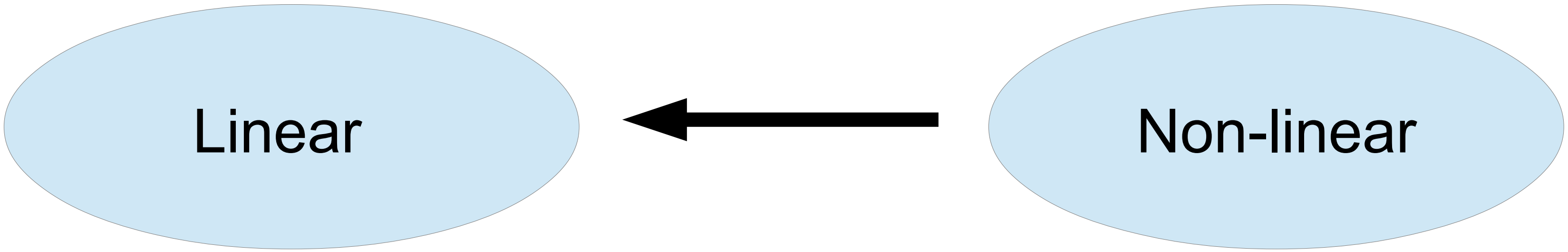
Occurs only in non-equal-time correlations!

More recent prehistory

- M. Czachor - 1989 (unpublished talk at „Problems in Quantum Physics II, Gdańsk'89”)

  Modification of a reduced density matrix in a linear system by „mobility of states” induced by a non-linear evolution in a correlated system

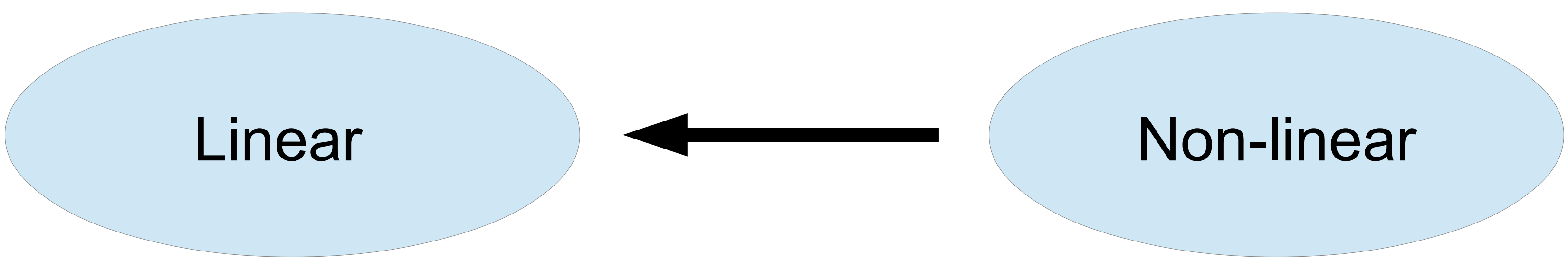
  Faster than light telegraph acting by noninvariance of partial trace under nonlinear evolution + entanglement (works in the oposite direction than the Gisin telegraph)



More recent prehistory

- J. Polchinski - 1989 (unpublished comments on Weinberg's nonlinear QM)

  Non-commutativity of observables in separated systems described a la Wienberg.



- J. Polchinski – 1991. The first paper where a partial solution was proposed: It eliminated effects based on noncommutativity and mobility, but not on projection-at-a-distance. The Gisin problem remained.

  J. Polchinski, Phys. Rev. Lett. **66**, 397 (1991)

A solution

- M. Czachor, H.-D. Doebner - 2002 (generalization of the Polchinski trick to non-equal-time correlations)

  M. Czachor, H.-D. Doebner, Phys. Lett. A **301**, 139-152 (2002)

- It eliminates all the telegraphs (Gisin's included)
- It shows that the difficulty is not related to QM, but occurs in all theories where nonlinear evolutions of probability are combined with reductions of probability at-a-distance (via correlations)
- It shows how to modify the projection postulate in nonlinear QM (it reduces to the usual one if the dynamics is linear)
- Russian roulette with a cheating player is an example of a classical probabilistic game where all these subtleties occur

Example: 1-particle „Nonlinear Schrödinger equation"

$$
\begin{aligned}
i\frac{\partial \psi_t(x)}{\partial t} &= -\frac{\partial^2 \psi_t(x)}{\partial x^2} + \epsilon |\psi_t(x)|^2 \psi_t(x) \\
&= \left(-\frac{\partial^2}{\partial x^2} + u[\psi_t(x)] \right) \psi_t(x) \\
&= \left(-\frac{\partial^2}{\partial x^2} + u_t(\psi_0, x) \right) \psi_t(x)
\end{aligned}
$$

Essentially this is a S. eq. with time-dependent potential

So $\frac{\partial \langle \psi_t | \psi_t \rangle}{\partial t} = 0$ but $\frac{\partial \langle \psi_t | \tilde{\psi}_t \rangle}{\partial t} \neq 0$

The „mobility effect"

for two different solutions $\psi_t(x)$ and $\tilde{\psi}_t(x)$

2-particle extensions

Naive extension (Weinberg'89;Białynicki-Birula-Mycielski'76)

$$i\frac{\partial\Psi_t(x,y)}{\partial t} = -\frac{\partial^2\Psi_t(x,y)}{\partial x^2} - \frac{\partial^2\Psi_t(x,y)}{\partial y^2} + (\epsilon_1 + \epsilon_2)|\Psi_t(x,y)|^2\Psi_t(x,y)$$

2-particle extensions

Generates malignant nonlocality

Naive extension (Weinberg'89;Białynicki-Birula-Mycielski'76)

$$i\frac{\partial\Psi_t(x,y)}{\partial t} = -\frac{\partial^2\Psi_t(x,y)}{\partial x^2} - \frac{\partial^2\Psi_t(x,y)}{\partial y^2} + (\epsilon_1+\epsilon_2)|\Psi_t(x,y)|^2\Psi_t(x,y)$$

Polchinski extension (Polchinski, 1991)

$$i\frac{\partial\Psi_t(x,y)}{\partial t} = -\frac{\partial^2\Psi_t(x,y)}{\partial x^2} - \frac{\partial^2\Psi_t(x,y)}{\partial y^2} + \Big(\epsilon_1\rho_t(x)+\epsilon_2\rho_t(y)\Big)\Psi_t(x,y)$$

M.Czachor, Nonlocal-looking equations can make nonlinear quantum dynamics local, PRA 57, 4122 (1998)

$$\rho_t(x) = \int |\Psi_t(x,y)|^2 dy, \quad \rho_t(y) = \int |\Psi_t(x,y)|^2 dx$$

Reduced density matrices of the subsystems depend only on parameters and initial conditions of these subsystems. Although mobility effect is still present, it becomes nonmalignant. But what about the Gisin argument?

Gisin's argument: A problem with non-equal-time correlations

2-qubit example

$$i|\dot{\psi}_1\rangle = A\langle\psi_1|\sigma_z|\psi_1\rangle\sigma_z|\psi_1\rangle$$

$$i|\dot{\psi}_2\rangle = B\langle\psi_2|\sigma_z|\psi_2\rangle\sigma_z|\psi_2\rangle$$

- At $t = t_1$ the state is

$$|\Psi(t_1)\rangle = \underbrace{e^{-iA\langle\sigma_z(0)\rangle_1\sigma_z t_1}}_{V_1(\Psi_0,t_1)} \otimes \underbrace{e^{-iB\langle\sigma_z(0)\rangle_2\sigma_z t_1}}_{V_2(\Psi_0,t_1)} |\Psi_0\rangle.$$

- At $t = t_1$ project with $E_1^{\pm} \otimes I_2$ and normalize

$$|\Psi(t_1)\rangle \mapsto \frac{E_1^{\pm} \otimes I_2|\Psi(t_1)\rangle}{\| E_1^{\pm} \otimes I_2|\Psi(t_1)\rangle \|} =: |\Psi_{\pm}(t_1)\rangle.$$

- Evolve the resulting state for $t_1 < t < t_2$ but starting at t_1 with

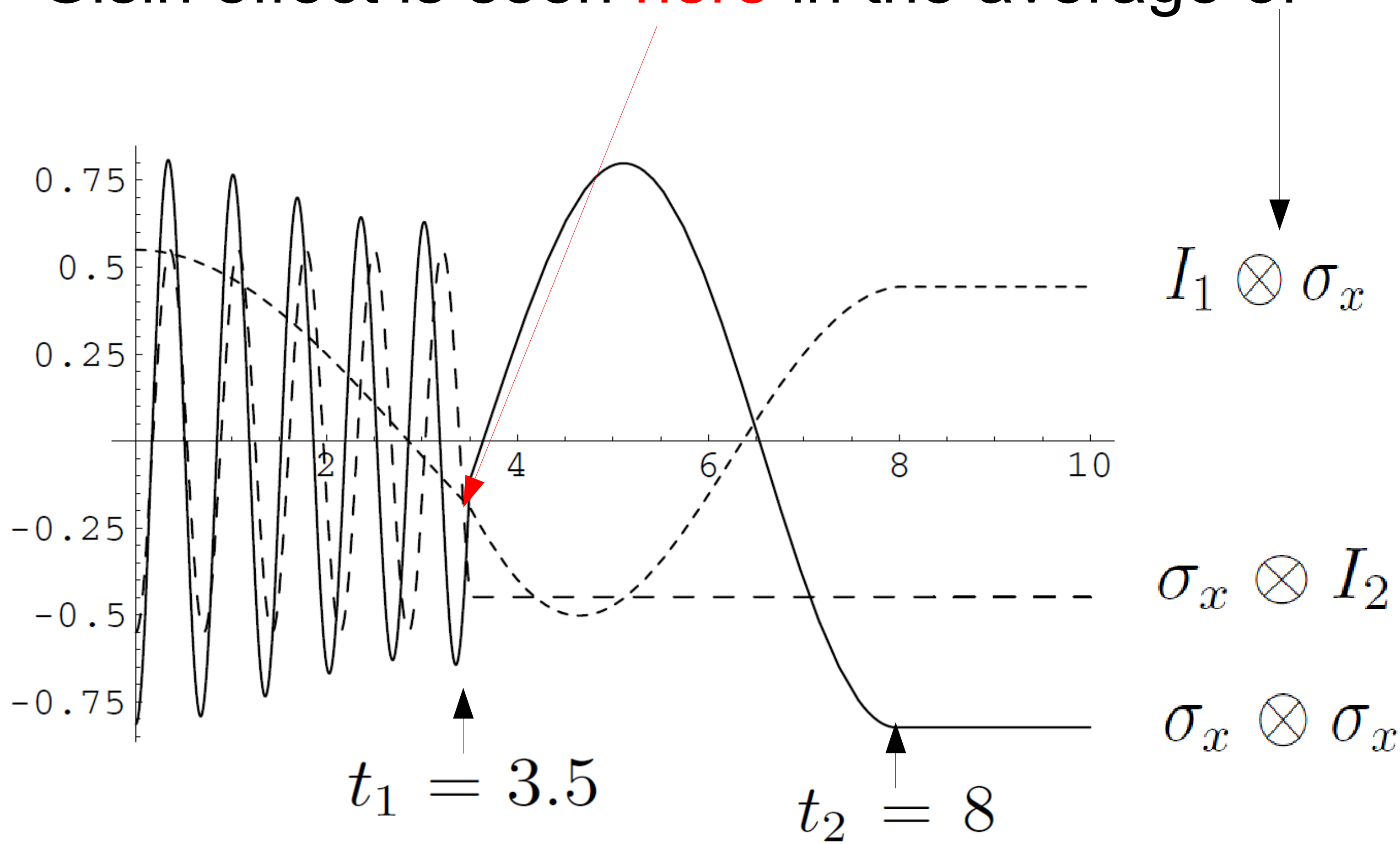
$$|\Psi_{\pm}(t_2)\rangle = I_1 \otimes \underbrace{e^{-iB\langle\Psi_{\pm}(t_1)|I_1\otimes\sigma_z|\Psi_{\pm}(t_1)\rangle\sigma_z(t_2-t_1)}}_{V_2\left(\Psi_{\pm}(t_1),t_2-t_1\right)} |\Psi_{\pm}(t_1)\rangle$$

Concrete initial state

$$|\Psi_0\rangle = \frac{1}{3}|1\rangle|2\rangle - \frac{2\sqrt{2}}{3}|2\rangle|1\rangle$$

$$|1\rangle = \begin{pmatrix} \cos(\pi/8) \\ \sin(\pi/8) \end{pmatrix}, \quad |2\rangle = \begin{pmatrix} -\sin(\pi/8) \\ \cos(\pi/8) \end{pmatrix}$$

Gisin effect is seen here in the average of



The whole reasoning revisited

Joint probabilities in 2-particle systems
(Heisenberg picture in linear QM)

Directly measurable probabilities (2-time correlation functions)

- Probability of the result "yes" for E_1 on particle #1

$$P[E_1(t_1)] = \langle \Psi_0 | E_1(t_1) \otimes I_2 | \Psi_0 \rangle,$$

- Probability of the result "yes" for E_2 on particle #2

$$P[E_2(t_2)] = \langle \Psi_0 | I_1 \otimes E_2(t_2) | \Psi_0 \rangle,$$

- Joint probability of results "yes" for both particles

$$P[E_1(t_1) \cap E_2(t_2)] = \langle \Psi_0 | E_1(t_1) \otimes E_2(t_2) | \Psi_0 \rangle.$$

Conditional probabilities can be deduced directly from the Bayes rule without any state-vector reduction (but are consistent with it)

$$P[E_2(t_2)|E_1(t_1)] = \frac{P[E_1(t_1) \cap E_2(t_2)]}{P[E_1(t_1)]}$$

Can we do the same in nonlinear QM?

No! There is no Heisenberg picture in nonlinear QM!

So reverse the question: Can we perform the same calculation directly in the Schroedinger picture?

So reverse the question: Can we perform the same calculation directly in the Schroedinger picture?

Yes. Solve Schroedinger equation with the time-dependent Hamiltonian parametrized by moments of „freezing the dynamics” (open system with detectors in environment) $\theta(t)=0$ for $t>0$

$$H_{t_1,t_2}(t) = \theta(t-t_1)H_1 \otimes I_2 + \theta(t-t_2)I_1 \otimes H_2$$

The Schroedinger dynamics becomes

$$|\Psi_{t_1,t_2}(t)\rangle = e^{-iH_1 \otimes I_2 \int_0^t \theta(\tau - t_1)d\tau - iI_1 \otimes H_2 \int_0^t \theta(\tau - t_2)d\tau}|\Psi_0\rangle$$

For *t* later than the moments of detection we get the same probabilities as in the Heisenberg picture

$$P[E_1(t_1) \cap E_2(t_2)] = \langle \Psi_{t_1,t_2}(t)|E_1 \otimes E_2|\Psi_{t_1,t_2}(t)\rangle$$

This trick can be employed also in nonlinear QM.
Combined with the Polchinski 2-particle extension it solves the Gisin problem.

Again the Gisin problem for 2 qubits

2-qubit example

$$i|\dot{\psi}_1\rangle = A\langle\psi_1|\sigma_z|\psi_1\rangle\sigma_z|\psi_1\rangle$$

$$i|\dot{\psi}_2\rangle = B\langle\psi_2|\sigma_z|\psi_2\rangle\sigma_z|\psi_2\rangle$$

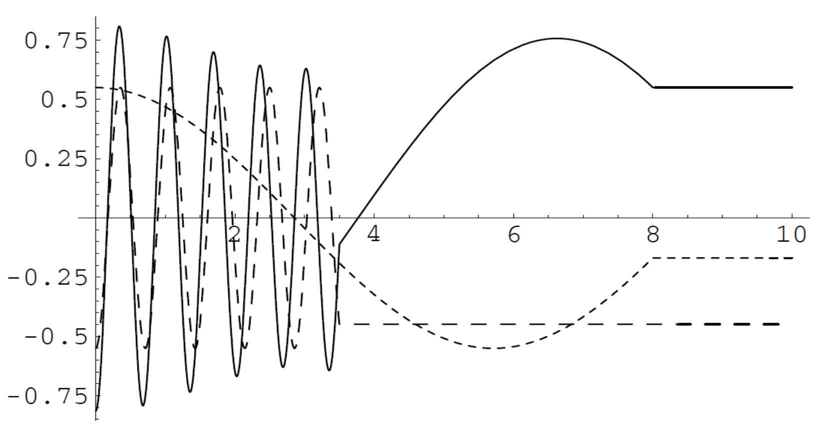
Polchinski 2-particle extension parametrized by the moments of detection

$$i|\dot{\Psi}\rangle = \Big(\theta(t-t_1)A\langle\Psi|\sigma_z\otimes I|\Psi\rangle\sigma_z\otimes I + \theta(t-t_2)B\langle\Psi|I\otimes\sigma_z|\Psi\rangle I\otimes\sigma_z\Big)|\Psi\rangle$$

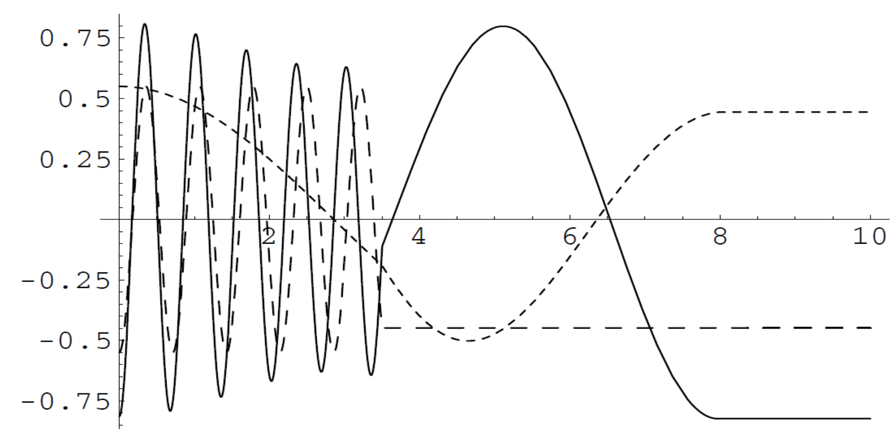
The solution

$$|\Psi_{t_1,t_2}(t)\rangle = e^{-iA\langle\Psi_0|\sigma_z\otimes I|\Psi_0\rangle\sigma_z\otimes I\int_0^t\theta(\tau-t_1)d\tau - iB\langle\Psi_0|I\otimes\sigma_z|\Psi_0\rangle I\otimes\sigma_z\int_0^t\theta(\tau-t_2)d\tau}|\Psi_0\rangle$$

Parametrized Polchinski (local)



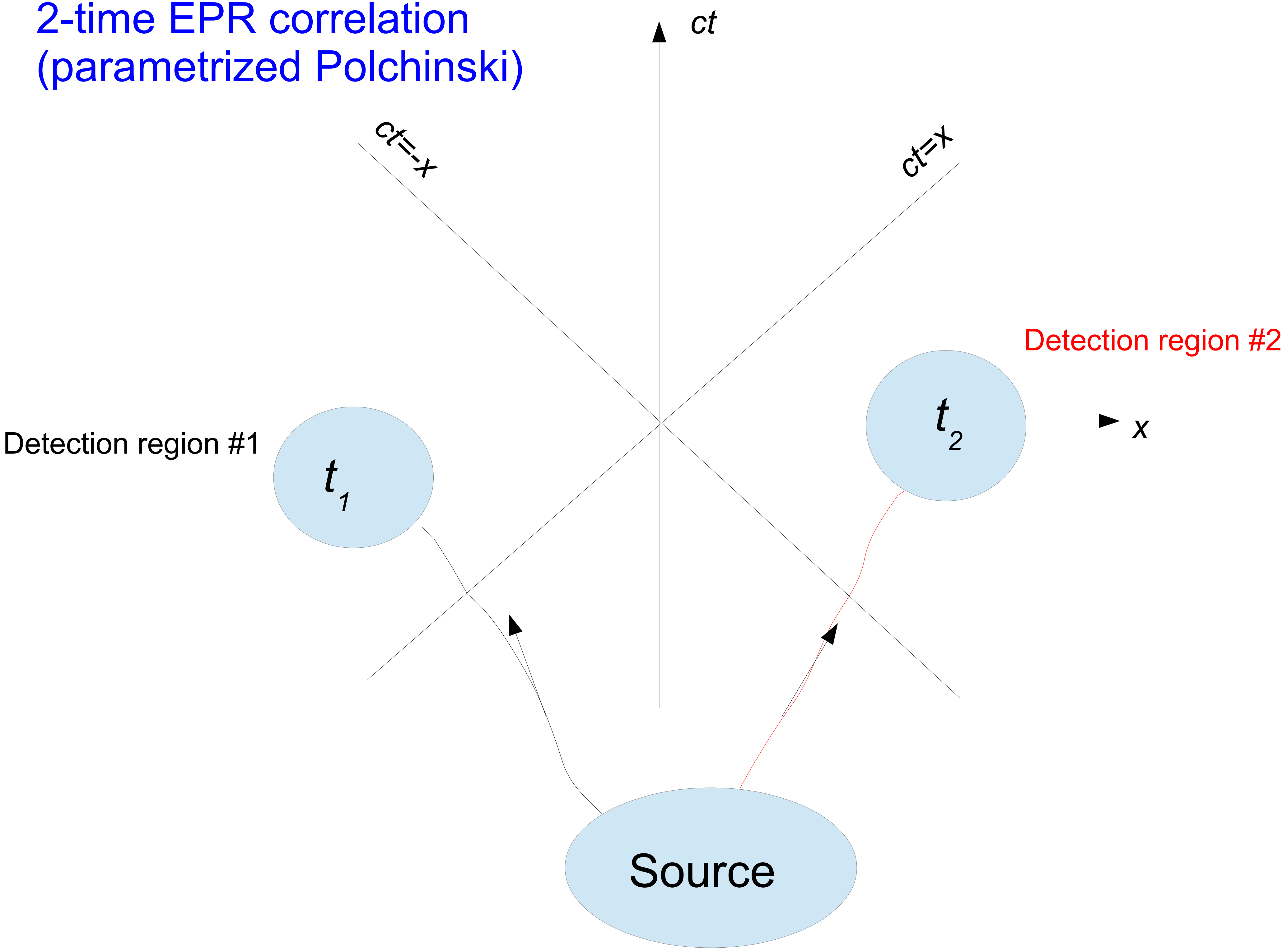
Gisin (nonlocal)



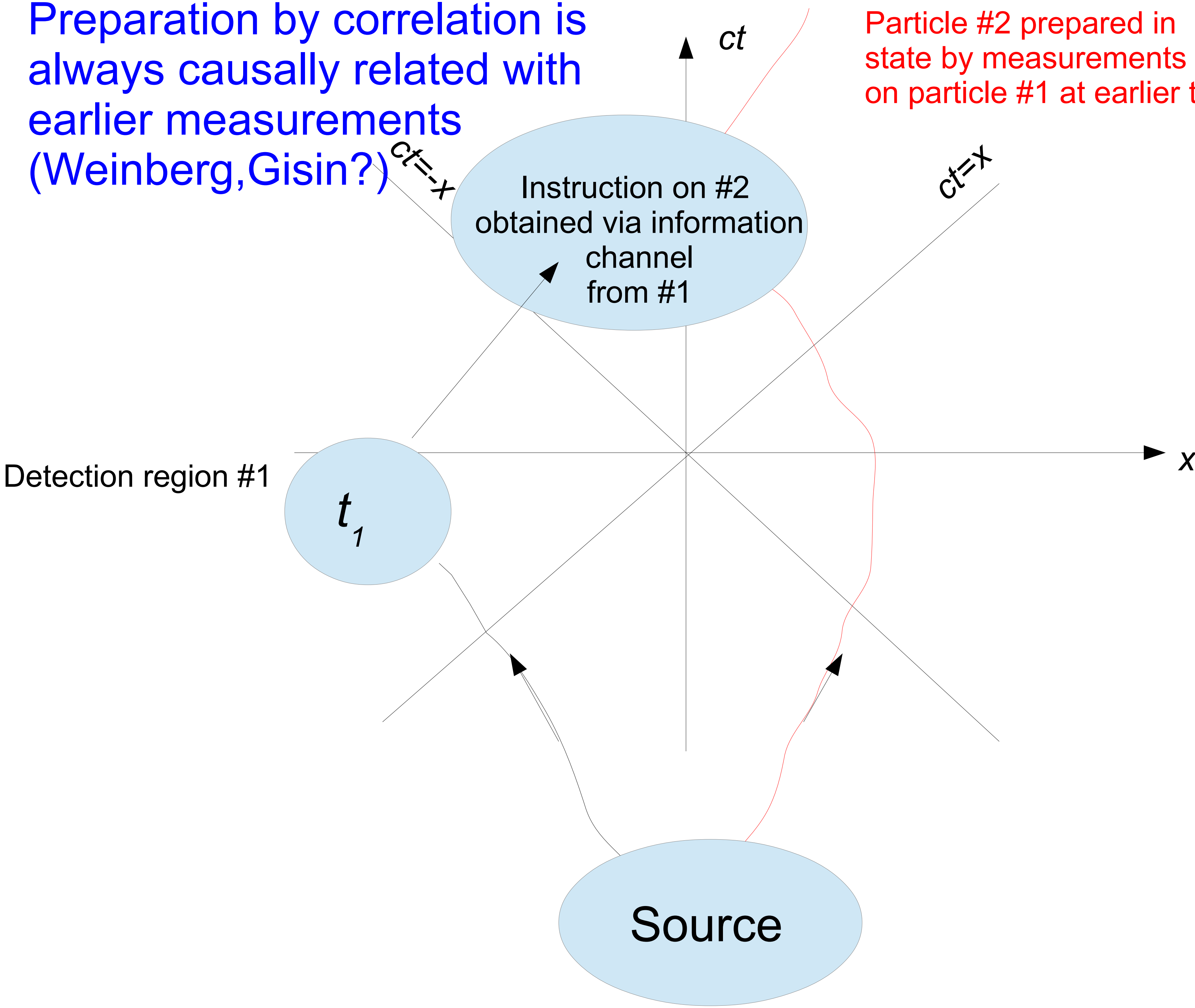
This is not yet the end of the story

What about preparation at a distance?
(e.g. in teleportation)

2-time EPR correlation (parametrized Polchinski)

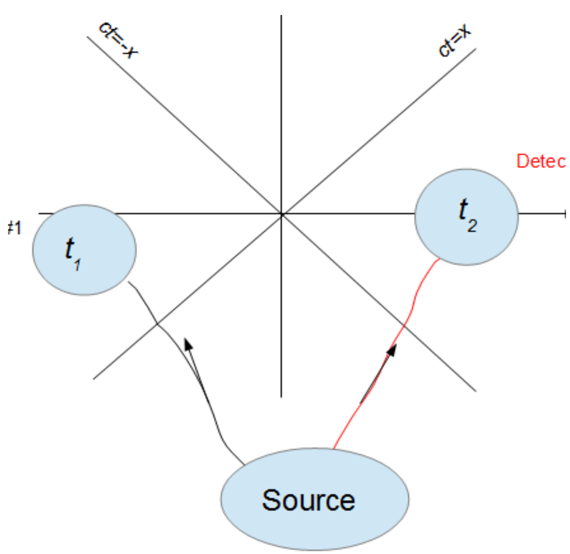


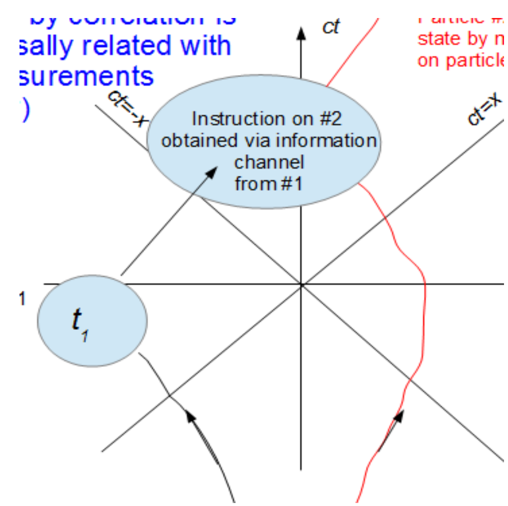
Preparation by correlation is always causally related with earlier measurements (Weinberg,Gisin?)



Russian roulette as classical system where identical subtleties occur

- We need a system where evolution of probability is probability-dependent
- There must be two parties
- We need an analogue of causally dependent or causally independent evolutions





The game

- Two players (Anna and Boris)
- Correlated by a gun where every second chamber is loaded (or a less deterministic rule)
- Boris begins – if he shoots himself then Anna can pull the trigger without risk of being killed
- If Boris survives then Anna will shoot herself unless she cheats and rotates the cylinder by one position

Now two variants

- Anna is informed about the result of Boris
- Anna is not informed about the result of Boris

- Let Anna be informed about the result of Boris

  If she cheats then the population of Annas will survive the game

- Let Anna be not informed about the result of Boris

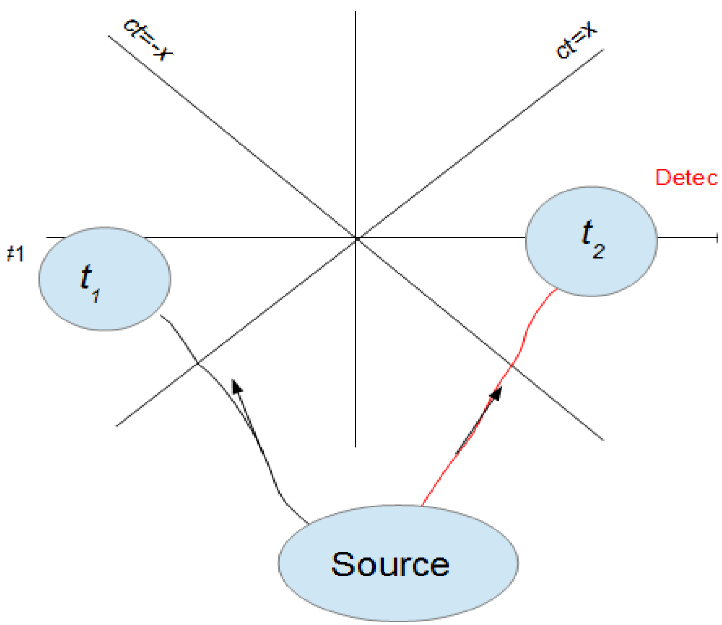
  Now cheating and non-cheating are statistically indistinguishable – in both cases 1/2 of the population of Annas will not survive

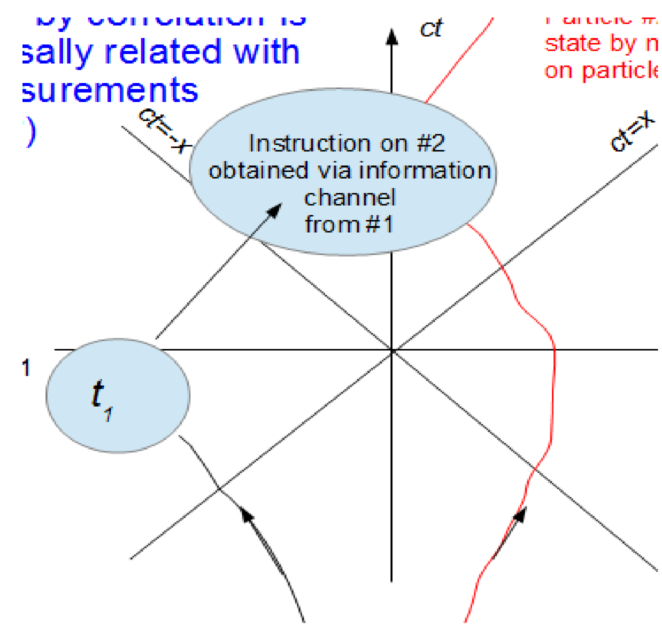
- The behavior of Anna depends on reduction of her conditional probability of getting shot

- But the reduction does not take place at the very moment Boris „makes his measurement” but only when this information reaches Anna

The moral

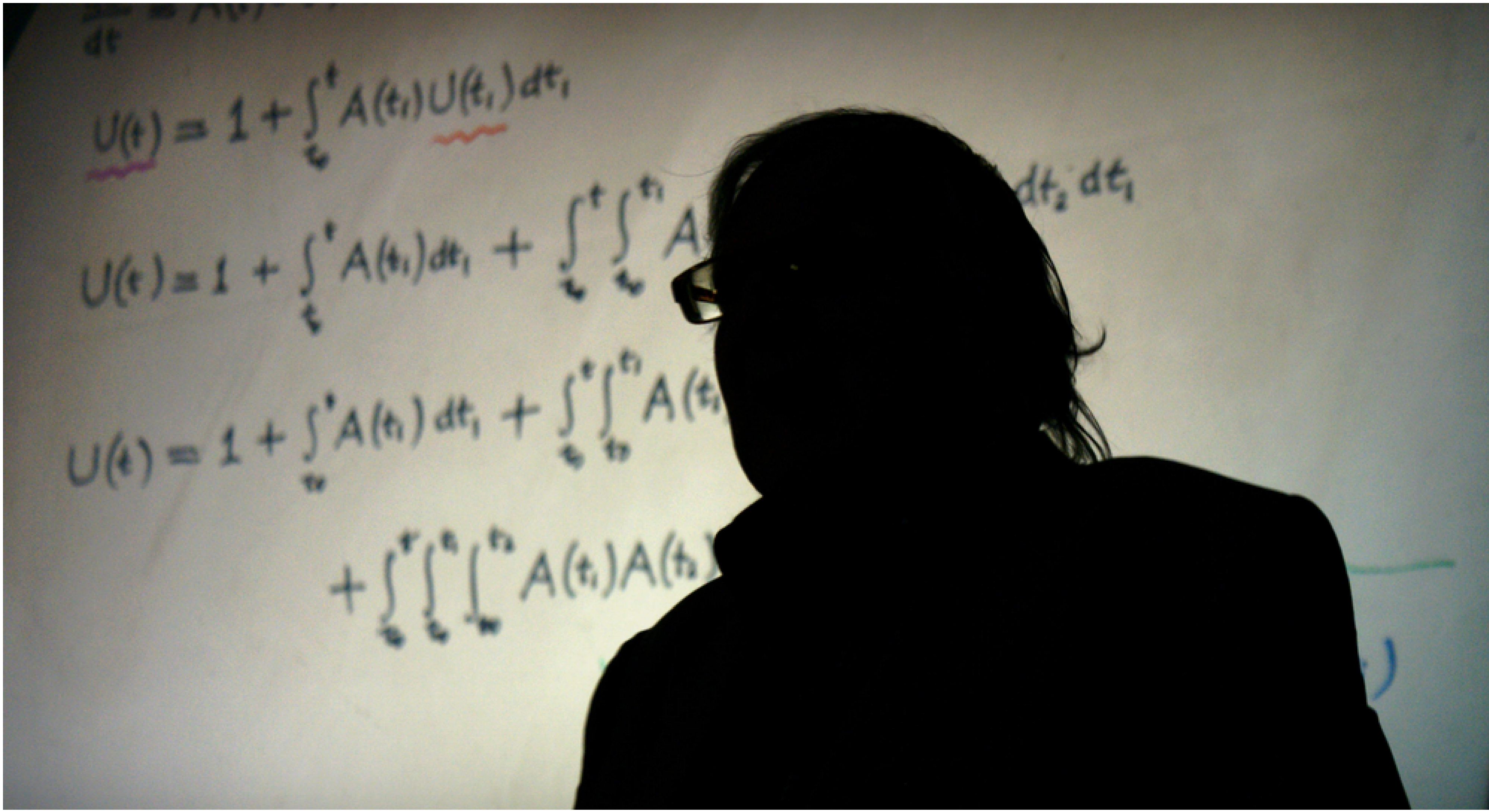
Here reduction of probability at the side of Boris does not influence „the generator” of Anna's evolution (parametrized Polchinski)





Here reduction of probability at the side of Boris does influence „the generator” of Anna's evolution (Weinberg, Gisin)

In memory of prof. Bogdan Mielnik, a man to whom I owe exceptionally much



A no-go theorem based on linearity of the Liouville equation
(essentially, Mielnik's convexity argument)

Mixed states should evolve in a linear way since the Liouville equation is always linear, even for nonlinear Hamiltonian systems. There is no consistent way of combining nonlinearity of S with linearity of vN.

Comments

- A nonlinear vN equation is a classical Hamiltonian system (formally similar to a rigid body, "Arnold top"). Any solution of a nonlinear vN equation is, in a dynamical sense, a *pure* state. Liouville equation must be treated in nonlinear QM in the same way as in other nonlinear Hamiltonian theories.
- Nonlinear vN equation is not an analog of the Liouville equation, but is a nonlinear Hamiltonan system whose solutions are Hamiltonian pure states.
- On the manifold of such states one can define a Liouville equation, which will be linear.
- Reduced density matrices obtained by reduction from pure entangled states are also pure in the Hamiltonian sense even if

$$\rho^2 \neq \rho$$

- In nonlinear QM vN equation is more fundamental than S equation!

No-go theorem based on the 0-homogeneity trick

M. Czachor, M. Kuna, "Complete positivity of nonlinear evolution: A case study", PRA 58, 128 (1998)

Step #1 (linear case)

Consider two systems 1 and 2 evolving by

$$\phi_1^t(a) = U_t a U_t^{-1} \qquad\qquad \phi_2^t(b) = b$$

with the initial state

$$\rho_{1+2}(0) = \sum_{ss'kl} \rho_{1+2}(0)_{ss'kl} |s\rangle\langle s'| \otimes |k\rangle\langle l|$$

$$= \sum_{k,l=1}^{m} a_{kl} \otimes |k\rangle\langle l|$$

The state evolves into

$$\sum_{k,l=1}^{m} a_{kl} \otimes |k\rangle\langle l| \to U_t \otimes 1_2 \left(\sum_{k,l=1}^{m} a_{kl} \otimes |k\rangle\langle l| \right) U_t^{-1} \otimes 1_2$$

The same in matrix notation

$$\rho_{1+2}(0) = \begin{pmatrix} a_{11} & \dots & a_{1m} \\ \vdots & \vdots & \vdots \\ a_{m1} & \dots & a_{mm} \end{pmatrix} \xrightarrow{\phi^t_{1+2}} \rho_{1+2}(t) = \begin{pmatrix} \phi^t_1(a_{11}) & \dots & \phi^t_1(a_{1m}) \\ \vdots & \vdots & \vdots \\ \phi^t_1(a_{m1}) & \dots & \phi^t_1(a_{mm}) \end{pmatrix}$$

The map ϕ^t_{1+2} is completely positive

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The map ϕ^t_{1+2} is completely positive

Step #2 (nonlinear case)

Theorem (Ando-Choi 1986)

A completely positive and 1-homogeneous ϕ^t_{1+2} *is linear*

Very strange!

WA Majewski, J.Phys.A, **23**, L359 (1990)

R Alicki, WA Majewski, Phys.Lett.A **148**, 69 (1990)

Any nonlinear S or vN equation can be modified according to $\hat{H}(\rho) \to \hat{H}\left(\frac{\rho}{\mathrm{Tr}\,\rho}\right)$

The resulting dynamics will be 1-homogeneous but unaffected on the orbit of normalized states.

Does it mean that any nonlinear S or vN dynamics will lead to negative probabilities when we trivially extend the system by adding a non-interacting time-independent “environment”?

If true it would exclude any reasonable nonlinear QM.

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The difficulty is at such a general level that it must be visible even in toy models.

Does it mean that any nonlinear S or vN dynamics will lead to negative probabilities when we trivially extend the system by adding a non-interacting time-independent “environment”?

If true it would exclude any reasonable nonlinear QM.

The difficulty is at such a general level that it must be visible even in toy models.

But the problem of faster-than-light communication in nonlinear QM has taught us how to combine non-interacting systems, and no problem with positivity has ever been observed...

So, where's the catch?

Toy model

$$i\dot{\rho}_1 = 2\frac{\mathrm{Tr}_1 h\rho_1}{\mathrm{Tr}_1 \rho_1}[h, \rho_1]$$

$$\dot{\rho}_2 = 0$$

Correct 2-particle Polchinski-type extension (no faster-than-light effect)

$$i\dot{\rho}_{1+2} = 2\frac{\mathrm{Tr}_{1+2} h \otimes 1_2 \rho_{1+2}}{\mathrm{Tr}_{1+2} \rho_{1+2}}[h \otimes 1_2, \rho_{1+2}]$$

Initial condition

$$\rho_{1+2}(0) = \begin{pmatrix} a & a & a & a \\ a & a+b & a+b & a \\ a & a+b & a+b & a \\ a & a & a & a \end{pmatrix}$$

Reduced density matrix

$$\rho_1(0) = \mathrm{Tr}_2 \rho_{1+2}(0) = 4a + 2b$$

1-homogeneous map

$$\rho_1(t) = U_t\big(\rho_1(0)\big)\rho_1(0)U_t\big(\rho_1(0)\big)^{-1}$$

0-homogeneous map

$$U_t\big(\rho_1(0)\big) = \exp\big[-2i\,\mathrm{Tr}_1\big(h\rho_1(0)\big)ht/\,\mathrm{Tr}_1\rho_1(0)\big]$$

One expects

$$\rho_{1+2}(t) = \begin{pmatrix} \phi_1^t(a) & \phi_1^t(a) & \phi_1^t(a) & \phi_1^t(a) \\ \phi_1^t(a) & \phi_1^t(a+b) & \phi_1^t(a+b) & \phi_1^t(a) \\ \phi_1^t(a) & \phi_1^t(a+b) & \phi_1^t(a+b) & \phi_1^t(a) \\ \phi_1^t(a) & \phi_1^t(a) & \phi_1^t(a) & \phi_1^t(a) \end{pmatrix}$$

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$$\phi_1^t(a) = U_t(a)aU_t(a)^{-1}$$ etc.

instead we get $U_t(4a+2b)aU_t(4a+2b)^{-1}$ etc.

The dynamics is not completely positive in mathematical sense, but satisfies all the physical requirements of a completely positive dynamics.

The definition of nonlinear CP maps is unphysical.

The Ando-Choi theorem is irrelevant.

Two basic facts about nonlinear S or vN dynamics

Fact #1 *For any solution of a nonlinear S or vN equation there exists a linear S or vN equation with time-dependent Hamiltionian which has the same solution.*

$$i\frac{d}{dt}|\psi_t\rangle = \hat{H}(\psi_t)|\psi_t\rangle$$

$$i\frac{d}{dt}|\psi_t\rangle = \hat{H}(t,\psi_0)|\psi_t\rangle$$

$$i\frac{d}{dt}|\psi_t\rangle = \hat{H}(t)|\psi_t\rangle$$

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Orbit-dependent Hamiltonian

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$$i\frac{d}{dt}|\psi_t\rangle \;=\; \hat{H}(t)|\psi_t\rangle$$

Time-dependent Hamiltonian (varying from orbit to orbit)

Two basic facts about nonlinear S or vN dynamics

Fact #2 *If all measurements are reducible to those of position (Feynman), then linearity is a gauge-dependent property (Doebner-Goldin 1992). Two theories that yield the same* $\rho_t(x)$ *are indistinguishable*

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Doebner-Goldin nonlinear gauge transformation

$$\begin{pmatrix} A \\ B \end{pmatrix} \mapsto \begin{pmatrix} 1 & 0 \\ \gamma & \lambda \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} =: \begin{pmatrix} A' \\ B' \end{pmatrix}$$

$$\psi = \exp[A + iB] \mapsto \exp[A' + iB'] =: \psi'$$

$$\psi(x) \mapsto$$
$$N_{\lambda,\gamma}[\psi](x) = |\psi(x)| \exp\left[i\lambda \arg\psi(x) + i\gamma \ln|\psi(x)|\right]$$

Example: $\psi' = N_{1,\gamma}[\psi]$

$$\rho_\psi = |\psi|^2 = |\psi'|^2 = \rho_{\psi'}$$

$$i\hbar\partial_t\psi' = \Big(-\frac{\hbar^2}{2m}\Delta + V\Big)\psi' + \frac{\hbar^2\gamma}{4m}\Big(iR_2 + 2R_1 - 2R_4\Big)\psi'$$
$$-\frac{\hbar^2\gamma^2}{8m}\Big(2R_2 - R_5\Big)\psi' - \frac{1}{2}\dot{\gamma}\ln\rho_{\psi'}\psi'$$

Białynicki-Birula—Mycielski logarithmic term

$$R_1 = R_1[\rho_{\psi'}, \vec{j}_{\psi'}] = \frac{m}{\hbar}\frac{\vec{\nabla}\cdot\vec{j}_{\psi'}}{\rho_{\psi'}}$$

$$R_2 = R_2[\rho_{\psi'}, \vec{j}_{\psi'}] = \frac{\Delta\rho_{\psi'}}{\rho_{\psi'}}$$

$$R_4 = R_4[\rho_{\psi'}, \vec{j}_{\psi'}] = \frac{m}{\hbar}\frac{\vec{j}_{\psi'}\cdot\vec{\nabla}\rho_{\psi'}}{\rho^2_{\psi'}}$$

$$R_5 = R_5[\rho_{\psi'}, \vec{j}_{\psi'}] = \frac{\vec{\nabla}\rho_{\psi'}\cdot\vec{\nabla}\rho_{\psi'}}{\rho^2_{\psi'}}$$

Doebner-Goldin gauge terms

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Doebner-Goldin gauge terms

But this is linear SE in nonlinear disguise!

PHYSICAL REVIEW A VOLUME 20, NUMBER 2 AUGUST 1979

Proposed neutron interferometer test of some nonlinear variants of wave mechanics

Abner Shimony

$$i\hbar \frac{\partial \psi(\vec{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\Delta + U(\vec{r},t)\right)\psi + F(|\psi|^2)\psi\,, \qquad (1)$$

$$i\hbar \frac{\partial \psi(\vec{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\Delta + U(\vec{r},t)\right)\psi + b\ln(a^n|\psi|^2)\psi\,. \qquad (2)$$

the phase shift:

$$\Delta \simeq \int_P^{P'} ds \left(\frac{2m}{\hbar^2}[E - F(|\alpha\psi_I|^2)]^{1/2} - \frac{2m}{\hbar^2}[E - F(|\psi_I|^2)]^{1/2}\right)$$

$$\simeq (d/\hbar)(m/2E)^{1/2}[F(|\psi_I|^2) - F(|\alpha\psi_I|^2)]$$

$$\simeq (\tau/\hbar)[F(|\psi_I|^2) - F(|\alpha\psi_I|^2)]\,,$$

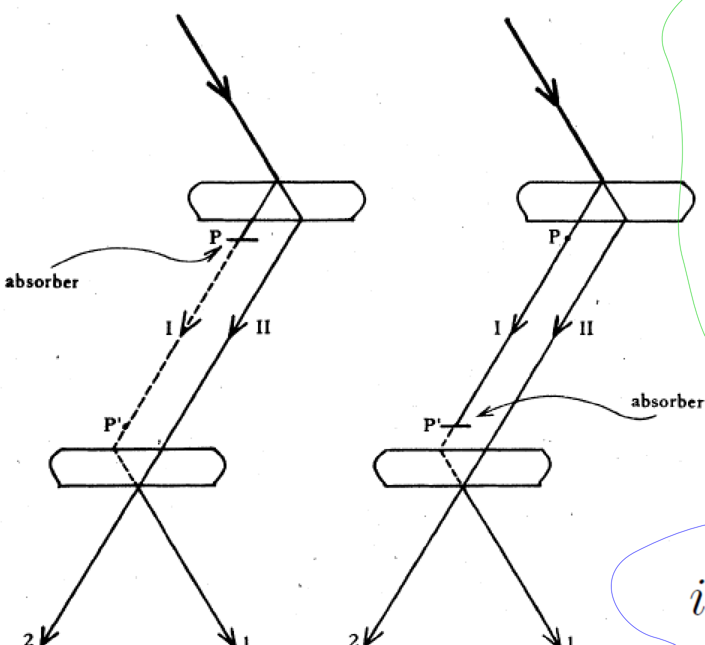

FIG. 1. Dashed line indicates the attenuation of the amplitude of a beam by a factor α. Output beams I and II are directed towards the neutron counters.

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Proposed neutron interferometer test of some nonlinear variants of wave mechanics

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$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}=\left(-\frac{\hbar^2}{2m}\Delta+U(\vec{r},t)\right)\psi+F(|\psi|^2)\psi\,, \quad (1)$$

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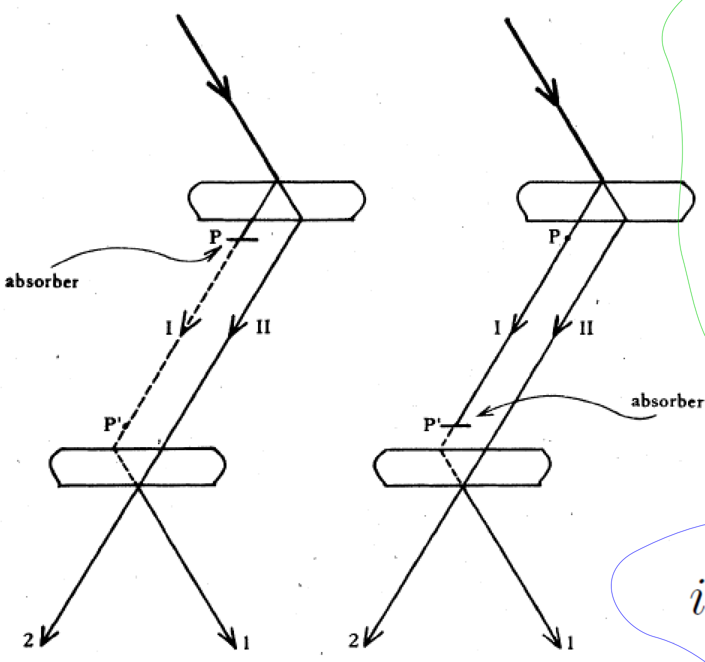

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Phase

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Argumentation based on homogeneity of *F* is dangerous

$$\cdots + F(|\psi(x)|^2)\psi(x) \longrightarrow \cdots + F\left(\frac{|\psi(x)|^2}{\langle\psi|\psi\rangle}\right)\psi(x)$$

does not change the dynamics of normalized states, but turns the Shimony phase shift into

$$F\left(\frac{|\psi(x)|^2}{\langle\psi|\psi\rangle}\right) - F\left(\frac{|\alpha\psi(x)|^2}{\langle\alpha\psi|\alpha\psi\rangle}\right) = 0$$

for any *F* !!!!

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Homework:

Extend the argument to nonlinear von Neumann equations...