XXXIX WORKSHOP ON GEOMETRIC METHODS IN PHYSICS

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Faculty of Mathematics, University of Białystok
Ciołkowskiego 1M, 15-245 Białystok, Poland

e-mail: wgmp@uwb.edu.pl

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LIST OF ABSTRACTS

Plenary lectures

1. **Almut BEIGE** — *University of Leeds, United Kingdom*

A fresh perspective on local photons and the Casimir effect

The Casimir effect [1] predicts the emergence of an attractive force between two parallel, highly reflecting plates in vacuum. This effect plays a vital role in various fields of physics, from quantum field theory and cosmology to nanophotonics and condensed matter physics. Nevertheless, Casimir forces still lack an intuitive explanation and current derivations still have severe weaknesses. For example, the standard derivation relies on regularisation procedures to remove infinities. Moreover, current cavity models often impose boundary conditions on the mirror surface, thereby restricting the electromagnetic field inside a cavity to standing wave modes with discrete frequencies. Such standing wave mode models cannot reproduce the typical behaviour of Fabry–Perot cavities, since they cannot account for the direction from which light enters [2]. They also contradict recent nano-cavity experiments which place light inside cavities with mirror distances well below optical wavelengths [3].

In this talk, I will discuss alternative approaches to light quantisation [4, 5, 6]. Starting from special relativity and treating space and time coordinates equivalently, we overcome no-go theorems of quantum electrodynamics and obtain a local relativistic quantum description of the electromagnetic field in free space [7]. When extended to cavities, our approach can be used for example to calculate Casimir forces without the introduction of cut-off frequencies directly in position space. Our local relativistic description provides new insight into the Casimir effect by attributing it to interference effects of evanescent fields.

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2. **Kenny DE COMMER** — *Vrije Universiteit Brussel, Belgium*

Quantizing real semisimple Lie groups

Let $\mathfrak{g}$ be a semisimple complex Lie algebra with compact real form $\mathfrak{u}$. The real forms of $\mathfrak{g}$ are in one-to-one connection with the Lie algebra involutions of $\mathfrak{u}$, which in turn are determined by the inclusion of their associated fixed point subalgebra $\mathfrak{k}$ in $\mathfrak{u}$. The latter inclusions are called compact symmetric pairs, and work of (among others) Noumi, Letzter and Kolb has shown that they admit quantizations as coideal subalgebras of Drinfeld–Jimbo quantized enveloping algebras. This has led to a very rich theory with connections to topological quantum field theory and the categorification programme. In this talk, we will show how quantum symmetric pairs lead to a general theory of quantized semisimple real Lie groups. We illustrate the general theory with the particular case of quantum $SL(2,\mathbb{R})$. This is partly based on joint work with Joel Dzokou Talla.

3. **Paweł KASPRZAK** — *Uniwersytet Warszawski, Poland*

Quantum correlations on quantum spaces: RFD, LP, HR and SEP

In the theory of operator algebras the concepts of residual finite dimensional (RFD) and lifting property (LP) are well known and often studied: e.g. the universal C^*-algebra $C^*(F_2)$ of a free group on two generators is RFD and has LP. Hyperrigidity (HR) in turn refers to a position of an operator system S in a C^*-algebra A: for example position of the canonical generating operator system S inside $C^*(F_2)$ is HR. In my talk I will explain how RFD, LP and HR arise in the theory of quantum correlations (QC). Moreover I will introduce yet another property which we call SEP (Strong Extension Property) and explain its role in the theory of QC.

Virtual plenary lectures

1. **Esteban ANDRUCHOW** — *Universidad Nacional de General Sarmiento and Instituto Argentino de Matemática "Alberto P. Calderón", Argentina*

Grassmann geometry in spaces of functions

First, we will briefly survey the main aspects of the Grasmann manifold of a Hilbert space. Let $\mathcal{H}$ be a complex Hilbert space, to each closed subspace $\mathcal{S} \subset \mathcal{H}$ corresponds a unique orthogonal projection $P_{\mathcal{S}}$ onto $\mathcal{S}$. Thus, $Gr(\mathcal{H})$, the set of closed subspaces of of $\mathcal{H}$, can be parametrized

$$Gr(\mathcal{H}) \simeq \{P \in \mathcal{B}(\mathcal{H}) : P = P^2 = P^*\},$$

by means of $P \sim R(P)$ (the range of P). Here $\mathcal{B}(\mathcal{H})$ denotes the algebra of bounded linear operators in $\mathcal{H}$. The benefit of this approach is that $Gr(\mathcal{H})$ is naturally a closed subset of a Banach space: $Gr(\mathcal{H})$ turns out to be a complemented real analytic submanifold of $\mathcal{B}(\mathcal{H})$, and therefore the tangent spaces of $Gr(\mathcal{H})$ inherit a natural ambient norm. In geometric terms, $Gr(\mathcal{H})$ has a natural Finsler metric (non Riemannian, non smooth). The natural action of the unitary group of $\mathcal{U}(\mathcal{H})$ of $\mathcal{H}$ on subspaces, namely

$$U \bullet \mathcal{S} = U(\mathcal{S}) \quad (U \in \mathcal{U}(\mathcal{H}), \mathcal{S} \subset \mathcal{H})$$

translates into

$$U \bullet P_{\mathcal{S}} = U P_{\mathcal{S}} U^*.$$

A natural linear connection was introduced in $Gr(\mathcal{H})$ ([5], [4]). The geodesics of this connection can be explicitely computed. They are of the form

$$\delta(t) = e^{itX} P e^{-itX},$$

where $X = X^*$ is *co-diagonal* with respect to P: $PXP = P^{\perp} X P^{\perp} = 0$. Porta and Recht [5] noted that if $\|P - Q\| < 1$ ($P, Q \in Gr(\mathcal{H})$) then there exists a unique geodesic joining P and Q, and this geodesic has minimal length (for the Finsler metric described above, i.e., for the usual norm in $\mathcal{B}(\mathcal{H})$). Later (see [1]) this was refined:

(a) There exists a geodesic between $P_{\mathcal{S}}$ and $P_{\mathcal{T}}$ ($\mathcal{S}, \mathcal{T} \subset \mathcal{H}$) if and only if

$$\dim(\mathcal{S} \cap \mathcal{T}^{\perp}) = \dim(\mathcal{S}^{\perp} \cap \mathcal{T}).$$

This (eventually non unique) geodesic can be chosen minimal. The length of this geodesic (if parametrized in the interval $[0,1]$) equals $\|[X,P]\|$ (here $[X,P] = XP - PX$).

(b) The geodesic is unique if and only if

$$\mathcal{S} \cap \mathcal{T}^{\perp} = \mathcal{S}^{\perp} \cap \mathcal{T} = \{0\}.$$

In the second part we examine concrete examples of Hilbert spaces and closed subspaces. This part of the talk is joint work with E. Chiumiento, G. Larotonda and A. Varela [2], [3]. For instance, we consider:

- $\mathcal{H} = L^2(\mathbb{T})$, $\mathcal{S}, \mathcal{T}$ of the form $\varphi H^2(\mathbb{T})$, where $H^2(\mathbb{T})$ is the Hardy space of the unit circle and φ is a continuous unimodular function. It is shown that there exists a unique geodesic between $\varphi H^2(\mathbb{T})$ and $\psi H^2(\mathbb{T})$ if and only if $w(\varphi) = w(\psi)$ (w=winding number), if and only if $N(T_{\bar{\varphi}\psi}) = \{0\}$. Here $T_{\bar{\varphi}\psi}$ is the Toeplitz operator with symbol $\bar{\varphi}\psi$. In such case a unique selfadjoint operator $X_{\varphi,\psi}$ is defined (the exponent in the geodesic joining both subspaces). The minimality propery of this geodesics permits one to establish operator norm inequalities which we shall specify in the talk.
- $\mathcal{H}$ a reproducing kernel Hilbert space of analytic functions: among these $H^2(\mathbb{T})$, or the Bergman space $B_2(\mathbb{D})$ of the unit disk; the subspaces are of the form

  $$\mathcal{Z}_{\mathbf{a}} = \{f \in \mathcal{H} : f|_{\mathbf{a}} = 0\},$$

  where $\mathbf{a} = \{a_1, a_2, \dots\}$ is a finite or infinite (countable) set. For finite sets, the answer to the question of existence of geodesics between subspaces of this type is affirmative for the Hardy space, if and only if both sets have the same cardinality; for the Bergman space, for $n \geq 3$, there are negative examples. We have some results for infinite sets, and again, operator norm inequalities are deduced from the minimality condition.

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2. **Iwo BIAŁYNICKI-BIRULA** — *Centum Fizyki Teoretycznej, PAN, Poland*

Phenomenon of Backflow

Backflow is a counterintuitive phenomenon discovered first in quantum mechanics. However I will show that, contrary to the statements made by many authors, the backflow is not a nonclassical effect. The backflow is a characteristic feature of solutions of the wave equations: quantum and classical. I will present simple solutions of the Maxwell equations and the Dirac equation where the backflow phenomenon is clearly seen. I will describe backflow in relativistic theories but this phenomenon can occur in the solutions of all kinds of wave equations: quantum and classical. I believe that it can be seen even in water waves on a pond.

3. **Hitoshi KONNO** — *Tokyo University of Marine Science and Technology, Japan*

Elliptic Quantum Toroidal Algebra $U_{q,t,p}(gl_{1,tor})$ and affine quiver gauge theories

We introduce the elliptic quantum toroidal algebra $U_{q,t,p}(gl_{1,tor})$. After giving some representations including the level $(0,0)$ representation realized by using the elliptic Ruijsenaars difference operator, we construct intertwining operators of the $U_{q,t,p}(gl_{1,tor})$-modules w.r.t. the Drinfeld comultiplication. We then show that $U_{q,t,p}(gl_{1,tor})$ gives a realization of the affine quiver W-algebra $W_{q,t}(\Gamma(\widehat{A}_0))$ proposed by Kimura–Pestun. This realization turns out to be useful to derive the Nekrasov instanton partition functions, i.e. the χ_y- and elliptic genus, of the 5d and 6d lifts of the 4d $\mathcal{N} = 2^*$ theories and provide a new Alday–Gaiotto–Tachikawa correspondence.

4. **Karl-Hermann NEEB** — *Friedrich-Alexander University, Department of Mathemtatics, Germany*

Geometric aspects of the modular theory of operator algebras

Modular theory is an important aspect of the theory of operator algebras and in the theory of local observables in Algebraic Quantum Field Theory (AQFT). It creates a one-parameter group of modular automorphisms from a single state and, sometimes, this group represents the flow of time (the dynamics) in a space-time domain. We study this question from a Lie group perspective by asking questions like: Which one-parameter groups of Lie groups can arise in this context as modular groups? This leads us to real standard subspaces of a complex Hilbert space, to antiunitary representations and to nets of standard subspaces on causal symmetric spaces.

This is joint work with Gestur Olafsson (Baton Rouge) and Vincenzo Morinelli (Rome).

5. **Gestur OLAFSSON** — *Louisiana State University, USA*

Reflection positivity on the sphere and related topics

6. **Tudor RATIU** — *Shanghai Jiao Tong University, China*

Cheeger–Simons differential character-valued momentum maps and applications

In order to address Donaldson's topological obstructions for the existence of momentum maps for certain diffeomorphism and gauge groups, an extension of the classical momentum map is introduced, capable to store discrete topological information. This talk will explain how this momentum map was introduced based on the model of the momentum map for Poisson Lie group actions and will discuss some of its applications.

7. **Sergei TABACHNIKOV** — *Pennsylvania State University, USA*

   Integrable transformations on centroaffine polygons

   One can interpret the famous Korteweg–de Vries equation as a completely integrable evolution on centroaffine curves. Accordingly, symmetries (the Bäcklund transformation) of the KdV equation also can be realized as transformations of centroaffine curves. I shall discuss a discrete version of these transformations, where the curves are replaced by polygons. The focus will be on the geometrical aspects of the problem.

8. **Alexander VESELOV** — *Loughborough University, United Kingdom*

   On integrability, geometrization and knots

   The talk is a review of the relation between Thurston's geometrization and Liouville integrability. Using as the main example the geodesic flows on the 3-folds with $SL(2,\mathbb{R})$-geometry in Thurston's sense, I will show that the corresponding phase space contains two open regions with integrable and chaotic behaviour respectively.

   A particular case of such 3-folds the modular quotient $SL(2,\mathbb{R})/SL(2,\mathbb{Z})$, which is known to be equivalent to the complement in 3-sphere of the trefoil knot. I will show that the remarkable results of Ghys about modular and Lorenz knots can be naturally extended to the integrable region, where these hyperbolic knots are replaced by the cable knots of trefoil.

   The talk is partly based on a recent joint work with Alexey Bolsinov and Yiru Ye.

9. **Xiaomeng XU** — *Peking University, China*

   Stokes phenomenon in geometry and physics

   This talk includes an introduction to the Stokes phenomenon of different equations, and then explores its relation with Yang–Baxter equations, integrable systems and so on.

Contributed lectures

1. **Krzysztof BARDADYN** — *Uniwersytet w Białymstoku, Poland*

   Simplicity of L_p-operator algebra crossed products

   In 2010's Phillips initiated a program whose aim is to generalize large parts of the modern C^*-algebra theory to operator algebras acting on L_p-spaces. Since then a number of papers appeared establishing some strong results in this direction, but there are still some fundamental open problems. One of them, explicitly stated in Phillips (2013) and Gardella-Lupini (2017), is a characterization of simplicity of L_p-crossed products, and in particular a generalization of the C^*-algebraic results from the seminal paper of Archbold and Spielberg (1993).

   In my talk I will present a solution to the aforementioned problem for an action of a discrete group on a locally compact Hausdorff space. The techniques work also for étale groupoids and give a characterization of the intersection property and simplicity of the correspodning L_p-operator algebras in terms of topological freeness.

   Based on joint work with Bartosz Kwaśniewski and Andrew McKee.

2. **Daniel BELTIŢĂ** — *Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Romania*

On the solvable Lie groups whose regular representation is a factor representation

We present an intrinsic characterization of the solvable Lie groups whose regular representation is a factor representation. The von Neumann algebras of these Lie groups turn out to be isomorphic to the hyperfinite factor of type II_∞. The key to these results is the relation between square-integrable representations and the coadjoint action of solvable Lie groups. The presentation is based on joint work with Ingrid Beltita.

3. **David BERMÚDEZ** — *CINVESTAV, Mexico*

Factorization method and new potentials from the inverted oscillator

One of the most important contributions of Prof. Bogdan Mielnik was contained in his seminal paper "Factorization method and new potentials with the oscillator spectrum". When we wrote this paper back in 2013, we intended to pay tribute to Prof. Mielnik legacy. In this work, we study the inverted (or repulsive) oscillator, we applied several of the known techniques of the factorization method to obtain new potentials. We end by commenting on the consequences of this work and more broadly on the legacy of Prof. Mielnik in Mexico.

4. **Petr BLASCHKE** — *Silesian University in Opava, Czech Republic*

Pedal coordinates, solar sail orbits and other force problems

Pedal coordinates provide a natural framework for studying force problems of classical mechanics in the plane. The trajectory of a test particle under the influence of central and Lorentz-like forces can be translated into pedal coordinates at once without the need of solving any differential equation. We generalize this result to cover more general force laws and also show the advantage of pedal coordinates in certain variational problems. These enable us to link together many dynamical systems as well as problems of the calculus of variation.

5. **Nora BRETON** — *CINVESTAV, Mexico*

Hidden Symmetries of the type D solutions of the Einstein Equations

In this contribution we present the Killing tensor and its associated conserved quantity; this allows to integrate in full form the geodesics for charged particles in the type D spacetime.

6. **Luca CAMPOBASSO** — *Łódź University of Technology, Poland*

Klein paradox in the Wigner phase space formalism

The Klein paradox is considered in the context of quantum mechanics in phase space. The external degrees of freedom are represented together with the internal degrees of freedom in the Hilbert space $L^2(\mathbb{R}) \otimes \mathbb{C}^2$. The tunneling coefficients are extrapolated with the help of a continuity equation newly formulated in terms of a density operator.

7. **Goce CHADZITASKOS** — *Czech Technical University in Prague, Czech Republic*

Asymmetric harmonic oscilator

The solution of one–dimensional asymmetric quantum harmonic oscillator is presented. We study the spectrum of a quantum harmonic oscillator, which has a spring constant k_- to the left of the equilibrium position and a spring constant k_+ to the right of the equilibrium position. The explicit form of eigenfunctions, the way how to calculate the eigenvalues, and the properties of the eigenfunctions are discussed.

8. **Alonso CONTRERAS-ASTORGA** — *CINVESTAV, Mexico*

Two aspects of Klein tunneling in graphene: supersymmetry and Dirac structure beyond the linear approximation

This talk is two-fold degenerate: first, we will present how the Super-Klein tunneling phenomenon emerges from supersymmetry; second, we will talk about the generalized Dirac structure beyond the linear regime, how those new terms have effects even at low energies in Klein tunneling, and why they could help us to model quantum gravitational analogs in tabletop experiments.

9. **Marek CZACHOR** — *Politechnika Gdańska, Poland*

Malignant and non-malignant nonlinearity in quantum mechanics

I will describe the current status of entangled states in nonlinear quantum mechanics.

10. **Tomasz CZYŻYCKI** — *Uniwersytet w Białymstoku, Poland*

Generalized Transforms of Affine Weyl Groups

Discrete transforms of Weyl orbit functions on finite fragments of shifted dual root lattices are established. The congruence classes of the dual weight lattices intersected with the fundamental domains of the affine Weyl groups constitute the point sets of the transforms. The shifted weight lattices intersected with the fundamental domains of the extended dual affine Weyl groups form the sets of labels of Weyl orbit functions. The coinciding cardinality of the point and label sets and the corresponding discrete orthogonality relations of Weyl orbit functions are demonstrated. The forward and backward discrete Fourier–Weyl transforms together with the associated interpolation and Plancherel formulas are presented. The unitary transform matrices of the discrete transforms are exemplified for the case A_2.

11. **Alina DOBROGOWSKA** — *Uniwersytet w Białymstoku, Poland*

Generalization of the concept of classical r-matrix to Lie algebroids

We present some new constructions of Lie algebroids starting from vector fields on manifold M. The tangent bundle TM possess a natural structure of Lie algebroid, but we use these fields to construct a collection of interesting new algebroid structures. Next, we show that these constructions can be used in a more general situation, starting from an arbitrary Lie algebroid over M. In the final step, we show that after limiting ourselves to Lie algebras these formulas as a special case contain brackets well known in theory of classical r-matrices. We can think of our constructions as extending the concept of classical r-matrices to Lie algebroids. Several examples illustrate the importance of these constructions.

12. **Ziemowit DOMAŃSKI** — *Poznań University of Technology, Institute of Mathematics, Poland*

Deformation quantization on the cotangent bundle of a Lie group

A complete theory of non-formal deformation quantization on the cotangent bundle of a Lie group is presented. The starting point of the construction is a pre-C*-algebra of observables being a deformation of the classical pre-C*-algebra of observables. The deformed product from this algebra (the star-product) is introduced by means of an appropriate integral formula. Basic properties of the star-product are proved and the pre-C*-algebra is extended to a Hilbert algebra and an algebra of distributions. The algebra of distributions contains functions representing all observables interesting from a physical point of view. An operator representation in position space is constructed. The theory is illustrated on the example of the quantization of a rigid body.

13. **Andrzej DRAGAN** — *Uniwersytet Warszawski, Poland*

Title to be announced

14. **Maciej DUNAJSKI** — *University of Cambridge, United Kingdom*

Quantum state reduction, and Newtonian twistor theory

I will discuss the equivalence principle in quantum mechanics in the context of Newton–Cartan geometry, and non–relativistic twistor theory. This is joined work with Roger Penrose.

15. **David FERNÁNDEZ** — *CINVESTAV, Mexico*

Factorization method: Bogdan Mielnik's contributions

We will briefly review the factorization method, which is the simplest algebraic technique to find exactly solvable models in quantum mechanics. We will describe Bogdan Mielnik's important contributions to the method, which have inspired the work of many people in the world, particularly at Physics Department of Cinvestav.

16. **Gerald GOLDIN** — *Rutgers University, United States*

Predicting "Anyons": Implications of History for Science

"Anyons" are quantum particles or excitations in two space dimensions whose exchange statistics can be intermediate between bosons and fermions. They are associated with surface phenomena in the presence of magnetic flux. Theoretical applications are numerous, and in 2020 experimentalists succeeded in creating anyonic excitations. Their prediction four decades earlier, which required fundamental changes in our understanding of quantum statistics, is often attributed exclusively and incorrectly to Frank Wilczek. I will outline the actual history, from predecessor ideas to the first clear, independent predictions in papers by Leinaas & Myrheim (1977) and Goldin, Menikoff, & Sharp (1980-81), followed by Wilczek's 1982 articles and subsequent important group-theoretical insights by Goldin, Menikoff, & Sharp (1983, 1985). Then I shall discuss some wider implications for physics teaching, presentation to the public, and integrity in science. Why did such an easy concept elude physicists for so long? What then led to independent predictions within a short time of each other? What can we learn from this about science education? I shall conclude by addressing the painful implications of scientists' and journalists' systemic failure or refusal to accurately attribute scientific achievements — breaches of integrity occurring even when there is no dispute. The “anyon” case is not unique. The social consequences of such failure include non-recognition and career obstacles disproportionately hurting women, minorities, and scientists in developing countries, as well as intimidation and disillusionment of younger scientists.

17. **Md Fazlul HOQUE** — *Czech Technical University in Prague, Czech Republic*

Quadratic algebras and spectrum of superintegrable systems

Algebraic methods are powerful tools in classical and quantum mechanics. Superintegrable systems are an important class of classical and quantum systems which can be solved using algebraic approaches. In this talk, I present higher rank quadratic algebra of the N-dimensional quantum Smorodinsky–Winternitz system, which is a maximally superintegrable and exactly solvable model. It is shown that the model is multiseparable and the wave function can be expressed in terms of Laguerre and Jacobi polynomials. We present a complete symmetry algebra $\mathcal{SW}(N)$ of the system, which it is a higher-rank quadratic one containing Racah algebra $\mathcal{R}(N)$ as subalgebra. The substructures of distinct quadratic $Q(3)$ algebras and their related Casimirs are also studied. The energy spectrum of the N-dimensional Smorodinsky–Winternitz system is obtained algebraically via the different set of subalgebras based on the Racah algebra $\mathcal{R}(N)$.

18. **Jiří HRIVNÁK** — *Czech Technical University in Prague, Czech Republic*

Electron in triangular graphene dots

Two types of honeycomb lattice Fourier–Weyl transforms associated to the irreducible crystallographic root system A_2 are utilized to study electronic properties of triangular graphene quantum dots. The triangular dots with armchair and zigzag edges are represented by two fundamentally different geometric configurations of the honeycomb lattice inside the fundamental domain of the A_2 affine Weyl group. The Schrödinger equations produced by tight-binding models of electron propagation with the nearest and next-to-nearest couplings are exactly solved through armchair and zigzag honeycomb Fourier–Weyl transforms. The inclusion of boundary conditions in the tight-binding Hamiltonians provides four types of electronic stationary states expressed via the honeycomb Weyl orbit functions. The contrasting behavior of the armchair and zigzag electronic probability densities is demonstrated. This is a joint work with Lenka Motlochová.

19. **Jerzy KIJOWSKI** — *Center for Theoretical Physics, Poland*

Arrival time in quantum mechanics: my long-standing disputes with Bogdan Mielnik

20. **Andrzej KRASIŃSKI** — *N. Copernicus Astronomical Center, Polish Academy of Sciences, Poland*

Memories about Bogdan Mielnik

This will be a collection of my personal memories, somewhat extended toward Bogdan’s extra-scientific activities. I was at first a student at Bogdan’s lecture course, then, for one semester, his teaching assistant, and then, on several occasions for many years, a fellow-participant at seminars and conferences. The talk will be a modest contribution to Bogdan’s portrait as a human being.

21. **Milan NIESTIJL** — *Delft University of Technology, Netherlands*

Positive Energy Representations of Gauge Groups With Support at a Fixed Point

Let $\mathcal{K} \to M$ be a locally trivial smooth bundle of Lie groups equipped with an action of some Lie group P by bundle automorphisms. Complementing recent progress B. Janssens and K.H. Neeb on the case where the P-action on M has no fixed-points, projective unitary representations $\overline{\rho}$ of the locally-convex Lie group $\mathcal{G} := \Gamma_c(\mathcal{K})$ are studied which are of "positive energy" and factor entirely through the germs at some fixed point $a \in M$ of the P-action. Under suitable assumptions, it is shown that the kernel of a particular quadratic form on $\mathbb{R}[[x_1, \cdots, x_d]]$ generates an ideal in $\mathfrak{G} := \mathcal{L}\rangle\rceil(\mathcal{G})$ on which the derived representation $d\rho$ must vanish. This leads in particular to sufficient conditions for $d\rho$ to factor through a finite jet space $J^k_x(\mathcal{K})$ or through $J^\infty_a(N, \mathcal{K})$ for some usually lower-dimensional submanifold N. Some examples are considered. If time permits, we will have a closer look at the special case where $P = S^1$.

22. **Petr NOVOTNÝ** — *Czech Technical University in Prague, Czech Republic*

Quantum particle on G_2 dual weight lattice

23. **Anatolij PRYKARPATSKI** — *Cracow University of Technology, Poland*

On the Dark Equations, the related integrability theory and applications

Some twenty years ago, a new class of nonlinear dynamical systems, called "dark equations" was introduced by Boris Kupershmidt, and shown to possess unusual properties that were not particularly well-understood at that time. Later, in related developments, some Burgers-type and also Korteweg–de Vries type dynamical systems were studied in detail, and it was proved that they have in many cases a finite number of conservation laws, a linearization and in some sense degenerate Lax representations, among other properties. In our lectures, we provide a description of a class of self-dual dark-type (or just, dark, for short) nonlinear dynamical systems, which a priori allows their quasi-linearization, whose integrability can be ffectively studied by means of a geometrically motivated gradient-holonomic approach. Moreover, we study a slightly modified form of a self-dual nonlinear dark dynamical system on a functional manifold, whose integrability was recently analyzed by other methods. Not only did this dynamical system appear to be Lax integrable, it also proved to have a rich mathematical architecture including compatible Poisson structures and an infinite hierarchy of nontrivial mutually commuting conservation laws. We also will demonstrate and prove these properties using the gradient-holonomic integrability scheme devised in our prior work with several collaborators. Finally, we will summarize presented results and indicate some possible future research directions and applications in control theory and other fields of modern mathematical physics.

24. **Maciej PRZANOWSKI** — *Politechnika Łódzka, Poland*

Canonical photon position operator with commuting components

It is shown that, up to unitary transformation, there exists a uniquely defined canonical photon position operator with commuting components.This operator is unitary equivalent to the one found by Margaret Hawton in 1999. It has a simple geometrical interpretation as defined by a Weitzenboeck connection in tangent bundle or by a flat connection on some trivial vector bundle. However, physical interpretation of such a photon position operator is not clear.

25. **Stefan RAUCH-WOJCIECHOWSKI** — *Linköping University, Sweden*

When knowledge of one integral of motion is sufficient for integrability?

A system of differential equations is integrable by quadratures when solutions can be expressed using integrations, algebraic operations and taking inverse functions. A general autonomous system of n equations requires knowledge of $n-1$ integrals of motion and one extra integration determines time dependence of solutions.

Usually the notion of integrability is associated with Hamiltonian integrability in $2n$ dimensional phase space when knowledge of only n independent and involutive integrals of motion is sufficient for Liouville integrability. This is due to the special nature of the vector-field which is determined by one function, the Hamiltonian.

But there are known systems of equations when only 2 or 1 integral of motion suffice for integrability due to special algebraic form of the equation. There is a trade off between number of integrals and algebraic features of equations.

The purpose of this talk is to make you aware of elegant, little known classes of n 2^{nd} order Newton equations for which only 1 **quadratic** in velocities integral of motion implies existence of **further** $n-1$ **integrals**. This renders equations integrable and solvable by quadratures through separation of variables. These equations have the triangular form: $d^2q_r/dt^2 = M_r(q_1,\dots,q_r)$, $r=1,\dots,n$ where the r^{th} equation depends only on the preceding variables q_j, $j=1,\dots,r$.

26. **Jaromir TOSIEK** — *Łódź University of Technology, Poland*

1-D Dirac equation in phase space quantum mechanics

A phase space counterpart od the 1-D Dirac equation is presented. The continuity equation in phase space quantum mechanics in both: the nonrelativistic and the relativistic case is discussed.

27. **Alice Barbora TUMPACH** — *Laboratoire Painlevé & Pauli Institut, Austria*

Banach Poisson Lie groups

We investigate the Banach Poisson Lie group structures on a family of unitary groups consisting of unitary operators on a separable Hilbert space, where the model space is the space of skew-Hermitian operators belonging to some Schatten classes, or some combination of Schatten classes. We will emphasize the following examples:

(a) the unitary group of operators which differs from the identity by a Hilbert-Schmidt operator,

(b) the restricted unitary group and the relation between its Poisson geometry and the KdV hierarchy,

(c) the whole unitary group of bounded operators.

28. **Elwira WAWRENIUK** — *Uniwersytet w Białymstoku, Poland*

Symplectic realizations of $\mathbf{e}(3)^*$

The Lie–Poisson space $\mathbf{e}(3)^* \cong \mathbb{R}^3 \times \mathbb{R}^3$ dual to the Lie algebra $\mathbf{e}(3)$ of the Euclidean group $E(3)$ is the phase space of a heavy top system. We consider the dense open submanifold $\mathbb{R}^3 \times \dot{\mathbb{R}}^3 \subset \mathbf{e}(3)^*$ of $\mathbf{e}(3)^*$ consisting of all 4-dimensional symplectic leaves ($\vec{\Gamma}^2 > 0$) and its two 5-dimensional submanifolds:

(i) submanifold of $\mathbb{R}^3 \times \dot{\mathbb{R}}^3$ defined by $\vec{J}\cdot\vec{\Gamma} = \mu||\vec{\Gamma}||$,

(ii) submanifold of $\mathbb{R}^3 \times \dot{\mathbb{R}}^3$ defined by $\vec{\Gamma}^2 = \nu^2$,

where $(\vec{J}, \vec{\Gamma}) \in \mathbb{R}^3 \times \mathbb{R}^3 \cong \mathbf{e}(3)^*$, μ, ν are some real fixed parameters and $\dot{\mathbb{R}}^3 := \mathbb{R}^3 \backslash \{0\}$. Basing on $U(2,2)$-invariant symplectic structure of the Penrose twistor space we find full and complete $E(3)$-equivariant symplectic realizations of these submanifolds. Lifts of the integrable Hamiltonian systems on $\mathbf{e}(3)^*$ to these symplectic realizations give a large family of integrable Hamiltonian systems.

29. **Karol ŻYCZKOWSKI** — *Jagiellonian University / CTP PAS, Poland*

Extremal quantum states & combinatorial designs

A *quantum combinatorial design* is composed of quantum states, arranged with a certain symmetry and balance. Such a constellation of states determines distinguished quantum measurements and can be applied for quantum information processing. Negative solution to the famous problem of 36 officers of Euler implies that there are no two orthogonal Latin squares of order six. We show that the problem has a solution, provided the officers are entangled, and construct orthogonal quantum Latin squares of this size. The solution can be visualized on a chessboard of size six, which shows that 36 officers are split in nine groups, each containing of four entangled states. It allows us to construct a pure nonadditive quhex quantum error detection code and four-party states with extremal entanglement properties.

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30. **Tomasz ŻYNDA** — *Warsaw Military University of Technology, Poland*

Reproducing kernels and minimal solutions of elliptic equations

Suppose that the set of square-integrable solutions of an elliptic equation which take value equal to c in some given point is non-empty. Then there is exactly one element with minimal L^2 norm. Moreover such an element depends in continuous way on a weight of integration and on a domain of integration. These theorems can be proved by using the theory of reproducing kernels.

Virtual contributed lectures

1. **Maram ALOSSAIMI** — *Sheffield University, UK*

The Poisson spectrum of the first part of the second case of the classification

Poisson algebras can be defined as Lie algebras that satisfy the Leibniz rule. In our research, we classified a large class of Poisson algebras $A = (D, \alpha, \beta, c, u)$ that appears in [3, Lemma 1.3]. In specific, our class $A = K[t][x, y] = (K[t], \alpha, \beta, c, u)$ is a Poisson polynomial algebra in two variables x and y with coefficients on the Poisson polynomial algebra $K[t]$, where K is an algebraic closure field with zero characteristic. Our aim is to classify some finite dimensional Poisson modules over class A. Also, we are interested in the Poisson spectrum of A, minimal and maximal Poisson ideals of A. The classification of A consists of three main cases and each case has several subcases. In this talk, I will give some background in Poisson algebras, introduce the Poisson algebra class A, talk about the first part of the second case and its Poisson spectrum.

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2. **Sara CRUZ Y CRUZ** — *Instituto Politécnico Nacional, Mexico*

Darboux transformations for the design of gradient-index optical waveguides

In this work we present a brief overview on the applications of the factorization method and Darboux transformations in the design of gradient-index optical architectures. Some particular cases of exactly solvable refractive index landscapes supporting the propagation of non-decaying TE modes are constructed and their underlying algebras are identified.

3. **Vladimir DRAGOVIĆ** — *The University of Texas at Dallas, United States*

Chebyshev dynamics, isoharmonic deformations, and constrained Schlesinger systems

The talk is based on interrelations between integrable billiards, extremal polynomials, Riemann surfaces, potential theory, and isomonodromic deformations. We discuss injectivity properties of rotation and winding numbers. We study dynamics of Chebyshev polynomials on several intervals and introduce a notion of isoharmonic deformations. We study their isomonodromic properties and formulate a new class of constrained Schlesinger systems. We provide explicit solutions to these systems. The talk is based on joint results with Vasilisa Shramchenko.

4. **Galina FILIPUK** — *Uniwersytet Warszawski, Poland*

Hamiltonians of the Painlevé and quasi-Painlevé equations

In this talk I shall present some recent results on the Hamiltonians of the Painlevé and quasi-Painlevé equations. This is a joint work with A. Dzhamay, A. Ligeza, A. Stokes on the Painlevé equations and T. Kecker on the quasi-Painlevé equations.

5. **Gerardo HERRERA CORRAL** — *CINVESTAV, Mexico*

Personal conversations with Bogdan and the manipulation of Schrödinger's particle by time dependent potentials

6. **Safiqul ISLAM** — *King Faisal University, Saudi Arabia*

A further analysis of a $(2+1)$-dimensional combined cosmological model in $f(R,T)$ gravity

This research is an extension of my published $(2+1)$ dimensional cosmological models in $f(R,T)$ gravity with $\Lambda(R,T)$ (IOP Conf. Series: Journal of Physics: Conf. Series 1258 , (2019) 012026). A new class of cosmological model in $f(R,T)$ modified theories of gravity is studied, and hence we define the cosmological constant Λ as a function of the trace of the stress-energy momentum tensor T and the Ricci scalar R, and call such a model $\Lambda(R,T)$ gravity. Here we have specified a certain form of $\Lambda(R,T)$. $\Lambda(R,T)$ is also defined in the perfect fluid as well as dust case. We intend to search for a combined model that satisfies the equation of state for dark energy matter or quintessence matter or perfect matter fluid. Some geometric and physical properties of the model are also discussed. The pressure, density and energy conditions are studied both when Λ is zero and when $\Lambda = \Lambda(r)$, a function of the radial parameter, r. We study the effective mass function and also the gravitational redshift function, bo th of which are found to be positive as per latest observations. The cosmological model is studied in $f(R,T)$ modified theory of gravity, where the gravitational Lagrangian is expressed both in terms of the Ricci scalar R and the trace of the stress-energy tensor T. The equation of state parameter is discussed in terms of ω corresponding to the three cases mentioned above. The behaviour of the cosmological constant is separately examined in presence of quintessence matter, dark energy matter and perfect fluid matter.

7. **Igor KANATTSIKOV** — *University of Exeter, United Kingdom*

Precanonical Quantum Teleparallel Gravity

The following topics will be covered:

1. The De Donder–Weyl (DW) Hamiltonian formulation and precanonical quantization of scalar fields.
2. The relation with the functional Schroedinger picture in QFT.
3. Teleparallel equivalent of General Relativity (TEGR) and its Palatini formulation in vielbein variables.
4. DW Hamiltonian formulation of TEGR and the analysis of constraints.
5. Precanonical quantization of TEGR and its first results in quantum cosmology.

8. **John KLAUDER** — *University of Florida, United States*

Unifying Classical and Quantum Physics + Quantum Fields and Gravity

Clearly, the classical and quantum realms appear to be well separated, and thus difficult to easily pass between one another. However, there are two bridges, built using the right tools and material, that can unite the two realms in a straightforward manner. This procedure works well for both canonical and affine quantization procedures. An additional topic illustrates the wonderful quantization results for covariant field theories plus Einstein's gravity using affine quantization.

9. **Gabriel LAROTONDA** — *Universidad de Buenos Aires and CONICET, Argentina*

Differentiable and Kähler geometry of the coadjoint orbit of a nuclear operator

We will discuss recent developments concerning the topology of the unitary orbit of a nuclear operator and of its closure. Then we will focus on the geometry of a weak

Riemannian metric in the orbit, compatible with the complex structure and the KKS symplectic form. The talk is based on joint work with D. Beltiţă (Bucharest) on one hand, and T. Goliński (Białystok) on the other.

10. **Fernand PELLETIER** — *Université de Savoie Mont Blanc, France*

On partial Banach-Lie algebroid structure: some motivations

In finite dimension it is well known that the cotangent bundle of a Poisson manifold is provided with a natural structure of Lie algebroid. On the other hand the prolongation of a Lie algebroid can be provided with a structure of Banach Lie algebroid. After looking for problems we meet for an adaptation for such results in the Banach setting, first, we will propose a generalization of finite Poisson manifold by the notion partial Poisson Banach manifold M, which is defined by a Poisson anchor on a weak subbundle $T^\flat M$ of T^*M. In this context, it will appear that we obtain only a "partial structure of Banach-Lie algebroid" on $T^\flat M$ and not a Banach-Lie algebroid structure even if $T^\flat M = T^*M$.

For analogous reasons, although the prolongation of a Banach Lie algebroid is naturally provided with an anchor, the Lie bracket can be lift onto the prolongation but gives rise only a "partial of Banach-Lie algebroid" structure on this prolongation but is not a Banach-Lie algebroid structure.

11. **Oscar ROSAS-ORTIZ** — *CINVESTAV, Mexico*

Factorization: Little or great algorithm?

In this talk, we discuss some important details of the history of the factorization method that are reported in the review article "Factorization: little or great algorithm?", written with Bogdan Mielnik. We also discuss the current state of the art on this topic and comment on some of the emerging trends.

12. **Artur SERGYEYEV** — *Silesian University in Opava, Czech Republic*

Integrable systems in higher dimensions: a new perspective

The search for partial differential systems in four independent variables that are integrable in the sense of soliton theory is a problem of longstanding interest, as according to general relativity our spacetime is four-dimensional and for a long time it appeared that such systems are very scarce. We show that this is not really the case. Namely, we will present an explicit effective construction that produces inter alia two infinite families of integrable systems in four independent variables as well as e.g. the first example of a system of this kind with nonisospectral Lax pair which is algebraic, rather than rational, in the spectral parameter.

13. **Elizaveta VISHNYAKOVA** — *Universidade Federal de Minas Gerais, Brazil*

Donagi–Witten construction and a graded covering of a supermanifold

In the paper "Super Atiyah classes and obstructions to splitting of supermoduli space", Donagi and Witten suggested a construction of a first obstruction class for splitting of a supermanifold via differential operators. We generalize this idea. As a result we obtain a family of embeddings of the category of supermanifolds into the category of iterated vector bundles and into the category of graded manifolds. It was shown that the images of a supermanifold in these categories satisfy universal properties of a graded covering or a graded semicovering.

In our talk we will discuss these functors in the case of a Lie supergroup and a Lie superalgebra.

LIST OF PARTICIPANTS

1. ALOSSAIMI, Maram — Sheffield University
Sheffield, UK
E-mail : maram.alosaimi@gmail.com

2. ANDRUCHOW, Esteban — Universidad Nacional de General Sarmiento and Instituto Argentino de Matemática "Alberto P. Calderón"
Buenos Aires, ARGENTINA
E-mail : eandruch@ungs.edu.ar

3. BARDADYN, Krzysztof — Uniwersytet w Białymstoku
Białystok, POLAND
E-mail : kbardadyn@math.uwb.edu.pl

4. BEIGE, Almut — University of Leeds
Leeds, UNITED KINGDOM
E-mail : a.beige@leeds.ac.uk

5. BELTIŢĂ, Daniel — Institute of Mathematics "Simion Stoilow" of the Romanian Academy
Bucharest, ROMANIA
E-mail : Daniel.Beltita@imar.ro

6. BERMÚDEZ, David — CINVESTAV
Mexico City, MEXICO
E-mail : david.bermudez@cinvestav.mx

7. BIAŁYNICKI-BIRULA, Iwo — Centum Fizyki Teoretycznej, PAN
Warszawa, POLAND
E-mail : birula@cft.edu.pl

8. BLASCHKE, Petr — Silesian University in Opava
Opava, CZECH REPUBLIC
E-mail : Petr.Blaschke@math.slu.cz

9. BRETON, Nora — CINVESTAV
Mexico City, MEXICO
E-mail : nora@fis.cinvestav.mx

10. CAMPOBASSO, Luca — Łódź University of Technology
Łódź, POLAND
E-mail : lcampobasso@gmail.com

11. CHADZITASKOS, Goce — Czech Technical University in Prague
Praha, CZECH REPUBLIC
E-mail : goce.chadzitaskos@fifj.cvut.cz

12. CONTRERAS-ASTORGA, Alonso — CINVESTAV
Mexico City, MEXICO
E-mail : alonso.contreras@cinvestav.mx

13. CRUZ Y CRUZ, Sara — Instituto Politécnico Nacional
Mexico City, MEXICO
E-mail : sgcruzc@ipn.mx

14. CZACHOR, Marek — Politechnika Gdańska
Gdańsk, POLAND
E-mail : mczachor@pg.edu.pl

15. CZYŻYCKI, Tomasz — Uniwersytet w Białymstoku
Białystok, POLAND
E-mail : tomczyz@math.uwb.edu.pl

16. DE COMMER, Kenny — Vrije Universiteit Brussel
Brussels, BELGIUM
E-mail : kenny.de.commer@vub.be

17. DOBROGOWSKA, Alina — Uniwersytet w Białymstoku
Białystok, POLAND
E-mail : alina.dobrogowska@uwb.edu.pl

18. DOMAŃSKI, Ziemowit — Poznań University of Technology, Institute of Mathematics
Poznań, POLAND
E-mail : ziemowit.domanski@put.poznan.pl

19. DRAGAN, Andrzej — Uniwersytet Warszawski
Warszawa, POLAND
E-mail : andrzejdragan@gmail.com

20. DRAGOVIĆ, Vladimir — The University of Texas at Dallas
Richardson, UNITED STATES
E-mail : Vladimir.Dragovic@utdallas.edu

21. DUNAJSKI, Maciej — University of Cambridge
Cambridge, UNITED KINGDOM
E-mail : m.dunajski@damtp.cam.ac.uk

22. FERNÁNDEZ, David — CINVESTAV
Mexico City, MEXICO
E-mail : david@fis.cinvestav.mx

23. FILIPUK, Galina — Uniwersytet Warszawski
Warszawa, POLAND
E-mail : Filipuk@mimuw.edu.pl

24. FUENTES, Jesús — University of Luxembourg
Belval, LUXEMBOURG
E-mail : jesus.fuentes@uni.lu

25. GARCIA-COMPEAN, Hugo — CINVESTAV
Mexico City, MEXICO
E-mail : compean@fis.cinvestav.mx

26. GOLDIN, Gerald — Rutgers University
New Brunswick, NJ, UNITED STATES
E-mail : geraldgoldin@dimacs.rutgers.edu

27. GOLIŃSKI, Tomasz — Uniwersytet w Białymstoku
Białystok, POLAND
E-mail : tomaszg@math.uwb.edu.pl

28. HERRERA CORRAL, Gerardo — CINVESTAV
Mexico City, MEXICO
E-mail : gherrera@fis.cinvestav.mx

29. HOQUE, Md Fazlul — Czech Technical University in Prague
Praha, CZECH REPUBLIC
E-mail : hoquemdf@fjfi.cvut.cz

30. HOROWSKI, Maciej — Uniwersytet w Białymstoku
Białystok, POLAND
E-mail : horowski@math.uwb.edu.pl

31. HRIVNÁK, Jiří — Czech Technical University in Prague
Praha, CZECH REPUBLIC
E-mail : jiri.hrivnak@fjfi.cvut.cz

32. ISLAM, Safiqul — King Faisal University
Al Hofuf, SAUDI ARABIA
E-mail : sislam@kfu.edu.sa

33. JAKIMOWICZ, Grzegorz — Uniwersytet w Białymstoku
Białystok, POLAND
E-mail : g.jakimowicz@uwb.edu.pl

34. KANATTSIKOV, Igor — University of Exeter
Exeter, UNITED KINGDOM
E-mail : kanattsi@gmail.com

35. KASPRZAK, Paweł — Uniwersytet Warszawski
Warszawa, POLAND
E-mail : pawel.kasprzak@fuw.edu.pl

36. KIELANOWSKI, Piotr — CINVESTAV
Mexico City, MEXICO
E-mail : kiel@fis.cinvestav.mx

37. KIJOWSKI, Jerzy — Center for Theoretical Physics
Warszawa, POLAND
E-mail : kijowski@cft.edu.pl

38. KLAUDER, John — University of Florida
Ossining, UNITED STATES
E-mail : klauder@ufl.edu

39. KONNO, Hitoshi — Tokyo University of Marine Science and Technology
Tokyo, JAPAN
E-mail : hkonno0@kaiyodai.ac.jp

40. KRASIŃSKI, Andrzej — N. Copernicus Astronomical Center, Polish Academy of Sciences
Warszawa, POLAND
E-mail : akr@camk.edu.pl

41. KWAŚNIEWSKI, Bartosz — Uniwersytet w Białymstoku
Białystok, POLAND
E-mail : bartoszk@math.uwb.edu.pl

42. LAROTONDA, Gabriel — Universidad de Buenos Aires and CONICET
Buenos Aires, ARGENTINA
E-mail : glaroton@dm.uba.ar

43. MCKEE, Andrew — Uniwersytet w Białymstoku
Białystok, POLAND
E-mail : a.mckee@uwb.edu.pl

44. NEEB, Karl-Hermann — Friedrich-Alexander University, Department of Mathemtatics
Erlangen, GERMANY
E-mail : neeb@math.fau.de

45. NETTO, Clarice — Instituto Nacional de Matemática Pura e Aplicada
Rio de Janeiro, BRASIL
E-mail : `claricenetto@id.uff.br`

46. NIESTIJL, Milan — Delft University of Technology
Delft, NETHERLANDS
E-mail : `milan.niestijl@gmail.com`

47. NOVOTNÝ, Petr — Czech Technical University in Prague
Praha, CZECH REPUBLIC
E-mail : `petr.novotny@fjfi.cvut.cz`

48. OLAFSSON, Gestur — Louisiana State University
Baton Rouge, USA
E-mail : `olafsson@math.lsu.edu`

49. ORLOV, Aleksandr — Institute of Oceanology, Institute of Theoretical and Experimental Physics
Moscow, RUSSIA
E-mail : `orlovs55@mail.ru`

50. OSTASZEWSKA, Urszula — Uniwersytet w Białymstoku
Białystok, POLAND
E-mail : `uostasze@math.uwb.edu.pl`

51. PELLETIER, Fernand — Université de Savoie Mont Blanc
Le Bourget du Lac, FRANCE
E-mail : `fernand.pelletier@univ-smb.fr`

52. PRYKARPATSKI, Anatolij — Cracow University of Technology
Kraków, POLAND
E-mail : `pryk.anat@cybergal.com`

53. PRZANOWSKI, Maciej — Politechnika Łódzka
Łódź, POLAND
E-mail : `maciej.przanowski@p.lodz.pl`

54. RAUCH-WOJCIECHOWSKI, Stefan — Linköping University
Linköping, SWEDEN
E-mail : `sterau10@yahoo.com`

55. RATIU, Tudor — Shanghai Jiao Tong University
Shanghai, CHINA
E-mail : `tudor.ratiu@epfl.ch`

56. ROSAS-ORTIZ, Oscar — CINVESTAV
Mexico City, MEXICO
E-mail : `oscar.rosas@cinvestav.mx`

57. SERGYEYEV, Artur — Silesian University in Opava
Opava, CZECH REPUBLIC
E-mail : `artur.sergyeyev@math.slu.cz`

58. SLIŻEWSKA, Aneta — Uniwersytet w Białymstoku
Białystok, POLAND
E-mail : `anetasl@uwb.edu.pl`

59. SZAJEWSKA, Marzena — Uniwersytet w Białymstoku
Białystok, POLAND
E-mail : `m.szajewska@math.uwb.edu.pl`

60. TABACHNIKOV, Sergei
Pennsylvania State University
State College, USA
E-mail : tabachni@math.psu.edu

61. TOSIEK, Jaromir
Łódź University of Technology
Łódź, POLAND
E-mail : tosiek@p.lodz.pl

62. TUMPACH, Alice Barbora
Laboratoire Painlevé & Pauli Institut
Vienna, AUSTRIA
E-mail : alice-barbora.tumpach@univ-lille.fr

63. VESELOV, Alexander
Loughborough University
Loughborough, UNITED KINGDOM
E-mail : A.P.Veselov@lboro.ac.uk

64. VISHNYAKOVA, Elizaveta
Universidade Federal de Minas Gerais
Belo Horizonte, BRAZIL
E-mail : vishnyakovae@googlemail.com

65. WAWRENIUK, Elwira
Uniwersytet w Białymstoku
Białystok, POLAND
E-mail : e.wawreniuk@uwb.edu.pl

66. WRIGHT, Michael
Archive Trust for Research in Mathematical Sciences
Bristol, UNITED KINGDOM
E-mail : michael@archmathsci.org

67. XU, Xiaomeng
Peking University
Beijing, CHINA
E-mail : xxu@bicmr.pku.edu.cn

68. YOSHIOKA, Akira
Tokyo University of Science
Tokyo, JAPAN
E-mail : yoshioka_a@rs.tus.ac.jp

69. ŻYCZKOWSKI, Karol
Jagiellonian University / CTP PAS
Kraków / Warszawa, POLAND
E-mail : karol@cft.edu.pl

70. ŻYNDA, Tomasz
Warsaw Military University of Technology
Warszawa, POLAND
E-mail : tomasz.zynda@wat.edu.pl