XXXVIII WORKSHOP ON GEOMETRIC METHODS IN PHYSICS

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LIST OF ABSTRACTS

Plenary lectures

1. **Francesca ARICI** — *Max Planck Institute for Mathematics in the Sciences, Germany*

   Circle and sphere bundles in noncommutative geometry

   In this talk I will recall how Pimsner algebras of self Morita equivalences can be thought of as total spaces of quantum circle bundles, and the associated six term exact sequence in K-theory can be interpreted as an operator algebraic version of the classical Gysin sequence for circle bundles. After reviewing some results in this direction, I will report on work in progress concerning the construction of higher dimensional quantum sphere bundles in terms of Cuntz–Pimsner algebras of sub-product systems. Based on (ongoing) joint work with G. Landi and J. Kaad.

2. **Oleg CHALYKH** — *University of Leeds, United Kingdom*

   Quiver varieties and integrable systems

   I will first review the formalism of double Poisson and quasi-Poisson brackets, due to Van den Bergh. This provides a method for performing (quasi-)Hamiltonian reduction leading to quiver varieties (going back to Nakajima) and their multiplicative version (due to Crawley-Boevey and Shaw). The (multiplicative) quiver varieties thus form a rich class of Poisson varieties, and it is natural to ask whether they accommodate interesting integrable systems. I will explain how the classical Calogero–Moser system and its variants can be produced starting from fairly simple quivers. Based on joint works with A. Silantyev (Dubna) and M. Fairon (Leeds).

3. **Giordano COTTI** — *Max-Planck-Institut für Mathematik, Germany*

   Algebraic geometric aspects of Isomonodromic Deformations: Helix structures in Quantum Cohomology of Fano Varieties

   In occasion of the 1998 ICM in Berlin, B. Dubrovin conjectured an intriguing connection between the enumerative geometry of a Fano manifold X with algebro-geometric properties of exceptional collections in the derived category $\mathcal{D}^b(X)$. Under the assumption of semisimplicity of the quantum co- homology of X, the conjecture prescribes an explicit form for local invariants of $QH^\bullet(X)$, the so-called "monodromy data", in terms of Gram matrices and characteristic classes of objects of exceptional collections. In this talk, a refinement of this conjecture will be presented, and particular attention will be given to the case of complex Grassmannians. At points of small quantum cohomology, these varieties manifest a coalescence phenomenon, whose occurrence and frequency is surprisingly subordinate to the distribution of prime numbers. A priori, the analytical description of these Frobenius structures cannot be obtained from an immediate application of the theory of isomonodromy deformations. The speaker will show how, under minimal conditions, the classical theory of M. Jimbo, T. Miwa, K. Ueno (1981) can be extended to describe isomonodromy deformations at a coalescing irregular singularity. Furthermore, a property of quasi-periodicity of Stokes matrices associated to the points of small Quantum Cohomology of complex Grassmannians will be discussed. Based on joint works with B. Dubrovin and D. Guzzetti.

4. **Huijun FAN** — *Peking University, China*

Fukaya category of Landau–Ginzburg model

I will report the construction of the mathematical foundation of the Floer theory (i. e., open-string A-side) of Landau–Ginzburg (LG) model via Witten equation. We introduce the concept of Regular tame exact Landau-Ginzburg system on a non-compact Kaehler manifold, and define the notion of LG brane, as the object of the category. The study on Witten equation in our context gives the Fukaya category of LG model. This is the joint work with Wenfeng Jiang and Dingyu Yang.

5. **Alexey GLUTSYUK** — *Ecole Normale Supérieure de Lyon, France*

On curves with Poritsky property and Liouville net

To each planar convex closed curve C the string construction associates a family of bigger closed curves $G(t)$ whose billiards have C as a caustic. Each curve $G(t)$ induces a dynamical system on C: given two tangent lines to C through the same point in $G(t)$, the tangency point of the left tangent line is sent to the tangency point of the right tangent line. A curve C is said to have Poritsky property, if this dynamical system has an invariant length element on C that is the same for all t. Curves with Poritsky property are closely related to integrable billiards. H. Poritsky have shown that the only planar curves with Poritsky property are conics. We extend his result to surfaces of constant curvature. We consider curves with Poritsky property on arbitrary Riemannian surface and obtain the following results: 1) a formula for the invariant length element in terms of the geodesic curvature and the natural length parameter; 2) each germ of curve with Poritsky property is completely determined by its 4-jet.

We also present the following very recent joint result with Sergei Tabachnikov and Ivan Izmestiev. Given a curve C bounding a topological disc on a Riemannian surface. Then the following statements are equivalent:

1) The curve C has Poritsky property.

2) The corresponding family of string constructions has Graves property: smaller curves are caustics for bigger curves.

3) The metric on the concave side of C is Liouville.

6. **Jimmie LAWSON** — *Louisiana State University, USA*

The Road to the Operator Geometric Mean with Stops Along the Way

We trace the path of development of the matrix and operator geometric mean from its beginnings as a two-variable mean to a multi-variable mean and barycentric map on integrable measures on the cone of positive elements of an arbitrary unital C^*-algebra. We point out a variety of major mathematical tools and theories, primarily geometric ones, that have been crucial at varying stages to make possible these developments: the theory and stochastics of non-positively curved (CAT_0) metric spaces, Riemannian geometry, the Thompson metric, a new class of power means, and the Wasserstein metric on probability measures, to mention some of the key ones.

7. **José MOURÃO** — *Instituto Superior Técnico, Portugal*

Imaginary time Hamiltonian flows and applications to Quantization, Kahler geometry and representation theory

The formalism to complexify time in the flow of a vector field on a complex manifold is reviewed. The complexified flow, besides acting on M, changes also the complex structure. We will review the following applications:

- For a compact Kahler manifold the imaginary time Hamiltonian flows correspond to Mabuchi geodesics in the infinite dimensional space of Kahler metrics on M that play a very important role in the study of stability of Kahler manifolds. A nontrivial example on the two-dimensional sphere will be described.
- Let the compact connected Lie group G act in an Hamiltonian and Kahler way on a Kahler manifold M so that its action extends to G_C. Then, by taking geodesics of Kahler structures generated by convex functions of the G-moment map to infinite geodesic time, one gets (conjecturally always, proved on several important examples) a concentration of holomorphic sections of holomorphic line bundles on inverse images of coadjoint orbits under the G-momentum map. A nontrivial toric example and the case of $M = G_C$ will be described.

On work in collaboration with T. Baier, J. Hilgert, O. Kaya and J. P. Nunes.

8. **Atsushi NAKAYASHIKI** — *Tsuda University, Japan*

Sato Grassmannian approach to degenerations of tau functions

We study degenerations of solutions (tau functions) of the Kadomtsev–Petviashvili (KP) equation corresponding to non-singular algebraic curves when they degenerate to singular curves. We use Sato Grassmannian to study this problem. It reduces the problem to simpler problem on frames. We show that, in the case of trigonal curves, degenerate tau functions are expressed as sums of derivatives of non-degenerate tau functions corresponding to lower genus curves. The talk is partly based on the joint work with J. Bernatska and V. Enolski.

9. **Sergey NESHVEYEV** — *University of Oslo, Norway*

Quantization of subgroups of the affine group

Although the problem of quantization of Lie bialgebras in the purely algebraic (formal) setting was solved in full generality in the 1990s by Etingof and Kazhdan, the list of noncompact Lie bialgebras admitting a quantization in the analytic (operator algebraic) setting is still quite short. In my talk I will present a quantization procedure for a class of semidirect product groups G defined by an action of a locally compact group Q on a locally compact abelian group V. In fact, three equivalent procedures will be given. The first is via reflection across the Hopf–Galois object defined by a distinguished irreducible representation of G. The second is via cocycle twisting related to the Kohn–Nirenberg quantization. The third is via a bicrossed product defined by a pair of groups isomorphic to Q. In the simplest case of the $ax + b$ group over the reals the equivalence of the second and the third constructions imply that certain quantum groups defined by Baaj–Skandalis and Stachura are indeed isomorphic. I will also show that the first construction makes sense in greater generality. Specifically, every locally compact group with group von Neumann algebra isomorphic to the algebra of bounded operators on a Hilbert space admits a canonical quantization which is neither commutative nor cocommutative. (Joint work with Pierre Bieliavsky, Victor Gayral and Lars Tuset.)

10. **Luis M. NIETO** — *Universidad de Valladolid, Spain*

Some recent results on point supported potentials

11. **Zoran RAKIĆ** — *University of Belgrade, Serbia*

On Non-local Modified Gravity

Many significant gravitational phenomena have been discovered and predicted using general theory of relativity. Despite to all of this it is not a complete theory. One of actual approaches towards more complete theory of gravity is its nonlocal modification. We consider nonlocal modification of the Einstein theory of gravity in framework of the pseudo-Riemannian geometry. The nonlocal term has the form $\mathcal{H}(R)\mathcal{F}(\Box)\mathcal{G}(R)$, where $\mathcal{H}$ and $\mathcal{G}$ are differentiable functions of the scalar curvature R, and $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ where f_n are is an analytic function of the d'Alembert operator $\Box$. We are paid our attention to the case where $\mathcal{H}(R) = \mathcal{G}(R) = \sqrt{R - 2\,\Lambda}$. Using calculus of variations we derived the corresponding equations of motion. The variation of action is induced by variation of the metric tensor $g_{\mu\nu}$. We consider several models of the above mentioned type, as well as the case when the scalar curvature is constant. Moreover, we consider space-time perturbations of the de Sitter space. It was shown that gravitational waves are described in the class of nonlocal models $\mathcal{H}(R)\mathcal{F}(\Box)\mathcal{G}(R)$, with respect to Minkowski metric by the same equations as in general relativity.

This is joint work with Ivan Dimitrijević, Branko Dragović and Jelena Stanković. The work was partially supported by the project ON174012 of MPNTR of Republic of Serbia.

12. **Alexander SERGEEV** — *Saratov State University / Higher School of Economics, Russia*

Supergroup $Osp(2, 2n)$ and super Jacobi polynomials Jacobi polynomials

Coefficients of super Jacobi polynomials of type $B(1, n)$ are rational functions in three parameters k, p, q. At the point $(-1, 0, 0)$ these coefficient may have poles. Let us set $q = 0$ and consider pair (k, p) as a point of A^2. If we apply blow up procedure at the point $(-1, 0)$ then we get a new family of polynomials depending on parameter t from projective space. If $t = \infty$ then we get supercharacters of Kac (Euler) modules for Lie supergroup $Osp(2, 2n)$ and supercharacters of irreducible modules can be obtained in the same way for nonnegative integer t depending on highest weight. Besides we obtained supercharcters of projective covers as specialisation of some nonsingular modification of super Jacobi polynomials.

13. **Adam SKALSKI** — *Polish Academy of Sciences, Poland*

On C*-completions of discrete quantum group rings

We will discuss just infiniteness of C*-algebras associated to discrete quantum groups and relate it to the C*-uniqueness of the quantum groups in question, i.e. to the uniqueness of a C*-completion of the underlying Hopf *-algebra. We will show that duals of q-deformations of compact Lie groups are never C*-unique, and on the other hand present an example of a discrete quantum group which is not locally finite and yet is C*-unique. Joint work with Martijn Caspers.

14. **Makoto YAMASHITA** — *University of Oslo, Norway*

Quantum symmetric pairs, cyclotomic KZ-equations, and module categories

The reflection equation (RE) in various incarnations gives quantum homogeneous spaces through Tannaka–Krein duality principle. In this talk, I explain a conjectural C*-categorical correspondence of the solutions to RE arising from quantizations of symmetric spaces: the Knizhnik–Zamolodchikov scheme (Leibman, Golubeva–Leksin, Enriquez) on the one hand and the coideal scheme (Balagovic–Kolb) on the other. In the formal setting, we can show that such structures are indeed classifiable via computation of coHochschild cohomology groups.

Based on joint works with K. De Commer, S. Neshveyev, and L. Tuset.

Contributed lectures

15. **Krzysztof BARDADYN** — *Uniwersytet w Białymstoku, Poland*

C*-algebraic approach to spectral analysis of weighted composition operators

Fundamental spectral information concerning operators acting on a Hilbert space depends only on the C*-algebra they generate. In the case of weighted composition operators with a full one-sided shift on a Cantor set, the C*-algebra in question is Cuntz algebra, which in turn depends only on the underlying dynamical system.

Motivated by the above remarks, we introduce axiomatically a large class of operators associated with irreversible dynamics. The spectral properties of these operators can be efficiently analyzed using C*-algebraic methods, and their spectra can be completely described in dynamical terms. (Based on joint work in progress with Bartosz Kwaśniewski.)

16. **Daniel BELTIŢĂ** — *Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Romania*

The standard groupoid of a von Neumann algebra

We discuss a certain groupoid that encodes a few key elements of the standard form and of the modular theory of an arbitrary von Neumann algebra. That groupoid does share some features of the Lie groupoids although its space of units is not a smooth manifold. Using this groupoid model we investigate the relation between the standard forms of von Neumann algebras and certain duality phenomena that occur in Poisson geometry. This presentation is based on joint work with Anatol Odzijewicz.

17. **Tomasz BRZEZIŃSKI** — *Uniwersytet w Białymstoku / Swansea University, Poland / United Kingdom*

On classical and deformed Hopf fibrations

The θ-deformed Hopf fibration $\mathbb{S}^3_\theta \to \mathbb{S}^2$ over the commutative 2-sphere is compared with its classical counterpart. It is shown that there exists a natural isomorphism between the corresponding associated module functors and that the affine spaces of classical and deformed connections are isomorphic. The latter isomorphism is equivariant under an appropriate notion of infinitesimal gauge transformations in these contexts. Gauge transformations and connections on associated modules are studied and are shown to be sensitive to the deformation parameter. Joint work with James Gaunt and Alexander Schenkel.

18. **Gustavo CORACH** — *Instituto Argentino de Matemática, Argentina*

Poincaré disk of a C*-algebra

We study the Poincaré disk of a unital C*-algebra A as a homogeneous reductive space of the group of all Moebius transformations with entries in A. We present a geometric interpretation of the partial isometry of an element of the disk in terms of a convenient operator cross ratio, related to some construction by M.I. Zelikin.

19. **Alina DOBROGOWSKA** — *Uniwersytet w Białymstoku, Poland*

Second order difference equations solvable by factorization method

This talk addresses an investigation on a factorization method for difference equations. Under some assumptions we find a general solution of the factorization problem for a family of second order difference equations.

20. **Laszlo FEHER** — *University of Szeged / Wigner Research Centre for Physics, Hungary*

Spin Ruijsenaars–Schneider–Sutherland models from Hamiltonian reduction

We shall report our recent results on spin generalizations of the classical integrable many–body models of Sutherland and Ruijsenaars–Schneider type. Our models are obtained via Hamiltonian reductions of free systems on cotangent bundles of Lie groups and their Poisson–Lie analogues. In some cases these two sources lead to the same reduced equations of motion and to a pair of compatible Poisson brackets. The talk will be based on the papers [1,2] and ongoing research.

[1] L. Feher, Poisson–Lie analogues of spin Sutherland models, arXiv:1809.01529 [math-ph]

[2] L. Feher, Bi-Hamiltonian structure of a dynamical system introduced by Braden and Hone, arXiv:1901.03558 [math-ph]

21. **Gerald GOLDIN** — *Rutgers University, USA*

On a General Formulation of Classical Nonlinear Lagrangian Electrodynamics with Conformal Symmetry

This talk summarizes some results of joint with Steven Duplij and Vladimir Shtelen, with formulas for a general formulation of classical nonlinear Lagrangian electrodynamics with conformal symmetry. The theory is based on nonlinear constitutive equations expressed in terms of conformal-invariant functionals of the field strengths. Some specific examples are presented.

22. **Kentaro HARA** — *Tokyo University of Science, Japan*

Hermitian–Einstein metrics from noncommutative $U(1)$ solutions

We show that Hermitian–Einstein metrics can be locally constructed by a map from (anti-)self-dual two-forms on Euclidean $\mathbb{R}^4$ to symmetric two-tensors introduced in "Gravitational instantons from gauge theory", H.S. Yang and M. Salizzoni, Phys. Rev. Lett. (2006) 201602, [hep-th/0512215]. This correspondence is valid not only for a commutative space but also for a noncommutative space. We choose $U(1)$ instantons on a noncommutative C2 as the self-dual two-form, from which we derive a family of Hermitian–Einstein metrics. We also discuss the condition when the two-forms are not instantons but they are solutions to the Yang–Mills equations.

23. **Mahouton Norbert HOUNKONNOU** — *University of Abomey-Calavi, Benin*

2-hom-associative bialgebras and hom-left symmetric dialgebras

From the definition and properties of unital hom-associative algebras, and the use of the Kaplansky's construction, we develop new algebraic structures called 2-hom-associative bialgebras, 2-hom-bialgebras, and 2-2-hom-bialgebras. Besides, we devise a construction of hom-left symmetric dialgebras and discussed their main relevant properties.

24. **Jiří HRIVNÁK** — *Czech Technical University in Prague, Czech Republic*

Graphene dots via discretizations of Weyl-orbit functions

The application of two fundamental discretizations of Weyl-orbit functions to an electron propagation on the graphene triangular dots are presented. Symmetries of the point and labels sets inside dual weight and root lattices of root systems are provided by affine and extended affine Weyl groups. The discrete orthogonality relations of the Weyl-orbit functions over the dual weight and root point sets induce four types of complex discrete Fourier-Weyl transforms. Subtractively combining the transforms of the A_2 group induces two types of extended Weyl-orbit functions and their corresponding discrete transforms on the fragment of the honeycomb lattice. Special types of extended Weyl-orbit functions represent stationary states of the electron propagation on the triangular graphene dot with armchair boundaries. Further extension of the presented approach to the triangular graphene dots with zigzag boundaries is discussed. This is a joint work with Lenka Motlochová.

25. **Grzegorz JAKIMOWICZ** — *Uniwersytet w Białymstoku, Poland*

Symmetries of the space of connections on a principal G-bundle and related symplectic structures

We investigate the G-invariant symplectic structures on the cotangent bundle T^*P of a principal G-bundle $P(M,G)$ canonically related to connections on $P(M,G)$ as well as to the elements of the group $Aut_{TG}TP$ of automorphisms of the tangent bundle TP which covers the identity map of P and commute with the action of TG on TP. The classical reduction procedure for these symplectic structures is described and possible applications are discussed.

26. **Pedro Raúl JIMÉNEZ MACíAS** — *CINVESTAV, Mexico*

Laguerre–Gaussian optical beams as boson realizations of either $SU(1,1)$ or $SU(2)$ groups

The paraxial wave equation for the propagation of light in a weakly guiding inhomogeneous medium with quadratic refractive index can be addressed via the eigenvalue problem that includes the (stationary) guided Laguerre–Gaussian modes as eigenvectors and the spectrum of propagation constants as the set of eigenvalues. We are interested in non stationary wave-packet solutions of the above problem to construct the representation space of either $SU(1,1)$ or $SU(2)$ Lie groups. It is found that the generators of the related Lie algebras can be expressed in terms of the Laguerre–Gaussian intertwining operators, so the related coherent states can be constructed such that their propagation factors depend on the discrete parameters that define the group representation. Joint work with S. Cruz y Cruz, Z. Gress and O. Rosas-Ortiz.

27. **Maria KARMANOVA** — *Sobolev Institute of Mathematics / Novosibirsk State University, Russia*

Area Formula for Graph Surfaces in sub-Lorentzian Geometry

The aim of the talk is to describe in sub-Lorentzian geometry surface images of graph mappings constructed using intrinsically Lipschitz mappings of Carnot groups. Sub-Lorentzian geometry can be considered as a sub-Riemannian version of well-known Minkowski geometry. The research in sub-Lorentzian geometry began only in recent years including its applications to physics. Moreover, structures with multi-dimensional time are also studied now.

We study graph mappings constructed from classes of intrinsically Lipschitz mappings of Carnot groups. Recall that intrinsically Lipschitz mappings on Carnot groups are Holder in the classical sense, nevertheless, they are differentiable in sub-Riemannian sense. Regarding the graphs of such mappings, it is known that they are not differentiable both in classical and in sub-Riemannian sense. We invent a special tool, the polynomial sub-Riemannian differentiability, that enables us to derive "differential" properties of graph mappings.

We introduce the following sub-Lorentzian distance on the image of graph-mappings.

Definition. If $w = \exp\Big(\sum\limits_{j=1}^{N+\widetilde{N}} y_j Y_j\Big)(v)$ then *squared sub-Lorentzian distance* is equal to

$$\mathfrak{d}_2^2(v,w) = \max_{k=1,\dots,\widehat{M}} \Bigg\{ \operatorname{sgn}\Big(\sum_{j:\, Y_j \in \widehat{V}_k^+} y_j^2 - \sum_{j:\, Y_j \in \widehat{V}_k^-} y_j^2 \Big) \cdot \Big| \sum_{j:\, Y_j \in \widehat{V}_k^+} y_j^2 - \sum_{j:\, Y_j \in \widehat{V}_k^-} y_j^2 \Big|^{1/k} \Bigg\}.$$

The corresponding sub-Lorentzian Hausdorff measure is defined by applying Caratheodory construction.

The main result is the theorem on sub-Lorentzian measure of graph surfaces.

Theorem. *The following area formula*

$$\int\limits_{\Omega} {}^{SL}\mathcal{J}(\varphi, v)\, d\mathcal{H}^{\nu}(v) = \int\limits_{\varphi_\Gamma(\Omega)} d\ {}^{SL}\mathcal{H}^{\nu}_{\Gamma}(y)$$

is valid with sub-Lorentzian Jacobian ${}^{SL}\mathcal{J}(\varphi, x)$ *equal to*

$$\sqrt{\det\big(E_{\dim V_1} + \big(\widehat{D}\varphi^+\big)^*_{\widetilde{V}_1,V_1}(x)\big(\widehat{D}\varphi^+\big)_{\widetilde{V}_1,V_1}(x) - \big(\widehat{D}\varphi^-\big)^*_{\widetilde{V}_1,V_1}(x)\big(\widehat{D}\varphi^-\big)_{\widetilde{V}_1,V_1}(x)\big)} \times \\ \times \prod_{j=2}^{M} \sqrt{\det\big(E_{\dim V_j} - \big(\widehat{D}\varphi^-\big)^*_{\widetilde{V}_j,V_j}(x)\big(\widehat{D}\varphi^-\big)_{\widetilde{V}_j,V_j}(x)\big)}.$$

The publication was supported by the Ministry of Education and Science of the Russian Federation (the Project number 1.3087.2017/4.6).

28. **Bartosz KWAŚNIEWSKI** — *Uniwersytet w Białymstoku, Poland*

Stone duality and quasi-orbit spaces for C*-inclusions

We define and analyze quasi-orbit space and a quasi-orbit map for a general C*-inclusion. This is done by exploiting duality between sober spaces and spacial locales, as well as the Galois connection between restriction and induction ideals. The results apply to cross section C*-algebras of Fell bundles over locally compact groups, regular C*-inclusions, tensor products, relative Cuntz–Pimsner algebras, and crossed products for actions of locally compact Hausdorff groupoids and quantum groups. (Based on joint work with Ralf Meyer)

29. **Andrey MIRONOV** — *Novosibirsk State University, Russia*

Angular billiard

We introduce a dynamical system which we call Angular billiard. It acts on the exterior points of a convex curve in Euclidean plane. In a neighborhood of the boundary curve this system turns out to be dual to the Birkhoff billiard. Using this system we get results on algebraic Birkhoff conjecture on integrable billiards. The results were obtained with Misha Bialy.

30. **Ali MOSTAFAZADEH** — *Koc University, Turkey*

Quantum system with a dynamical state space and a geometric extension of quantum mechanics

We address the problem of defining the energy observable for a quantum system whose state space depends on time. The solution leads to a moderate geometric extension of QM where the role of the Hilbert space and the Hamiltonian operator is played by a complex Hermitian vector bundle E endowed with a metric-compatible connection and a global section of a real vector bundle determined by E. The axioms of QM are not replaced by others but elevated to the level of the relevant bundles. The standard description of quantum systems in terms of a Hilbert space and a Hamiltonian operator, which respectively determine the kinematical and dynamical properties of the system, is recovered locally, i.e., in local patches of E. A major part of this work is conducted during WGMP XXXVI which was held in Białowieża in 2017.

Reference: Phys. Rev. D 98, 046022 (2018), arXiv: 1803.04175.

31. **Zouhaïr MOUAYN** — *Sultan Moulay Slimane University, Morocco*

Nonlinear coherent states associated with a measure on the positive real half line

We construct a class of generalized nonlinear coherent states by means of a newly obtained class of 2D complex orthogonal polynomials. The associated coherent states transform is discussed. A polynomials realization of the basis of the quantum states Hilbert space is also given. Here, the entire structure owes its existence to a certain measure on the positive real half line, of finite total mass, together with all its moments. We illustrate this construction with the example of the measure $r^{\beta}e^{-r}dr$, which leads to a new generalization of the true-polyanalytic Bargmann transform.

Joint work with S.T. Ali and K. Ahbli.

32. **Vladimir NAZAIKINSKII** — *Ishlinsky Institute for Problems in Mechanics RAS, Russia*

Fock Quantization of Canonical Transformations and Semiclassical Asymptotics for Degenerate Problems

The linear theory of run-up of long waves on a shallow beach involves differential operators degenerating in a special way on the boundary of the domain where the problem is considered (e.g. the velocity in the wave equation vanishes on the boundary as the square root of the distance from the boundary). The construction of semiclassical asymptotics for such problems is given by a version of Maslov's canonical operator based on a peculiar phase space geometry and using the Hankel transform to express rapidly oscillating solutions near the boundary. Note that the Hankel transform arises here as the Fock quantization of a classical canonical transformation regularizing the Hamiltonian system associated with the problem. We use this example to discuss the approach to degenerate problems in which new classes of operators arise by quantization of degenerate classical objects.

33. **Yury NERETIN** — *Pauli Institute, Russia*

Rank one transformations of measure spaces

34. **Aleksandr ORLOV** — *Institute of Oceanology / High School of Economics, Russia*

Quantum chaos, integrals of tau functions, 2D Y-M and Hurwitz numbers

35. **Fernand PELLETIER** — *Université de Savoie Mont Blanc, France*

On the Finslerian entropy of smooth distributions and Stefan–Sussman foliations

Starting from the definition of the entropy of a growing family of distances on a compact metric space, we define the Finslerian entropy of a smooth distribution and of a Stefan–Sussman foliation. This notion of entropy is a generalization of most classical topological entropies on a Riemannian compact manifold: the entropy of a flow ([2]), of a regular foliation ([4]), of a regular distribution ([1]) and more generally of a "geometrical structure" ([5]). In this way, we obtain the nullity of the Finslerian entropy of a controllable distribution and of a singular Riemannian foliation.

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36. **Anatolij PRYKARPATSKI** — *Cracow University of Technology, Poland*

On the current algebra representations and quantum many-particle integrable Hamiltonian models

There is developed the G. Goldin's current algebra representation scheme for reconstructing quantum Hamiltonian and symmetry operators in case of quantum integrable spatially many- and one-dimensional Schrödinger type dynamical systems.

In the report we are interested mainly in studying local current algebra representations in suitably renormalized Fock spaces and their applications to constructing the related finite-particle factorized representations for corresponding secondly-quantized many-particle Hamiltonian operators. As examples we have studied in detail the factorized structure of Hamiltonian operators, describing such quantum integrable spatially many- and one-dimensional models as generalized oscillatory, Calogero–Sutherland, Coulomb type and Nonlinear Schrödinger dynamical systems of spin-less bose-particles.

Main topics to be discussed are as follows:

(a) An integrable many-particle oscillatory quantum model

(b) A generalized integrable many-particle oscillatory quantum model

(c) The Calogero–Sutherland quantum model: the current algebra representation, the Hamiltonian reconstruction and integrability

(d) An integrable many-particle Coulomb type quantum model on axis

(e) Quantum many-particle Hamiltonian dynamical system on axis with $\beta\delta$-interaction, its quantum symmetries and integrability

Conclusion: In the work we succeeded in developing an effective algebraic scheme of constructing density operator and functional representations for the canonical local quantum current algebra and its application to quantum Hamiltonian and symmetry operators reconstruction. We analyzed the corresponding factorization structure for quantum Hamiltonian operators, governing spatially many- and one-dimensional integrable dynamical systems. The quantum generalized oscillatory, Calogero–Sutherland, Coulomb type and Nonlinear Schrödinger models of spin-less bose-particles were analyzed in detail.

(jointly with: G. Goldin, D. Blackmore and D. Prorok)

37. Claudia QUINTANA — *CINVESTAV, Mexico*

Entangled bipartite qubit systems coupled to photon baths: in search of the master equation

The time-evolution of pure states ($\rho^2 = \rho$) associated with a closed bipartite system $\mathcal{S} = \mathcal{S}_A + \mathcal{S}_B$ is unitary and obeys the Heisenberg equation. By summing the degrees of freedom of the subsystem B we obtain the reduced state ρ_A of subsystem A, which is a mixed state ($\rho_A^2 \neq \rho_A$) in general. One may consider $\mathcal{S}_A$ as an open system that interacts with $\mathcal{S}_B$ in controlled form. The law of motion for $\mathcal{S}_A$ is not represented by the Heisenberg equation anymore, so that a master equation is necessary. We consider a closed tetra-partite system $\mathcal{S}$ composited by two entangled qubits and two quantized single-mode radiation fields; two isolated QED cavities contain a pair (qubit + field) each one. Our interest is addressed to investigate the master equation for the different configurations of subsystems $\mathcal{S}_A$ and $\mathcal{S}_B$ that can be obtained from $\mathcal{S}$. For instance, one of the configurations identifies $\mathcal{S}_A$ and $\mathcal{S}_B$ with the systems (qubit + field) in the cavities, other considers $\mathcal{S}_A$ as the subsystem qubit + qubit and $\mathcal{S}_B$ as the subsystem field + field, and so on. That is, we look for the master equation that provides ρ_A in agreement with the reduction of the pure states of $\mathcal{S}$ for each of the above mentioned configurations.

38. Oscar ROSAS-ORTIZ — *CINVESTAV, Mexico*

Parity-time symmetry in supersymmetric quantum mechanics: An interplay between Riccati, Ermakov and Schrodinger equations

Nonlinear Riccati and Ermakov equations are combined to pair the energy spectrum of two different quantum systems via the supersymmetric approach. One of the systems is assumed Hermitian, exactly solvable, with discrete energies in its spectrum. The other system is characterized by a complex-valued potential that inherits all the energies of the former one and includes an additional real eigenvalue in its spectrum. The balanced gain and loss profile of such potential gives rise to (but it is not restricted to) parity-time symmetry in the supersymmetric quantum systems so constructed.

39. **Andrzej SITARZ** — *Jagiellonian University, Poland*

On differential operators in noncommutative geometry.

The study of quantum (noncommutative) spaces requires better understanding of differential geometry adapted to this setup. I shall discuss the problem of distinguishing between the noncommutative counterparts of differential and pseudodifferential operators and demonstrate that many examples of constructed Dirac operators are, in the generalized sense, noncommutative differential operators.

40. **Stephen B. SONTZ** — *Centro de Investigacion en Matematicas, Mexico*

Coherent States for a Toeplitz Quantization of the Manin Plane

In the theory of Toeplitz quantization of algebras coherent states are defined as eigenvectors of a Toeplitz annihilation operator. These coherent states are studied in the case when the algebra is the generically non-commutative Manin plane. We introduce the resolution of the identity, upper and lower symbols as well as a coherent state quantization, which in turn quantizes the Toeplitz quantization. We thereby have a curious combination of quantization schemes which might be a novelty. We proceed by identifying a generalized Segal–Bargmann space $\mathcal{SB}$ of square-integrable, anti-holomorphic functions as the image of a coherent state transform. Then $\mathcal{SB}$ has a reproducing kernel function which allows us to define a secondary Toeplitz quantization, whose symbols are functions.

41. **Piotr STACHURA** — *SGGW, Poland*

On Zakrzewski category of groupoids

I will present (mostly the algebraic part) of an approach to groupoids started by S. Zakrzewski together with some results on formal properties of morphisms.

42. **Jaromir TOSIEK** — *Łódź University of Technology, Poland*

Phase space representation of a quantum particle with discrete internal degrees of freedom

The Weyl–Wigner–Moyal formalism for a quantum particle with discrete internal degrees of freedom is proposed. A one to one correspondence between operators in the Hilbert space $L^2(\mathbb{R}^3) \otimes \mathcal{H}^{(s+1)}$ and functions on the discrete phase space $\mathbb{R}^3 \times \mathbb{R}^3 \times \{0, \ldots, s\} \times \{0, \ldots, s\}$ is presented. The expressions for the Stratonovich–Weyl quantizer, star product and Wigner functions of such systems are obtained.

43. **Barbara TUMPACH** — *University of Lille, France*

Banach Poisson–Lie groups

We will present the theory of Banach Poisson–Lie groups and explain the main difficulties arising from the infinite-dimensional context.

44. **Alexander ZHEGLOV** — *Moscow State University, Russia*

Algebraic geometric properties of spectral surfaces of quantum integrable systems and their isospectral deformations

Quantum integrable systems, or rings of commuting partial differential operators, and their isospectral deformations admit a convenient algebraic geometric description if they are considered as subrings in a certain "universe" ring — a purely algebraic analogue of the ring of pseudodifferential operators on a manifold. This description is a natural generalization of the classification of rings of commuting ordinary differential or difference operators described in the works of Krichever, Novikov and Mumford.

Already in the case of dimension two there are significant restrictions on the geometry of spectral varieties, and therefore the question of their classification arises. I'll talk about recent results on possible types of smooth spectral surfaces. The talk is based on joint work with Vik. S. Kulikov.

45. **Henryk ŻOŁĄDEK** — *Uniwersytet Warszawski, Poland*

Birkhoff normalization, Bifurcations of Hamiltonian systems and the Deprits formula

We consider Hamiltonian autonomous systems with n degrees of freedom near a singular point. In the case of absence of resonances of order <5 we present a direct computation of the Birkhoff normal form. In the case of two degrees of freedom we study one-parameter deformations of the $0:1$; $1:1$ and $2:1$ resonant singularities. The obtained results are used in a direct derivation of the Deprits formula for the isoenergetic degeneracy determinant in the restricted three body problem.

Finally, we present a new case of Lyapunov stable libration point in the restricted four body problem.

Poster session

46. **Goce CHADZITASKOS** — *Czech Technical University in Prague, Czech Republic*

A Common Assessment Space for Different Sensor Structures

The study of the evolution process of our visual system indicates the existence of variational spatial arrangement; from densely hexagonal in the fovea to a sparse circular structure in the peripheral retina. Today's sensor spatial arrangement is inspired by our visual system. However, we have not come further than rigid rectangular and, on a minor scale, hexagonal sensor arrangements. Such a space is created by implementing a continuous extension of discrete Weyl Group orbit function transform which extends a discrete arrangement to a continuous one. The implementation of the space is demonstrated by comparing two types of generated hexagonal images from each rectangular image with two different methods of the half-pixel shifting method and virtual hexagonal method. In the experiment, a group of ten texture images were generated with variational curviness content using ten different Perlin noise patterns, adding to an initial 2D Gaussian distribution pattern image.

Joint work with Wei Wen, Ondřej Kajínek, Siamak Khatibi.

47. **Tomasz CZYŻYCKI** — *Uniwersytet w Białymstoku, Poland*

Generalized Multivariate Chebyshev Polynomials

Classes of generalized Chebyshev polynomials related to root systems of simple Lie algebras are introduced. The variables of the polynomials are given via the classical character functions of the Lie algebras. Depending on the type of the algebra there exist two, four or eight polynomial classes. The admissible shift of the weight lattice, which permits the constructions of the eight classes of the classical series C_n, directly generalizes the four classical kinds of Chebyshev polynomials. The discrete orthogonality relations over the sets of the generalized Chebyshev nodes are presented in detail and the forms of generating functions and cubature rules are discussed. This is a joint work with Jiří Hrivnák.

48. **Valeria GONZÁLEZ** — *Instituto Politécnico Nacional, Mexico*

Molien series and invariant polynomials in field theory

49. **Ashis MANDAL** — *Indian Institute of Technology Kanpur, India*

On Hom-Lie-Rinehart algebras

We describe the notion of hom-Lie-Rinehart algebras as an algebraic analogue of hom-Lie algebroids. We define modules (left and right) over this hom-structure and describe homology and cohomology complexes by considering coefficient modules. In the sequel, we consider some special classes of hom-Gerstenhaber algebras and their relationship with hom-Lie algebroids.

50. **Filip NEMEC** — *Czech Technical University in Prague, Czech Republic*

Using adiabatic quantum computers to solve SAT3

The adiabatic theorem in quantum mechanics enables us to derive new algorithms in quantum computation and gives us a new perspective on quantum computation. Using the adiabatic theorem we are able to find ground eigenstates of a complicated Hamiltonian H_1 by constructing a simpler time dependent Hamiltonian H_0 and letting H_0 adiabatically evolve to the given Hamiltonian H_1. We compare the basic concepts of adiabatic quantum computation (AQC) with more common approaches and demonstrate SAT3 solution using the AQC.

51. **Petr NOVOTNÝ** — *Czech Technical University in Prague, Czech Republic*

Classification of twisted cocycles

Classification of Lie algebras is a complicated task with difficulty rapidly growing with their dimension. Complete classification is known up to dimension 6 only. Even in low dimensional cases, whenever there appears parametric continuum, distinguishing non-isomorphic algebras is demanding. One of the tools which simplifies the isomorphism problem are invariant characteristics, for example dimensions of the vector spaces of the standard cohomology cocycles. In order to obtain more invariants, the definition of cocycles is modified. The complete classification of the resulting twisted cocycles up to dimension 3 is presented as well as their role in classification of low-dimensional Lie algebras.

52. **Stanislav SKOUPÝ** — *Czech Technical University in Prague, Czech Republic*

State transfer by means of discrete-time quantum walks

We introduce the scheme of discrete-time quantum walk algorithm for the state transfer on highly symmetric graphs with two marked vertices, sender and receiver. We consider the state transfer algorithm based on modified discrete-time quantum walk with coins search algorithm. We show how to simplify the calculation by finding the invariant subspace of the walk with respect to the evolution operator of the walk. We calculate the number of steps and we prove that the perfect state transfer is achieved on the star graph, on the complete bipartite graph and on the complete graph with one self loop at each vertex. The number of steps of the walk for each case scale as $O(\sqrt{N})$ where N is number of vertices.

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