XXXIV WORKSHOP ON GEOMETRIC METHODS IN PHYSICS

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XXXIV Workshop on Geometric Methods in Physics organized by Institute of Mathematics, University of Białystok, Ciołkowskiego 1M, 15-245 Białystok, Poland, e-mail: wgmp@uwb.edu.pl

LIST OF ABSTRACTS

Plenary lectures

1. **Alexander ALLDRIDGE** — *Universität zu Köln, Germany*

   Radial parts of superdifferential operators, and applications to invariants

   Supergroup actions, slices, and radial parts

   A theorem of Helgason states that if an action of a Lie group on a manifold admits a slice, then any differential operator admits a 'radial' component on the slice. For actions of Lie supergroups, this is no longer true in general, as the naive slice condition is too weak. We introduce a strong notion of slice that allows for the generalisation of Helgason's theorem, using the concept of 'orbits through generalised points'.

   We study in particular the isotropy group action for a Riemannian symmetric superspace. In this setting, the strong slice condition holds if and only if the supermanifold is of 'even type' (i.e. the slice in the naive sense is an ordinary manifold).

   To remedy this situation, we consider a weak notion of slices. We prove in general that at least any differential operator locally invariant under the transporter supergroup of the weak slice admits a radial part on the quotient of the weak slice by the transporter supergroup. This quotient is (mildly) singular in general. For the example of Riemannian symmetric superspaces, the transporter supergroup action factors through the Weyl group, which is a reductive Lie group of positive dimension in the cases not of odd type.

   We apply these results to the determination of invariant functions on Riemann symmetric superspaces and their flat degenerations.

   This work is joint with J. Hilgert and T. Wurzbacher, and with K. Coulembier, respectively.

2. **Andrei DOMRIN** — *Moscow State University, Russia*

   Local inverse scattering

   We develop a local version of the inverse scattering method for studying soliton equations of parabolic type (including KdV, NLS, Boussinesq, but not sine-Gordon, for example). The potentials are germs of holomorphic matrix-valued functions, without any boundary conditions. The scattering data are matrix-valued formal power series in the spectral parameter. We give a precise description of all possible scattering data and exact criteria for solubility of the local holomorphic Cauchy problem for a soliton equation of parabolic type in terms of the scattering data of the initial conditions. Applications include the strongest possible version of the Painleve property (global meromorphic extension in x of any local holomorphic solution), the issues related to trivial monodromy, and a proof of divergence of the Kontsevich–Witten series with respect to all higher times.

3. **David J. FERNÁNDEZ** — *CINVESTAV, Mexico*

   Painlevé equations and SUSY QM

   An algorithm to generate solutions to the Painlevé IV and V equation will be introduced, which is based on supersymmetric quantum mechanics applied to the

harmonic and radial oscillators respectively. The corresponding solutions turn out to be expressed in terms of confluent hypergeometric functions, which include some solution hierarchies associated to more specific special functions they are related with.

4. **François GAY-BALMAZ** — *CNRS Ecole Normale Superieure de Paris, France*

   Dirac structures and reduction for nonholonomic mechanical systems on Lie groups with broken symmetry

   We present the theory of Dirac reduction by symmetry for nonholonomic systems on Lie groups with broken symmetry. The reduction is carried out for the Dirac structures, as well as for the associated Lagrange–Dirac and Hamilton–Dirac dynamical systems. This reduction procedure is accompanied by reduction of the associated variational structures on both Lagrangian and Hamiltonian sides. The theory is illustrated with the help of finite and infinite dimensional examples. In particular, we show that the equations of motion for second order Rivlin–Ericksen fluids can be formulated as an infinite dimensional nonholonomic system.

5. **Toshihiro IWAI** — *Kyoto University, Japan*

   Change in energy eigenvalues against parameters

   The Dirac equations of space-dimension two is studied under the APS boundary condition,where the mass is considered as a parameter ranging over all real numbers and where APS is an abbreviation of Atiyah-Patodi-Singer and the boundary condition requires that the boundary values of eigenstates of the Dirac equation should belong to the subspace of eigenstates associated with positive or negative eigenvalues for a boundary operator. The spectral flow for a one-parameter family of operators is the net number of eigenvalues passing through zeros in the positive direction as the parameter runs. It is demonstrated that the spectral flow for the Dirac equation with the APS boundary condition is $\pm$, depending on the sign of the total angular momentum eigenvalue. The chiral bag boundary condition is also treated as comparison's sake. In addition, discrete symmetry for the associated energy bands is discussed. Related topics will be touched upon, including winding numbers for the corresponding "semi-quantum" Hamiltonian, rotation-vibration systems in quantum chemistry for isolated molecules, comparison with topological insulators, and further extension of the present model. This talk is based on joint works with B. Zhilinskii at Université du Littoral Côte d'Opale.

6. **Bas JANSSENS** — *Universiteit Utrecht, The Netherlands*

   Representation theory of gauge groups

   As the name suggests, gauge groups arise as transformation groups of gauge theories, such as (Q)ED, (Q)CD and WZW-models. We give classification results for various types of smooth projective unitary representations, from the point of view that gauge groups are Lie groups modelled on an infinite dimensional vector space.

7. **Tom LADA** — *North Carolina State University, USA*

   Homotopy Derivations of Infinity Algebras

   We will define strong homotopy derivations of A-infinity and L-infinity Algebras. We will show that symmetrization of a strong homotopy derivation of an A-infinity

algebra yields a strong homotopy derivation of the symmetrized L-infinity algebra. We will give examples of strong homotopy derivations that generalize inner derivations.

8. **Andrey LAZAREV** — *University of Lancaster, United Kingdom*

Derived localization and infinity local systems

Localization of rings and modules is one of the basic tools in commutative algebra. In contrast, localizing noncommutative rings is a much more delicate procedure, primarily because of the non-exactness of the corresponding functor. I will show how to construct localization in a noncommutative context in such a way that exactness is manifestly present. As an application, I will describe a higher, or topological, form of the Riemann-Hilbert correspondence (recall that the classical Riemann-Hilbert correspondence is an equivalence of categories between local systems on a manifold and flat vector bundles).

9. **Angela PASQUALE** — *Université de Lorraine, France*

Resonances for the Laplacian on Riemannian symmetric spaces

Let D be the Laplacian on a Riemannian symmetric space of the noncompact type $X = G/K$, and let $s(D)$ denote its spectrum. The resolvent $R(z) = (D - z)^{-1}$ is a holomorphic function on $C \setminus s(D)$, with values in the space of bounded operators on $L^2(X)$. We study the meromorphic continuation of R as a distribution valued map on a Riemann surface above $C \setminus s(D)$. If such a meromorphic continuation is possible, then the poles of the meromorphically extended resolvent are called the resonances. When all Cartan subgroups of G are conjugate, then there are no resonances. In other examples the resonances exist and can be explicitly determined. They are linked to the spherical principal series representations of G. This talk is based on joint works with Joachim Hilgert (Universität Paderborn) and Tomasz Przebinda (University of Oklahoma).

10. **Fernand PELLETIER** — *Université de Savoie, France*

Integrability of distributions on infinite dimensional manifolds and applications

In differential geometry, a distribution on a smooth manifold M is an assignment

$$\mathcal{D} : x \mapsto \mathcal{D}_x \subset T_x M$$

on M, where $\mathcal{D}_x$ is a subspace of $T_x M$. The distribution is integrable if, for any $x \in M$ there exists an immersed submanifold $f : L \to M$ such that x belongs to $f(L)$ and for any $z \in L$ we have $Tf(T_z L) = \mathcal{D}_{f(z)}$. On the other hand, $\mathcal{D}$ is called involutive if, for any vector fields X and Y on M which are tangent to $\mathcal{D}$, the Lie bracket $[X, Y]$ is also tangent to $\mathcal{D}$. The distribution is invariant if for any vector field X tangent to $\mathcal{D}$, the flow ϕ_t^X leaves $\mathcal{D}$ invariant. On finite dimensional manifold, when $\mathcal{D}$ is a sub-bundle of TM, the classical Frobenius Theorem gives an equivalence between integrability and involutivity. In the other case, the distribution is â€{singularâ€{ and even under assumptions of smoothness on $\mathcal{D}$, in general, the involutivity is not a sufficient condition for integrability (we need some more additional local conditions). These problems were clarified and resolved essentially in [Su], [St].

We will explain how these results in finite dimension can be extended to the context of Banach manifolds and also in the frame work of direct limit of Banach

manifolds. We will end by application of these results in the framework of Banach Poisson manifolds and of direct limits of Banach Lie algebroids.

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11. **Florin RĂDULESCU** — *University of Rome II/Institute of Mathematics of the Romanian Academy, Italy*

Unbounded Hochschild cohomology, Berezin quantization, free group factors

We present the construction of a canonical obstruction, coming from Berezin's deformation, in the study of the isomorphism problem for free group factors and related problems.

12. **Tatsuya TATE** — *Tohoku University, Japan*

Quantum walks in one dimension

The notion of quantum walks was originally given as quantum random walks in quantum physics in 1993 and was re-discovered, as in the form recently adopted, in computer science around 2001. It is defined as a unitary operator on a Hilbert space defined over graphs, and it is regarded as a non-commutative version of classical random walks. Non-commutativity affects asymptotic properties of quantum walks which are quite different from classical random walks. In this talk, after a review on the classical random walks, definitions and simple properties of one-dimensional quantum walks will be given. Some asymptotic results of quantum walks, which are quite different from classical one, will be explained. Recently an algebraic aspect of one-dimensional quantum walks has become considered. A concrete formula for the powers of quantum walks based on the algebraic structure will be explained.

13. **Siye WU** — *National Tsing Hua University, Taiwan*

Non-orientable surfaces and S-duality

In this talk, I explain the role of non-orientable surfaces in twisted $N = 4$ supersymmetric Yang-Mills theory in 4 dimensions whose compactification along orientable surfaces yields mirror symmetry and geometric Langlands program as studied by Kapustin and Witten. I relate the discrete electric and magnetic fluxes of 't Hooft in 4 dimensions to the topology of moduli spaces.

14. **Maxim ZABZINE** — *Uppsala University, Sweden*

Localization techniques in quantum field theory

15. **Nguyen Tien ZUNG** — *Université de Toulouse, France*

Action-angle variables: old and new

Contributed lectures

16. **Alexander BELAVIN** — *Landau Institute for Theoretical Physics RAS, Russia*

    Frobenius manifolds, Integrable systems and String theory

17. **Daniel BELTIŢĂ** — *Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Romania*

    Quantization and C^*-algebras of nilpotent Lie groups

    We will describe the image of operator valued Fourier transforms on general nilpotent Lie groups. Our approach relies on suitable stratifications of the duals of nilpotent Lie algebras and a Lie theoretic method of quantization of coadjoint orbits based on global canonical coordinates. We thus show that the C^*-algebra of any nilpotent Lie group is a solvable C^*-algebra with special spectral properties. A characterization of Heisenberg groups in terms of group C^*-algebras will be provided along these lines. The talk is based on joint work with Ingrid Beltiţă and Jean Ludwig.

18. **Andrew BRUCE** — *Instytut Matematyczny Polskiej Akademii Nauk, Poland*

    Higher order mechanics on graded bundles

    We discuss the applications of the recently discovered weighted Lie algebroids to the theory of higher order Lagrangian mechanics on graded bundles following the geometric ideas of Tulczyjew. As a particular example we will focus on higher order mechanics on Lie algebroids, which is motivated by reductions of higher order systems that posses symmetries.

19. **Tomasz BRZEZIŃSKI** — *Swansea University, United Kingdom*

    Rota-Baxter systems

    We introduce a pair of operators similar to and encompassing the Rota-Baxter operators, and we describe algebraic systems associated to such pairs.

20. **Goce CHADZITASKOS** — *České Vysoké Učení Technické v Praze, Czech Republic*

    Polytopes vibrations within Coxeter group symmetry

    A simple method of using the symmetry of polytopes in order to determine four resonant frequencies is presented. Knowledge of these frequencies, or at least their ratios can be used for control of some principal changes of polytopes.

21. **Andreas DESER** — *Institut für Theoretische Physik, Leibniz Universität Hannover, Germany*

    Star products on graded manifolds and deformations of Courant algebroids from string theory

    Deformations of Courant algebroids are of interest in both, string theory and mathematics. It was realized by Roytenberg, that Lie bialgebroids and their associated Courant algebroids can be characterized by a homological vector field on

the cotangent bundle of the parity reversed version of the underlying Lie algebroid. This lead to the introduction of the Drinfel'd double of a Lie bialgebroid. In a similar way, we show that the so-called C-bracket, a bi-linear operation governing the gauge algebra of double field theory, can be characterized by the Poisson structure on the Drinfel'd double of the underlying Lie-bialgebroid. Using this result, we are able to apply a graded version of the Moyal-Weyl star product to compute the first order deformation of the C-bracket. Remarkably, these coincide with the first order correction in the string coupling parameter found recently in string theory.

22. **Guillaume DHONT** — *Université du Littoral Côte d'Opale, France*

Symbolic interpretation of the Molien function: free and non-free modules of covariants

A problem from molecular physics led us to the investigation of the algebraic structures of the polynomials generated from the (x_i, y_i) components of n vectors in a plane with a common origin. The symmetry group is assumed to be $SO(2)$ and the irreducible representations (m) are labelled by the integer m. The ring of invariants $(m = 0)$ is Cohen-Macaulay. The module of covariants is free when $|m| < n$ but is non-free if $|m|$ is greater or equal than the number of vectors. We discuss the symbolic interpretation of the corresponding Molien functions in term of integrity bases and propose a graphical representation of the structure of the modules.

23. **Djelloul DJEBBOURI** — *Université Dr Taher Moulay – Saïda, Algeria*

Dualistic structures on generalized warped products

In this work, we generalize the dualistic structures on warped product manifold to the dualistic structure on doubly warped prodrct and generalized warped product manifolds. We construct a symmetric tensor field $G_{f_1 f_2}$ on product manifold and we give conditions under wich $G_{f_1 f_2}$ becomes a metric tensor. These a tensor field will be called the generalized warped product and then we develope an expression of curvature for the connexion of the generalized warped product in relation to those corresponding analogue of its base and fiber and warping functions. By construction a frame field in $M_1 * M_2$ with respect to the riemannian metric G, then we calculate the Laplace-Beltrami operator of a function on generalized warped product wich may be expressed in terme of the local restrictions of the functions to the base and fiber. Finally, we show also that the dualistic structures on the base M_1 and fiber M_2 manifold induce on the generalized warped product $M_1 * M_2$ a dualistic structure.

24. **Michał DOBRSKI** — *Politechnika Łódzka, Polska*

Generalized Fedosov algebras and noncommutative gravity

Simple generalization of Fedosov construction based on fairly general (not necessarily Moyal) fiberwise star product in the Weyl bundle will be presented. The construction is motivated by a possible application in noncommutative field theory, particularly in noncommutative gravity. The basic idea of this application is to introduce the spacetime metric into the deformation quantization scheme from the very begining, as the "symmetric part of the noncommutativity tensor". It turns out, that such approach can be covered by the considered generalization. Remarks on some particular noncommutative gravity model built within this framework will be provided.

25. **Vladimir DRAGOVIĆ** — *The University of Texas at Dallas, United States*

The Painleve VI equations, Poncelet polygons, and the Schlesinger equations

In 1990's Hitchin constructed explicit algebraic solutions to the Painlevé VI (1/8,-1/8,1/8,3/8) equation associated to the Poncelet polygons, inscribed in a conic and circumscribed about another conic. We will show that Hitchin's construction is the Okamoto transformation between Picard's solution and the general solution of the Painlevé VI (1/8,-1/8,1/8,3/8) equation and it can be formulated in an invariant way, in terms of an Abelian differential of the third kind on the associated elliptic curve. The last observation allows to obtain solutions to the corresponding Schlesinger system in terms of this differential as well. The solution of the Schlesinger system admits a natural generalization to higher genera, and it is related to higher-dimensional Poncelet polygons. This is a joint work with V. Shramchenko. The research is supported by NSF grant no. 1444147.

26. **Mahouton Norbert HOUNKONNOU** — *International Chair in Mathematical Physics and Applications, Benin*

Center-symmetric algebras and bialgebras: relevant properties and consequences

Lie admissible algebra structures, called center - symmetric algebras, are defined. Main properties and algebraic consequences are derived and discussed. Bimodules are given and used to build a center -symmetric algebra on the direct sum of a center - symmetric algebra and a vector space. Then the matched pair of center - symmetric algebras is established and related to the matched pair of sub - adjacent Lie algebras. Besides, Manin triple of center - symmetric algebras is defined and linked with their associated matched pairs. Further, center - symmetric bialgebras of center - symmetric algebras are investigated and discussed. Finally, a theorem yielding the equivalence between Manin triple of center - symmetric algebras, matched pairs of Lie algebras and center - symmetric algebras, and center - symmetric bialgebra is provided.

27. **Jiří HRIVNÁK** — *České Vysoké Učení Technické v Praze, Czech Republic*

Four types of orthogonal polynomials of affine Weyl groups

The link between the Fourier calculus of the four families of special functions associated to root systems, $C-$, $S-$, S^s- and S^l-functions, and the four families of the induced orthogonal polynomials is discussed. The affine Weyl groups corresponding to the root systems of simple Lie algebras are recalled and sign homomorphisms, which allow general explicit description of the orbit functions, are described. Both continuous and discrete orthogonality of the four types of orbit functions are detailed for each type and the weights, which label the orthogonal functions, are chosen for each type of function and orthogonality separately. The four types of Chebyshev-like orthogonal polynomials, induced by the four types of orbit functions, inherit the discrete and continuous orthogonality from the orbit functions. The discrete and continuous orthogonality of the polynomials are explicitly formulated; their application for the development of the related Fourier methods and possible physical applications are discussed.

28. **Hassan JOLANY** — *University of Lille 1, France*

Song-Tian theory in Birational geometry

Recently Gang Tian jointly with Song introduced a new method in Minimal model program in Birational geometry by using Kahler Ricci flow theory. He proposed

a program of finding canonical metrics on canonical models of projective varieties of positive Kodaira dimension. They gave an affirmative answer to finding canonical metrics for complex surfaces by using modified Kahler Ricci flow. They introduced generalized Kahler Einstein Metric by using Weil-Petersson metric on moduli space of Calabi Yau manifolds and considered the existence of generalized Kahler Einstein metrics. I try to explain this theory.

29. **Dmitry KAPARULIN** — *Tomsk State University, Russia*

Symmetries, conservation laws and Lagrange anchors in linear systems

30. **Vladimir MOLCHANOV** — *Derzhavin Tambov State University, Russia*

Canonical representations for hyperboloids: an interaction with an overalgebra

Let $\mathcal{X}$ be a homogeneous space G/H where $G = \mathrm{SO}_0(p,q)$, $H = \mathrm{SO}_0(p,q-1)$, a hyperboloid in $\mathbb{R}^n$, $n = p+q$. Canonical representations of G on the hyperboloid $\mathcal{X}$ are defined as restrictions to G of maximal degenerate series representations of an overgroup $\widetilde{G} = \mathrm{SL}(n,\mathbb{R})$. We write explicitly an interaction of Poisson and Fourier transforms for canonical representations with Lie operators of $\widetilde{G}$. For elements of the Lie algebra $\widetilde{\mathfrak{g}}$ of $\widetilde{G}$ not belonging to the Lie algebra $\mathfrak{g}$ of G, this interaction contains differential operators of the fourth, second and zero orders.

31. **Rafik NASRI** — *Université Dr Taher Moulay – Saïda, Algeria*

Warped Poisson brackets on warped products

In this work, we generalize the geometry of the product pseudo-Riemannian manifold equipped with the Poisson structure to the geometry of warped product of pseudo-Riemannian manifold equipped with a warped Poisson structure. We construct three bivector fields on a product manifold and we show that each of them under certain condition to a Poisson structure. One of these bivector fields will be called warped bivector field. For a warped product of pseudo-Riemannian manifolds equipped with a warped bivector field, we compute the corresponding contravariant Levi-Civita connexion and the curvatures associated with.

32. **Yurii NERETIN** — *Institute for Theoretical and Experimental Physics, Russia*

Thompson groups and mapping class groups

33. **Petr NOVOTNY** — *České Vysoké Učení Technické v Praze, Czech Republic*

Twisted Cocycles of Lie algebras

Standard cohomology cocycles corresponding to adjoint representation form a vector space which dimension is an invariant for considered Lie algebra. In order to get more invariants of original Lie algebra, we modify the definition of cocycles using complex parameters. The complete classification of these twisted cocycles for $q = 1, 2, 3$ will be presented.

34. **Zbigniew PASTERNAK-WINIARSKI** — *Politechnika Warszawska, Poland*

Ramadanov theorem for weighted Bergman kernels

We study the limit behavior of weighted Bergman kernels on a sequence of domains in complex space $\mathbb{C}^N$, and show that under some conditions on domains and weights, weighed Bergman kernels converge uniformly on compact sets. Then we give a weighted generalization of the theorem given by Skwarczyński, highlighting some special property of the domains, on which the weighted Bergman kernels converge uniformly. Moreover we will show that convergence of weighted Bergman kernels implies this property.

35. **Tomasz PRZEBINDA** — *University of Oklahoma, USA*

Howe's correspondence and characters

36. **Jakub REMBIELIŃSKI** — *Uniwersytet Łódźki, Poland*

Title to be announced

37. **Martin SCHLICHENMAIER** — *University of Luxembourg, Luxembourg*

N-point Virasoro algebras are multi-point Krichever–Novikov type algebras

We show how the recently again discussed N-point Witt, Virasoro, and affine Lie algebras are genus zero examples of the multi-point versions of Krichever–Novikov type algebras as introduced and studied by the speaker. Using this more general point of view, useful structural insights and an easier access to calculations can be obtained. The concept of almost-grading will yield information about triangular decompositions which are of importance in the theory of representations. Examples are the algebra of functions, vector fields, differential operators, current algebras, affine Lie algebras, Lie superalgebras and their central extensions. Of particular interest is also the 3-point situation.

38. **Armen SERGEEV** — *Steklov Mathematical Institute, Russia*

Adiabatic limit in Ginzburg-Landau and Seiberg-Witten equations

39. **Alexey SHARAPOV** — *Tomsk State University, Russia*

Variational tricomplex of a local gauge system

I shall present the concept of a variational tricomplex that can be defined for any gauge system. The variational tricomplex provides an efficient tool for describing such notions as global symmetries, conservation laws, and the Lagrange structures associated with gauge dynamics. It also allows us to establish a direct relationship between the BV and BFV-BRST formalisms as well as their non-Lagrangian and non-Hamiltonian counterparts. In pure algebraic terms, one can regard this relationship as that between the S_∞- and P_∞-algebras underlying the gauge dynamics.

40. **Ekaterina SHEMYAKOVA** — *State University of New York at New Paltz, USA*

Darboux Transformations for Differential Operators on the Superline

We give a full description of Darboux transformations of any order for arbitrary (non-degenerate) differential operators on the superline. We show that every Darboux transformation of such an operator factorizes into elementary Darboux transformations of order one. Similar statement holds for operators on the ordinary line. (Joint work with Ted Voronov and my student Sean Hill.)

41. **Aneta SLIŻEWSKA** — *Uniwersytet w Białymstoku, Poland*

Poisson geometry related to Atiyah sequences

We construct and investigate a short exact sequence of Poisson $\mathcal{V}B$-groupoids, which is canonically related to the Atiyah sequence of a G-principal bundle P. Our results include a description of the structure of the symplectic leaves of the Poisson groupoid $\frac{T^*P\times T^*P}{G} \rightrightarrows \frac{T^*P}{G}$.

42. **Alfonso Giuseppe TORTORELLA** — *Università degli Studi di Firenze, Italy*

Deformations of coisotropic submanifolds in abstract Jacobi manifolds

In our work [3], using the Atiyah algebroid and first order multi-differential calculus on non-trivial line bundles, we attach an L_∞-algebra to any coisotropic submanifold S in an abstract (or Kirillov's) Jacobi manifold. Our construction generalizes and unifies analogous constructions in [4] (symplectic case), [1] (Poisson case), [2] (locally conformal symplectic case). As a new special case, we attach an L_∞-algebra to any coisotropic submanifold in a contact manifold, including Legendrian submanifolds. The L_∞-algebra of a coisotropic submanifold S governs the (formal) deformation problem of S.

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[3] H.V. Lê, Y.-G. Oh, A.G. Tortorella, and L. Vitagliano, Deformations of coisotropic submanifolds in abstract Jacobi manifolds, arXiv:1410.8446.

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43. **Theodore VORONOV** — *University of Manchester, United Kingdom*

Microformal geometry

In search of L-infinity mappings between homotopy Poisson algebras of functions, we discovered a construction of a certain nonlinear analog of pullbacks. Underlying such "nonlinear pullbacks" there is a formal category which is a "formal thickening" of the usual category of (super)manifolds. It has the same set of objects, but more morphisms. These "thick" or "microformal" morphisms between supermanifolds are defined by particular formal canonical relations between the (anti)cotangent bundles. Thick morphisms have beautiful properties. For example, we can define the adjoint of a nonlinear morphism of vector bundles as a thick

morphism of the dual bundles, so that it becomes the ordinary adjoint in the linear case. (See preprints: `arxiv.org/abs/1409.6475` and `arxiv.org/abs/1411.6720`.)

In the talk, I will explain the construction of microformal morphisms and nonlinear pullbacks, and their applications to homotopy Poisson structures, vector bundles and L-infinity algebroids. Recently, a quantum version was also discovered, based on some oscillatory integral operators. (See `arxiv.org/abs/1506.02417`.)

44. **Paweł WÓJCICKI** — *Politechnika Warszawska, Poland*

Biholomorphically invariant metrics induced by weighted Bergman kernel

In around 1947 Stefan Bergman introduced biholomorphically invariant metric by means of Bergman kernel of the domain. A little later in 1969 (around) Maciej Skwarczyński introduced another biholomorphically invariant metric by means of Bergman kernel of a domain as well. It is still not clear whether these metrics are equivalent or not. In this talk we will introduce similar metrics induced by weighted Bergman kernels and try to characterize the domains of holomorphy by means of them. We will be mainly focused on weighted Skwarczyński distance.

Poster session

45. **Mohammed ABDELMALEK** — *Université Abou Bekr Belkaid Tlemcen, Algeria*

Transversal submanifolds in higher codimension

In this talk, we establishe a relantionship between generalized Newton transformations (GNT) of a submanifold and the transversality of this manifold to another one. In particular we generalize the case of transversal hypersurfaces.

46. **Yury GRIGORYEV** — *Saint Petersburg State University, Russia*

On bi-Hamiltonian formulation of the perturbed Kepler problem

The perturbed Kepler problem is shown to be a bi-Hamiltonian system in the domain of definition of the action-angle variables. The graph of the corresponding Hamilton function is not a hypersurface of translation and, therefore, it constitutes a counterexample to the original Fernandes theorem about the necessary and sufficient conditions for the existance of bi-Hamiltonian structures around a Liouville torus.

47. **Lenka HÁKOVÁ** — *České Vysoké Učení Technické v Praze, Czech Republic*

On generalization of Weyl group orbit functions

Weyl group orbit functions are defined in the context of Weyl groups of simple Lie algebras. They are multivariable complex functions possessing remarkable properties such as (anti)invariance with respect to corresponding Weyl group, continuous and discrete orthogonality. Crucial tool in their definition are so-called sign homomorphisms, which coincide with one-dimensional irreducible representations. In this work we generalize the definition of orbit functions using irreducible characters of representations of higher dimensions. We describe their properties and give examples.

48. **Grzegorz JAKIMOWICZ** — *Uniwersytet w Białymstoku, Poland*

Title to be announced

49. **Robert JANKOWSKI** — *Uniwersytet w Białymstoku, Poland*

The function of information processing intensity as a tool to estimate parameters of probability distributions

50. **Michał JÓŹWIKOWSKI** — *Institute of Mathematics, Polish Academy of Sciences, Poland*

Title to be announced

51. **Ondřej KAJÍNEK** — *České Vysoké Učení Technické v Praze, Czech Republic*

Generalizing convolution to orbits of Weyl groups of rank 2

Weyl group orbit functions provide generalization of common cosine, sine and exponential functions which are used for generalization of goniometric transforms. Convolution theorem uses goniometric transform to map convolution into multiplication. Generalized transforms give us the chance to explore different types of orbit convolution theorems and orbit convolutions themselves. We present three types of orbit convolutions and related orbit convolution theorems. We show several results of image filtering using orbit convolution as a demonstration of its possibilities.

52. **Hirosuke KUWABARA** — *Tokyo Metropolitan University, Japan*

A time dependent Pais–Uhlenbeck oscillator and its decomposition

The Pais–Uhlenbeck (PU) oscillator is the simplest model with higher time derivatives, and its properties were studied for a long time. In this paper, we extend the 4th order free PU oscillator to a non-trivial case, dubbed the 4th order time dependent PU(tdPU) oscillator, which has time dependent frequencies. We show that this model cannot be decomposed into two harmonic oscillators in contrast to the original PU oscillator by a linear coordinate canonical transformation showed by Smilga. As a result of sustaining canonicality of this transformation for the tdPU oscillator, an interaction is added.

53. **Barbara ŁUPIŃSKA** — *Uniwersytet w Białymstoku, Poland*

Title to be announced

54. **Justyna MAKOWSKA** — *Uniwersytet w Białymstoku, Poland*

One-sided invertibility of binomial functional operators in spaces L^p

One-sided invertibility condition of binomial functional operators in space L^p is given for operators generated by a Morse-Smale type mappings.

55. **Lenka MOTLOCHOVA** — *České Vysoké Učení Technické v Praze, Czech Republic*

Cubature formulas of multivariate polynomials arising from symmetric orbit functions

It is shown that symmetric orbit functions, known from irreducible representations of simple Lie groups, have applications in numerical analysis. In particular, the study of remarkable properties of these functions yields cubature formulas, approximating a weighted integral of any function connected to any simple Lie group, by a weighted finite sum of function values. We summarize ideas leading to such formulas and present explicit results for simple Lie groups of rank two. We also discuss an optimal and cubature approximation of any function by multivariate polynomials arising from symmetric orbit functions and provide examples related to the Lie group C_2.

56. **Morteza SOLTANI** — *Isfahan University, Iran*

Particle Creation in Presence of Medium

In this work particle creation is extended to a dissipative massless scalar field in 1+1 dimension satisfying a Robin boundary condition (BC) at a non-relativistic moving boundary. For this purpose we define a Lagrangian for scalar field and then constitutive equations for the scalar field and medium are obtained as the Euler-Lagrange equations using the Lagrangian of a total system. We derive input and output bosonic field operators. Which allows us to calculate the spectral distribution of created particles. In addition we applied our result for particle creation in presence of a dissipative medium by using Lorentz oscillator model of medium. And we will show that particle emission in presence of medium for Dirichlet boundary condition can be considerably increased and for Neumann one it can be decreased.

57. **Nasrin SOLTANI** — *Payame Noor University, Iran*

Entanglement generation in Graphene layer

In this work we will consider an electron in a graphene layer which is interacting with a spin impurity. In this way we will solve the Dirac equation and we will calculate the entanglement between impurity and the electron. We also will consider two localized impurity and calculate entanglement between these two impurities after interacting with an electron.

58. **Elwira WAWRENIUK** — *Uniwersytet w Białymstoku, Poland*

Classical and quantum Kummer shapes

Gérard G. Emch Memorial Session

Prof. Stephen L. Adler from the Institute for Advanced Studies sent us the following message:

"I first made contact with the work of Gerard Emch when I was interested in quaternionic quantum mechanics, and studied his seminal papers on the subject. In my book on Quaternionic Quantum Mechanics and Quantum Fields, I made extensive use of a line of reasoning initiated by Emch to prove that the S-matrix in quaternionic scattering is complex and to improve on his results for Poincaré group representations. (Page 34 of my book gives a list of sections where I used his method.) Later on I visited Gérard in Florida, where he was a most gracious host, and our conversations on quaternionic projective group representations led to a joint paper reconciling our different approaches to the topic. Gerard was an outstanding mathematical physicist and scientist; it was a privilege to know him."

Steve Adler

Plenary lectures

59. **Geoffrey SEWELL** — *Queen Mary University of London, United Kingdom*

Hyperbolic Quantum Flows and the Question of Quantum Chaos

Hyperbolic flows, as formulated by Anosov, are the prototype of classical dynamic systems that evolve chaotically. In this talk I shall discuss their quantum counterparts, as formulated by Emch, Narnhofer, Sewell and Thirring within the operator algebraic setting of quantum theory; and I shall discuss their bearing on the question of quantum chaos.

60. **Kalyan Bidhan SINHA** — *J.N. Centre for Advanced Scientific Research, India*

Hyponormal Operators and Krein's Trace Formula

There are nice Trace formulae related to Hyponormal operators and a comparison between these and the Krein's Trace formula is studied. Multi-operator variable cases also are considered.

Invited lectures

61. **Syed Twareque ALI** — *Concordia University, Canada*

Complex orthogonal polynomials and coherent states

We discuss a newly constructed method for obtaining a class of generalized complex orthogonal polynomials and their connection to certain families of coherent states.

62. **Fabio BAGARELLO** — *Universitá di Palermo, Italy*

Mathematical aspects of non self-adjoint Hamiltonians, with applications

We discuss some mathematical aspects of deformed canonical commutation and anti-commutation relations, and we show how these relations are related to some particular non self-adjoint Hamiltonians with real eigenvalues. We also discuss the dynamical problem for this kind of operators.

63. **Pierre BIELIAVSKY** — *Université Catholique de Louvain, Belgium*

On Zagier's product and Rankin-Cohen brackets

64. **Antoinette EMCH-DERIAZ** — *University of Florida, USA*

The Gerard I knew for sixty years

65. **Ichiro FUJIMOTO** — *Kanazawa Institute of Technology, Japan*

Entropy of completely positive maps and applications to quantum information theory

In scope of CP-convexity which describes quantum entanglement, we study mathematical structure of quantum interactions, and propose new entropy, which we call "atomic entropy", for completely positive maps. Applying this notion, we propose new information quantities which recover the natural meaning and inequalities in the quantum information theory.

66. **Gerald GOLDIN** — *Rutgers University, USA*

Some comments on indistinguishable particles and interpretation of the quantum-mechanical wave function

67. **Ryszard KOSTECKI** — *Perimeter Institute for Theoretical Physics, Canada*

Operator-algebraic quantum foundations revisited

One of the returning main themes in G.G. Emch's work was the programme of using operator algebras as a foundational framework for quantum theory. I will start from reviewing the reasons why this programme has not been completely successful. Next I will discuss some insights and results obtained in other approaches to quantum foundations. Finally, I will show how one can use these insights to revitalise the operator algebraic approach to quantum foundations. The main conceptual and structural changes that I will focus on are, respectively, a shift from ontic to epistemic interpretation of quantum states, and a shift from analysis of intrinsic structures of operator algebras to the geometric structures on the state spaces of these algebras.

XXXIV WORKSHOP ON GEOMETRIC METHODS IN PHYSICS

Białowieża, POLAND, 1 June – 4 July 2015

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