Integrability of distributions on infinite dimensional manifolds and applications

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In differential geometry, a distribution on a smooth manifold $M$ is an assignment $D : x \mapsto D_x \subset T_x M$ on $M$, where $D_x$ is a subspace of $T_x M$. The distribution is integrable if, for any $x \in M$ there exists an immersed submanifold $f : L \to M$ such that $x$ belongs to $f(L)$ and for any $z \in L$ we have $Tf(T_z L) = D_{f(z)}$. On the other hand, $D$ is called involutive if, for any vector fields $X$ and $Y$ on $M$ which are tangent to $D$, the Lie bracket $[X, Y]$ is also tangent to $D$. The distribution is invariant if for any vector field $X$ tangent to $D$, the flow $\phi^X_t$ leaves $D$ invariant. On finite dimensional manifold, when $D$ is a sub-bundle of $TM$, the classical Frobenius Theorem gives an equivalence between integrability and involutivity. In the other case, the distribution is ”singular” and even under assumptions of smoothness on $D$, in general, the involutivity is not a sufficient condition for integrability (we need some more additional local conditions). These problems were clarified and resolved essentially in [Su], [St].

We will explain how these results in finite dimension can be extended to the context of Banach manifolds and also in the frame work of direct limit of Banach manifolds. We will end by application of these results in the framework of Banach Poisson manifolds and of direct limits of Banach Lie algebroids.

References
