Graph C*-algebras with applications to quantum spaces III

XIV School on Geometry and Physics, Białystok
23–27.06.2025

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Recap

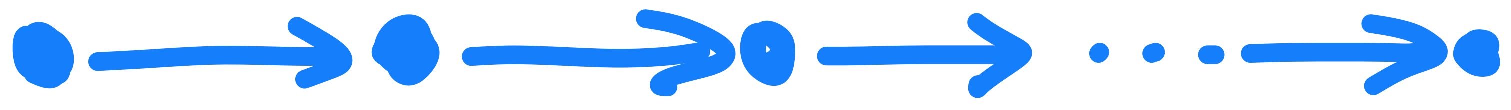
The C*-algebra of a row finite directed graph $E = (E^0, E^1, r, s)$ is the universal C*-algebra generated by pairwise orthogonal projections $\{p_v \mid v \in E^0\}$ and partial isometries $\{s_e \mid e \in E^0\}$ subject to the relations

(CK1) $s_e^* s_e = p_{s(e)}$ for every $e \in E^1$,

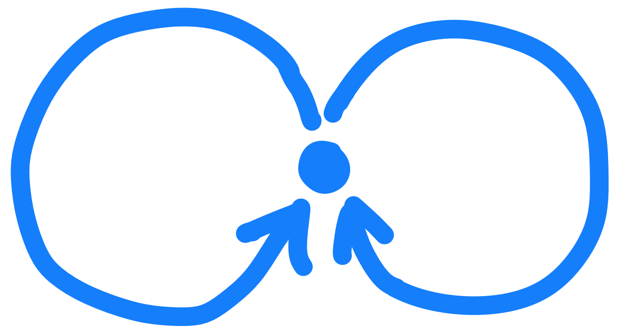
(CK2) $p_v = \sum_{e \in r^{-1}(v)} s_e s_e^*$ for every $v \in E^0$ that is not a source.

We've seen a few basic examples:

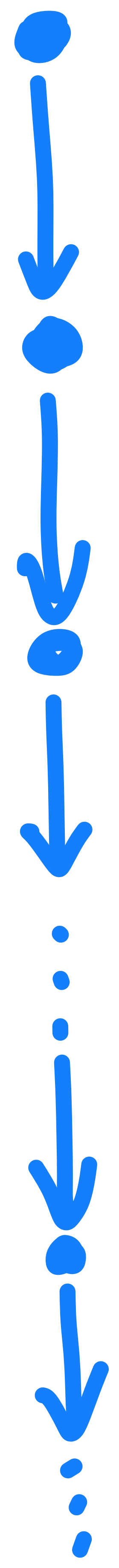
$C(\mathbb{T})$, continuous functions on the circle



$M_n(\mathbb{C})$, matrices



$\mathcal{O}_2$, Cuntz algebra



$\mathcal{K}$, compact operators

Gauge action

Before we see some more examples, let us turn to another important feature of graph C*-algebras: the gauge action.

Let E be a (row finite) graph. Let $\{S, P\}$ be a Cuntz–Krieger E-family.

For $z \in \mathbb{T}$, consider the operators $\{P_v \mid v \in E^0\}$, $\{zS_e \mid e \in E^1\}$. It is easy to check that these will also satisfy the Cuntz-Krieger relations and that they will generate the same C*-algebra as $\{S, P\}$.

This gives rise to an action of $\mathbb{T}$ on $C^*(\{S, P\})$, which in C*-world means a strongly continuous group homomorphism $\gamma : \mathbb{T} \to \mathrm{Aut}(C^*(\{S, P\}))$.

In particular, $\mathbb{T} \curvearrowright C^*(E)$. We call this action the **gauge action**.

Uniqueness theorems

The existence of a gauge action makes life easier! In particular, we have the so-called **gauge-invariant uniqueness theorem**:

Theorem: *Let E be a directed graph, and $\{T, Q\}$ a Cuntz–Krieger E-family in a C*-algebra B with each $Q_v \neq 0$. Suppse there is an action $\beta : \mathbb{T} \to \mathrm{Aut}(B)$ satisfying*

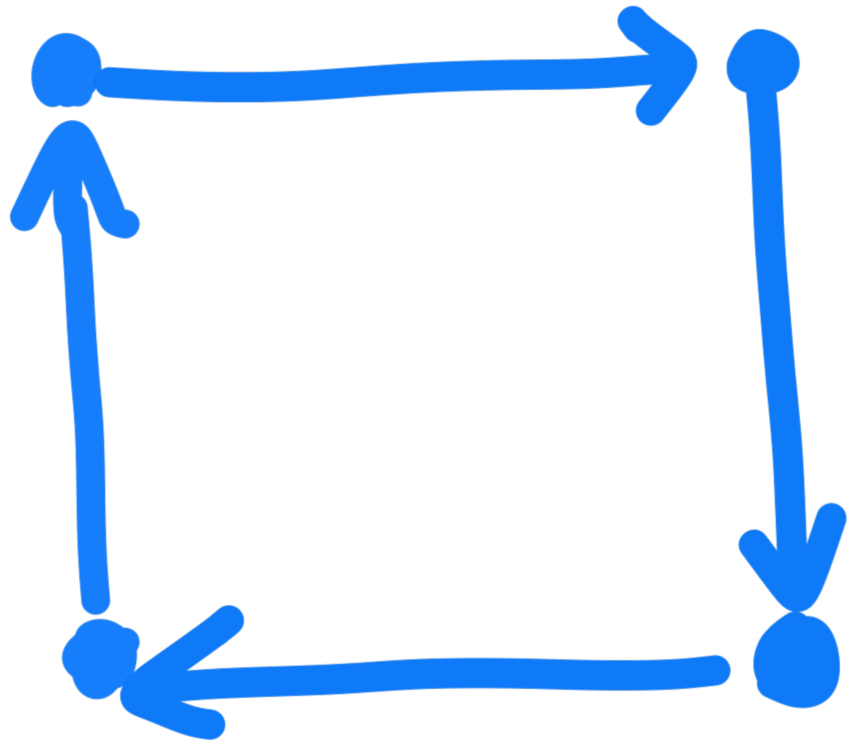
$$\beta_z(T_e) = zT_{e'} \quad \beta_z(Q_v) = Q_v$$

*for every $e \in E^1, v \in E^0$. Then $\pi_{T,Q} : C^*(E) \to C^*(T, Q)$ is a *-isomorphism.*

The gauge action also allows one to prove the **Cuntz–Krieger uniqueness theorem**:

Theorem. *Let E be a row-finite graph such that every cycle has an entry. Suppose $\{T, Q\}$ is a* Cuntz–Krieger E-family in a C*-algebra B with each $Q_v \neq 0$. Then the *-homomorphism $\pi_{T,Q} : C^*(E) \to B$ is a *-isomorphism of $C^*(E)$ onto $C^*(T, Q) \subset B$.

The uniqueness theorems allow us to produce more examples.



Define functions $P_{v_i}, S_{e_i} : \mathbb{T} \to M_4(\mathbb{C})$, $1 \leq i \leq 4$ by

$$P_{v_i}(z) = e_{i,i'} \qquad S_{e_i}(z) = \begin{cases} e_{i,i+1} \text{ if } 1 \leq i \leq 3 \\ ze_{1,4} \text{ if } i = 4. \end{cases}$$

Then $\{S, P\}$ is a Cuntz–Krieger E-family and $C^*(S, P) \cong C(\mathbb{T}, M_n)$. For $\lambda \in \mathbb{T}$, define

$$u_\lambda := \mathrm{diag}(\lambda, \lambda^1, \ldots, \lambda^4) \in \mathcal{U}(C(\mathbb{T}, M_n)).$$

Then $\beta : \mathbb{T} \to \mathrm{Aut(C(\mathbb{T}, M_n))}$ given by

$$\beta_\lambda(f)(z) = U_\lambda f(w^4 z) U_\lambda^*$$

defines a gauge action. It follows that $C^*(E) \cong C(\mathbb{T}, M_n)$

Beyond the row finite case

To tackle some of the examples from quantum spaces, we will need to go beyond the row finite case.

We immediately run into difficulties with the second Cuntz–Krieger relation:

(CK2) $P_v = \sum_{e \in r^{-1}(v)} S_e S_e^*$ whenever v is not a source.

If we allow v to receive infinitely many edges, $\sum_{e \in r^{-1}(v)} S_e S_e^*$ has no chance of converging in any C*-algebra, as it is the sum of pairwise orthogonal projections.

Note, however that (CK2) implies that $S_e S_e^* \leq P_v$ for every $e \in r^{-1}(v)$.

This still makes sense outside the row finite setting. And whenever v is not an infinite receiver, (CK2) still makes sense.

Beyond the row finite case

These two observations lead to the “right” relations outside the row finite setting.

Definition: Let $E = (E^0, E^1, r, s)$ be an arbitrary directed graph. The graph C*-algebra $C^*(E)$ is the universal C*-algebra generated by pairwise orthogonal projections $\{p_v \mid v \in E^1\}$ and partial isometries $\{s_e \mid e \in E^1\}$ subject to the following relations:

(E1) $s_e^* s_e = p_{s(e)}$ for every $e \in E^1$,

(E2) $s_e s_e^* \leq p_{r(e)}$ for every $e \in E^1$,

(E3) $p_v = \sum_{e \in r^{-1}(v)} s_e s_e^*$ for every $v \in E^0$ which is neither source nor infinite receiver.

Beyond the row finite case

It turns out that for any graph E, there exists a row finite graph F and a full projection $p \in C^*(F)$ such that $C^*(E) \cong pC^*(F)p$.

Recall that for a C*-algebra A and projection $p \in A$, the C*-subalgebra pAp is called a hereditary subalgebra called a corner.

A projection p is full if $\overline{ApA} = A$ or equivalently, if pAp is not contained in any proper ideal of A.

A full corner pAp shares many properties with A.

For example, for a C*-algebra B let $\mathcal{I}(B)$ denote the ideals in B. There exists a bijection

$\mathcal{I}(A) \to \mathcal{I}(pAp), I \mapsto pIp$ with inverse $\mathcal{I}(pAp) \to \mathcal{I}(A), J \mapsto \overline{AJA}$.

A C*-algebra and its full corners also have isomorphic K-theory.

So we can bootstrap many row finite results to the more general case.

Now onto some more interesting examples…

The final slides are based on joint work with Tomasz Brzeziński (Białystok/Swansea), Ulrich Krähmer (Dresden), and Réamonn Ó Buachalla (Prague)

Quantum spaces

The meaning of **quantum space** is a term that usually refers to a so-called q-deformation of the algebra of functions on a classical space which can be described by generators and relations.

Given a $q \in (0,1]$, one replaces certain commutations relations with relations involving generators and functions in q.

Noncommutative torus

For example,

$C(\mathbb{T}^2)$ is generated by u, v satisfying

$$u^*u = uu^* = v^*v = vv^* = 1, \text{ and } uv = vu.$$

For $q \in (0,1]$, the **noncommutative torus** $C_q(\mathbb{T}^2)$ is generated by u, v satisfying

$$u^*u = uu^* = v^*v = vv^* = 1, \text{ and } uv = qvu.$$

Similarly we may define the **noncommutative n-torus** $C_q(\mathbb{T}^n)$.

Quantum spaces from Lie groups

For compact connected simply connected Lie groups and their homogeneous spaces—for example the odd dimensional quantum spheres and quantum complex projective spaces—one can construct quantum spaces in a very precise way that allows one to keep much of the Lie theoretic structure, suitably interpreted.

For example, $C_q(S^{2n-1})$ admits a circle action from which one recovers $C_q(\mathbb{C}P^n)$ as the fixed point subalgebra. For $n = 2$, $C_q(\mathbb{C}P^1)$ is the well-known **Podleś sphere**, a q -deformation of the 2-sphere.

In fact, in the graph C*-algebra of Hong and Szymański, the canonical gauge action is precisely this circle action, recovering quantum projective space as the fixed point algebra of the corresponding quantum sphere.

Quantum 3-sphere

Another well-known example is the noncommutative 3-sphere, $C_q(S^3)$.

For $q \in (0,1]$, the quantum 3-sphere $C_q(S^3)$ is generated by two elements, α, γ, subject to the relations

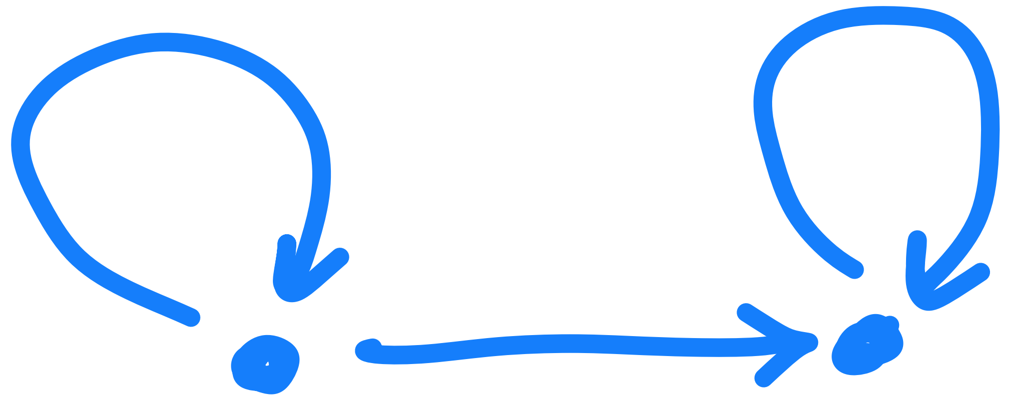
$$\begin{pmatrix} \alpha & -q\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \text{ is a unitary matrix.}$$

When $q = 1$ it is not difficult to check that this determines the usual sphere relations for S^3.

One can also define quantum odd-dimensional spheres $C_q(S^{2n-1})$.

Hong and Szymański showed that various quantum spaces could be realised as graph C*-algebras, for example:

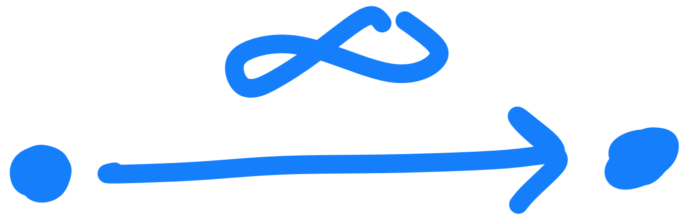
quantum odd-dimensional spheres



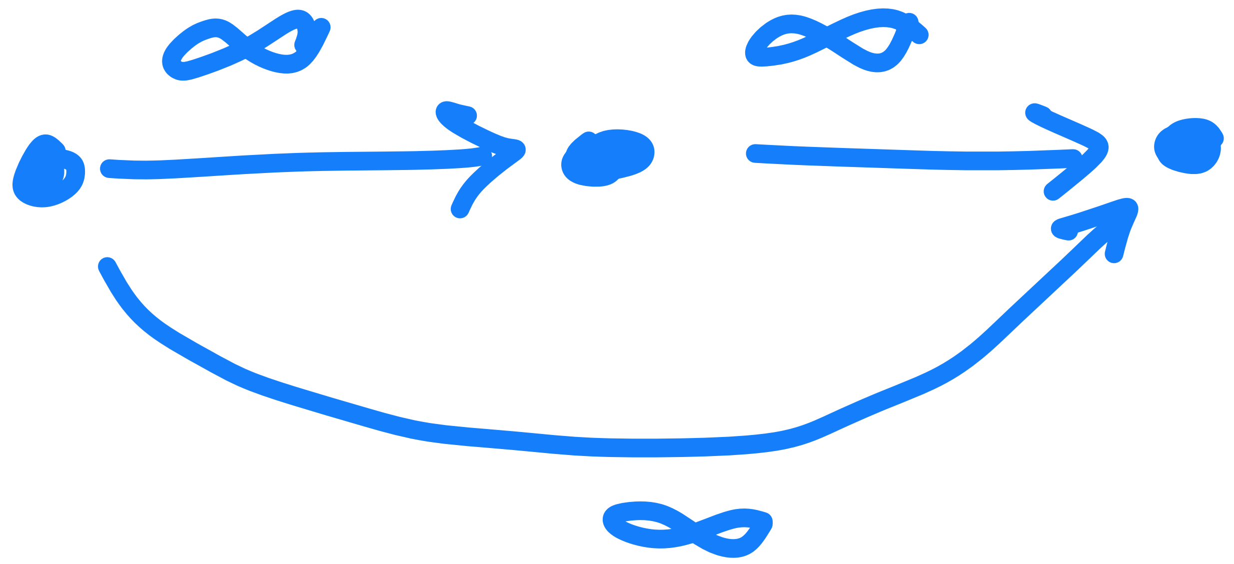
$C_q(S^3)$

$C_q(S^5)$

quantum complex projective spaces



$C_q(\mathbb{C}P^1)$



$C_q(\mathbb{C}P^3)$

Quantum spaces from Lie groups

For compact connected simply connected Lie groups and their homogeneous spaces—for example the odd dimensional quantum spheres and quantum complex projective spaces—one can construct quantum spaces in a very precise way that allows one to keep much of the Lie theoretic structure, suitably interpreted.

For example, $C_q(S^{2n-1})$ admits a circle action from which one recovers $C_q(\mathbb{C}P^n)$ as the fixed point subalgebra. For $n = 2$, $C_q(\mathbb{C}P^1)$ is the well-known **Podleś sphere**, a q-deformation of the 2-sphere.

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Quantum flag manifolds

Quantum projective spaces are examples of a larger class of quantum spaces called **quantum flag manifolds**.

In the classical setting, a flag manifold is a simply connected compact homogeneous Kähler manifold. In particular, they arise as quotients of simply connected compact semisimple Lie groups.

These Lie groups admit a particularly satisfying q-deformation via the enveloping algebra of their associated complex semisimple Lie algebra.

Drinfeld–Jimbo deformations

Given a complex semisimple Lie algebra $\mathfrak{g}$, its enveloping algebra $U(\mathfrak{g})$ has a q-deformation, $U_q(\mathfrak{g}_q)$, for $q \in (0,1)$, which also admits a Hopf *-algebra structure and has the same representation theory as $U(\mathfrak{g})$.

If $\mathfrak{g}$ is a Lie algebra of rank r, then $U_q(\mathfrak{g})$ is generated by elements E_j, F_j, K_j, $1 \leq j \leq r$ subject to certain relations,

$$K_iE_j = q_i^{a_{ij}}E_jK_i, \quad K_iF_j = q_i^{-a_{ij}}F_jK_i, \quad K_iK_j = K_jK_i, \quad K_iK_i^{-1} = K_i^{-1}K_i = 1,$$

$$E_iF_j - F_jE_i = \delta_{ij}\frac{K_i - K_i^{-1}}{q_i - q_i^{-1}},$$

along with the so-called quantum Serre relations.

Hopf *-algebra structure

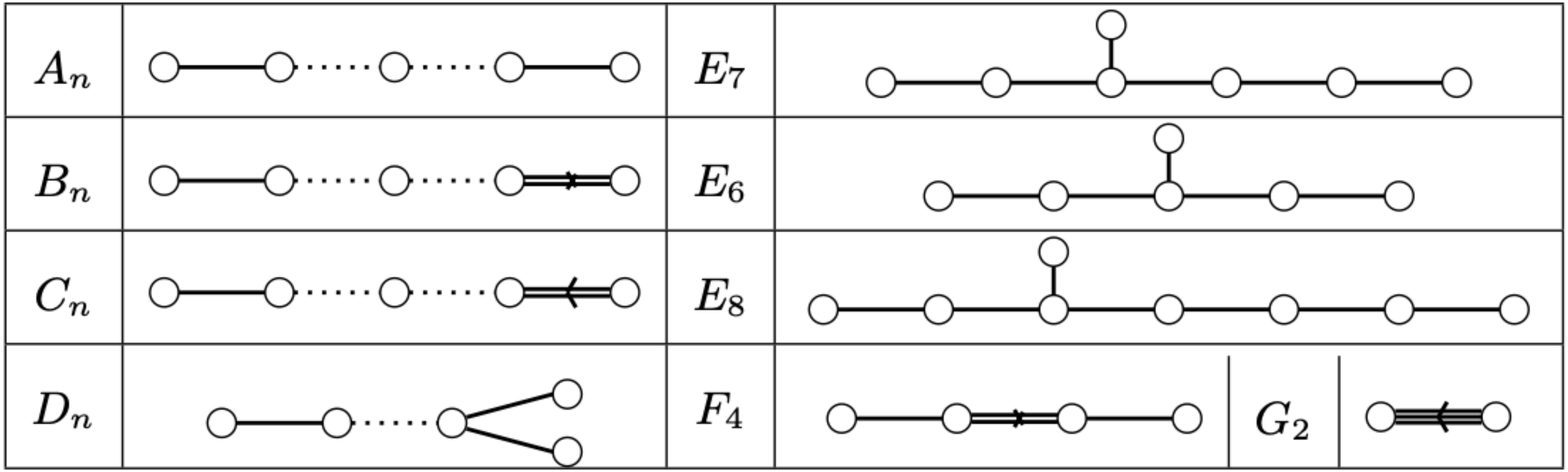
A **Hopf algebra** structure is defined on $U_q(\mathfrak{g})$ by

$$\Delta(K_i) = K_i \otimes K_i, \quad \Delta(E_i) = E_i \otimes K_i + 1 \otimes E_i, \quad \Delta(F_i) = F_i \otimes 1 + K_i^{-1} \otimes F_i,$$
$$S(E_i) = -E_iK_i^{-1}, \quad S(F_i) = -K_iF_i, \quad S(K_i) = K_i^{-1},$$
$$\epsilon(E_i) = \epsilon(F_i) = 0, \ \ \epsilon(K_i) = 1.$$

A **Hopf *-algebra** structure, called the **compact real form** of $U_q(\mathfrak{g})$, is defined by

$$K_i^* := K_i, \qquad E_i^* := K_iF_i, \qquad F_i^* := E_iK_i^{-1}.$$

If $\mathfrak{g}$ is a Lie algebra of rank r, then the relations for E_j, F_j, K_j, $1 \leq j \leq r$ can essentially be read off the corresponding Dynkin diagram.



Dynkin diagrams for complex semisimple Lie algebras

Coordinate algebras and C*-algebras

Dual to the quantum enveloping algebra is the **quantum coordinate algebra** $\mathcal{O}_q(G)$ which is a Hopf *-algebra admitting a C*-completion $C_q(G)$.

When $q = 1$, this is precisely $C(G)$ for G the **simply connected compact semisimple Lie group** with Lie algbera $\mathfrak{g}$. While $C_q(G)$ is no longer a Hopf algebra, the coproduct

$$\Delta : C_q(G) \to C_q(G) \otimes_{\min} C_q(G)$$

does extend to a *-homomorphism, and one can check that it is **a compact quantum group** in the sense of Woronowicz.

Example: quantum SU_2

For $\mathfrak{g} = \mathfrak{sl}_2$, the Dynkin diagram consists of a single node, so the quantum enveloping algebra $U_q(\mathfrak{sl}_2)$ is generated by E, F, K.

The dual $\mathcal{O}_q(G) = \mathcal{O}_q(SU_2)$ is generated by α, γ such that

$$\begin{pmatrix} \alpha & -q\gamma^* \\ \gamma & \alpha^* \end{pmatrix} \text{ is unitary.}$$

Thus we see that just as in the classical picture, we have $\mathcal{O}_q(SU_2) = \mathcal{O}_q(S^3)$ and hence also $C_q(SU_2) = C_q(S^3)$.

Constructing quantum flag manifolds

For a given Dynkin diagram, choose a subset of nodes, S.

Define a subalgebra $U_q(\mathfrak{l}_S) \subset U_q(\mathfrak{g})$, generated by K_j, $1 \leq j \leq r$ and E_j, F_j if the j-th node is in S.

$U_q(\mathfrak{l}_S) \subset U_q(\mathfrak{g})$ dualizes to a surjection $\mathcal{O}_q(G) \to \mathcal{O}_q(L_S)$, from which we construct a coaction

$$\Delta_R : \mathcal{O}_q(G) \to \mathcal{O}_q(G) \otimes \mathcal{O}_q(L_S).$$

The coordinate algebra of the quantum flag manifold is given by

$$\mathcal{O}_q(G/L_s) := \{a \in \mathcal{O}_q(G) \mid \Delta_R(a) = a \otimes 1\},$$

which admits a C*-completion, $C_q(G/L_S)$.

When $q = 1$, we recover the continuous functions on the associated flag manifold.

Detecting graph C*-algebras

Yesterday we saw that (unital) graph C*-algebras are classified by a K-theoretic invariant. While this result is remarkable, it unfortunately doesn't tell us how to tell if a C*-algebra we run into in the wild is actually a graph C*-algebra.

Mistaking a C*-algebra for a graph C*-algebra can sometimes have deadly consquences: graph C*-algebras are relatively docile and friendly to humans, while some C*-algebras will not hesitate to attack unassuming researcher.

Luckily, in some instances, it is possible to determine when we are indeed dealing with a graph C*-algebra:

Theorem [Eilers–Ruiz–Sørensen] *Let A be a C*-algebra with $\mathrm{Prim}(A)$ finite. Suppose that for each $x \in \mathrm{Prim}(A)$, the subquotient $A[x] \cong \mathscr{K}$, or $A[x] \cong \mathbb{C}$.* Then there exists an "amplified graph" E such that $A \cong C^*(E)$.

Some terminology

Recall that a *-representation of a C*-algebra on a Hilbert space H is a *-homomorphism $\pi : A \to \mathscr{B}(H)$.

A *-homomorphism is irreducible if there are no non-trivial $\pi(A)$-invariant subspaces in H.

An ideal is **primitive** if it is the kernel of an irreducible ideal.

$\mathrm{Prim}(A) = \{$primitive ideals of $A\}$.

$\mathrm{Prim}(A)$ is equipped with the hull-kernel topology: if $X \subset \mathrm{Prim}(A)$, then $\overline{X} = \{\rho \in \mathrm{Prim}(A) : \rho \supseteq \cap_{\pi \in X} \pi\}$.

When $A = C(X)$ for a compact Hausdorff space X, we have $\mathrm{Prim}(A) \cong X$.

Existence

Theorem (Eilers–Restorff–Sørensen): *Let A be a C*-algebra with $\mathrm{Prim}(A)$ finite. Suppose that for each $x \in \mathrm{Prim}(A)$, the subquotient $A[x] \cong \mathscr{K}$, or $A[x] \cong \mathbb{C}$.* Then there exists an "amplified graph" E such that $A \cong C^*(E)$.

A graph is called **amplified** if, for every $v \in E^0$, we have $|r^{-1}(v)| \in \{0,\infty\}$.

$A[x]$ is the simple subquotient defined as follows: For any open subset $U \subset \mathrm{Prim}(A)$, set $A[U] := \bigcap_{p \in \mathrm{Prim(A)} \setminus U} p$.

Let U, V be a pair of open sets with $V \subset U$ and $\{x\} = U \setminus V$. Set $A[x] = A[U]/A[V]$

Elementary *-representations of $C_q(G)$

For every $1 \leq i \leq \mathrm{rank}(\mathfrak{g})$, there is a map from $U_q(\mathfrak{sl}_2) \to U_q(\mathfrak{g})$ defined on generators by $K \mapsto K_i$, $E \mapsto E_i$ and $F \mapsto F_i$.

Dually, we get a surjective *-homomorphism $\sigma_i : C_q(G) \to C_q(SU_2)$.

For $s_i \in W_G$ we define a *-representation by

$$\pi_{s_i} := \rho \circ \sigma_i : C_q(G) \to \mathscr{B}(L^2(\mathbb{Z}_+)),$$

where $\rho : C_q(SU_2) \to \mathscr{B}(L^2(\mathbb{Z}_+))$ is given by

$$\rho(\alpha)(e_n) = (1 - q^{2n})^{1/2} e_{n-1}, \qquad \rho(\gamma)(e_n) = -q^n e_n .$$

Irreducible *-representations of $C_q(G/L_S)$

Given $C_q(G/L_S)$, let W_G denote the Weyl group of G and W_S the subgroup of W_G given by

$$W_S := < s_i \in W_G \mid i \in S > .$$

Let W^S be the set of W/W_S coset representatives of minimal length.

Given $w \in W^S$, let $w = s_{i_1} \cdots s_{i_k}$ be in reduced form. Define

$$\pi_w := \pi_{s_{i_1}} \otimes \cdots \otimes \pi_{s_{i_k}} \circ \Delta^{k-1} : C_q(G/L_S) \to \mathscr{B}(L^2(\mathbb{Z}_+^k)) .$$

Dijkhuisen and Stokman (following work of Soibelman) showed π_w does not depend on the choice of reduced word for w. Moreover, all *-representations of $C_q(G/L_S)$ are of this form.

From this description of the irreducible *-representations, we see that the primitive ideal space is finite, and moreover, it is straightforward to see that $\ker(\pi) \cong \mathscr{K}$ for all but one representation which has kernel $\mathbb{C}$.

It follows that $C^*(G/L_S) \cong C^*(E)$ for some amplified graph E.

How do we know if a given graph is the one we're looking for?

Theorem [Eilers–Ruiz–Sørensen] *Let A be a unital C*-algebra with $\mathrm{Prim}(A)$ finite and let E be an amplified graph with finitely many vertices. Then $A \cong \mathrm{C}^*(E)$ if and only if $\mathrm{Prim}^\tau(A) \cong \mathrm{Prim}^\tau(\mathrm{C}^*(E))$.*

...so now we just need to find the right graphs...

Ideal ingredients

Let E be a graph.

The set $H \subset E^0$ is **hereditary** if $w \in H$ and $w \leq v$ implies $v \in H$.

$S \subset E^0$ is **saturated** if, whenever $v \in E^0$ with $|r^{-1}(v)| < \infty$ and $s(r^{-1}(v)) \subset S$, then $v \in S$.

A graph $E = (E^0, E^1, r, s)$ satisfies **Condition (K)** if, for every $v \in E^0$one of the following is satisfied:

- If $r(e) = v$, then there is no loop μ with $e \in \mu$.
- There are two loops μ_1, μ_2 with $r(\mu_1) = r(\mu_2) = v$, and neither μ_1 nor μ_2 is an initial subpath of the other.

The graphs of the quantum flag manifolds will never have loops, so Condition (K) is always satisfied.

Given a subset $H \subset E^0$ which is both hereditary and saturated, the **breaking vertices** of S is the set of vertices

$$B_H := \{v \in E^0 \mid |r^{-1}(v)| = \infty \text{ and } 0 < |r^{-1}(v) \cap s^{-1}(E^0 \backslash H)| < \infty\}$$

Thus the set of breaking vertices for S consists of all infinite receivers with at least one, and at most finitely many, paths starting outside of H.

The graphs of quantum flag manifolds will contain infinite receivers, but for any hereditary and saturated subset H, we have $B_H = \emptyset$.

Let H be a saturated and hereditary subset and let $B \subset B_H$. For $v \in B$, define

$$p_{v,H} := \sum_{r(e)=v, s(e)\notin H} s_e s_e^* .$$

Let $J_{H,B}$ be the ideal of $C^*(E)$ generated by $\{p_v \mid v \in H\} \cup \{p_v - p_{v,H} \mid v \in B\}$.

If E satisfies Condition (K), the ideals of $C^*(E)$ are in one-to-one correspondence with pairs (H, B) of saturated and hereditary subsets H and subsets $B \subset H$ of breaking vertices.

If I is an ideal in $C^*(E)$ then $I = J_{H,B}$ with $H = \{v \in E^0 \mid p_v \in I\}$ and $B = \{v \in B_H(E) \mid p_v - p_{v,H} \in I\}$.

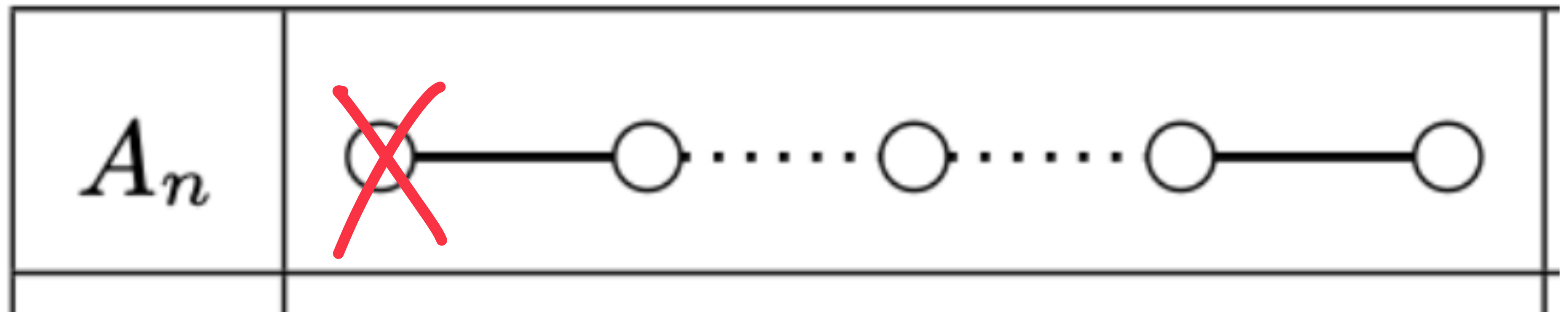
Let $E = (E^0, E^1, r, s)$ be a directed graph. A subset $M \subset E^0$ is a maximal tail if it satisfies the following three conditions:

- If $v \in E^0$ and $w \in M$, and $v \leq w$, then $v \in M$.
- If $v \in M$ is a regular vertex, then there exists an edge $e \in E^1$ such that $s(e) = v$ and $r(e) \in M$.
- For any $v, w \in M$, there is a $y \in M$ such that $v \leq y$ and $w \leq y$.

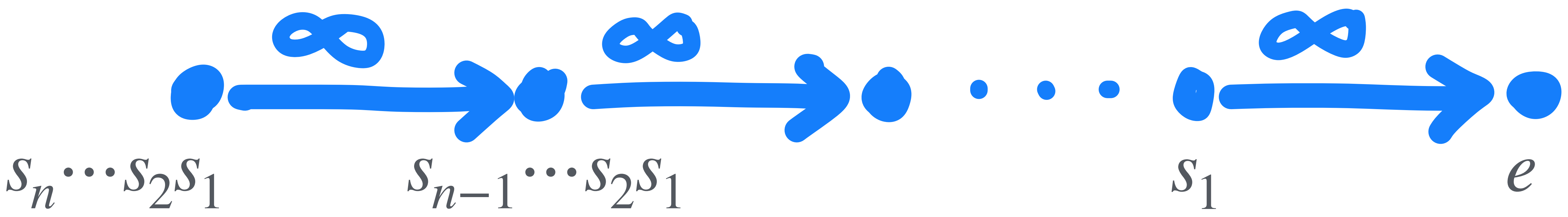
Given a hereditary saturated subset $H \subset E^0$ and $v \in B_H$, the ideal $J_{H,\{v\}} \subset \mathrm{C}^*(E)$ is primitive if and only if $E^0 \backslash H$ is a maximal tail.

Let $C_q(G/L_S)$ be a quantum flag manifold. The vertices of E_s are indexed by elements in W^S, and we draw infinitely many arrows from the vertex corresponding to $w \in W^S$ to the to vertex corresponding to $v \in W^S$, precisely when $w = s_i v$ or $w = v s_i$ for some generator s_i and $\ell(w) > \ell(s)$.

The Dynkin diagram representing $C_q(\mathbb{C}P^n)$ is

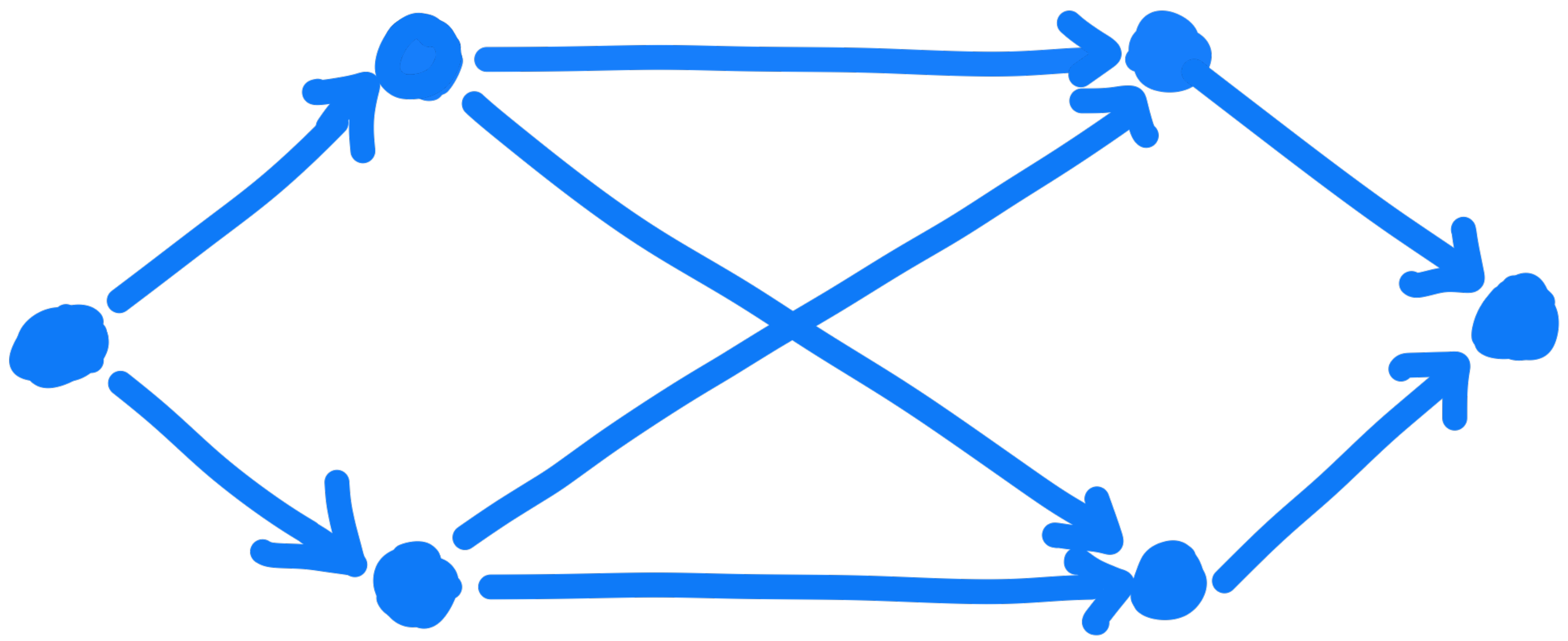


We have $W^S := \{e, s_1, s_2 s_1, s_3 s_2 s_1, \ldots, s_n \cdots s_2 s_1\}$, so the graph is



A second example is the full quantum flag manifold of quantum SU_3, $C_q(SU_3/\mathbb{T}^2)$, which is given by the Dynkin diagram of A_2 with all nodes crossed, so that $S = \emptyset$.

We have $W^S = S_3 = \{e, s_1, s_2, s_1s_2, s_2s_1, s_1s_2s_1\}$, which gives us the following graph:



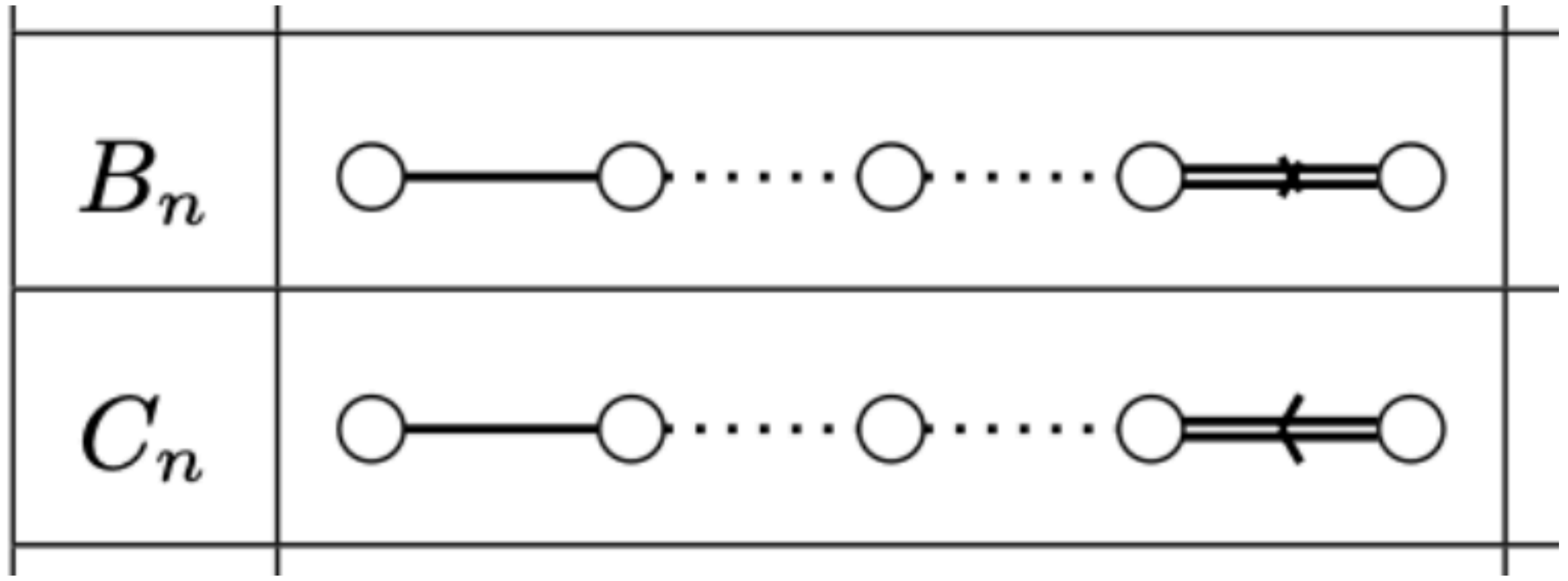
Consequences

From the graph C*-algebra picture, it is immediate to see that the C*-algebra of a quantum flag manifold does not depend on the parameter $q \in (0,1)$.

We can also calculate the K-theory, and see that they have same K-theory as their classical counterparts.

Consequences

We also find interesting isomorphisms. For example, for any fixed n, label the nodes of the B_n and C_n from left to right as $1, \ldots, n$.



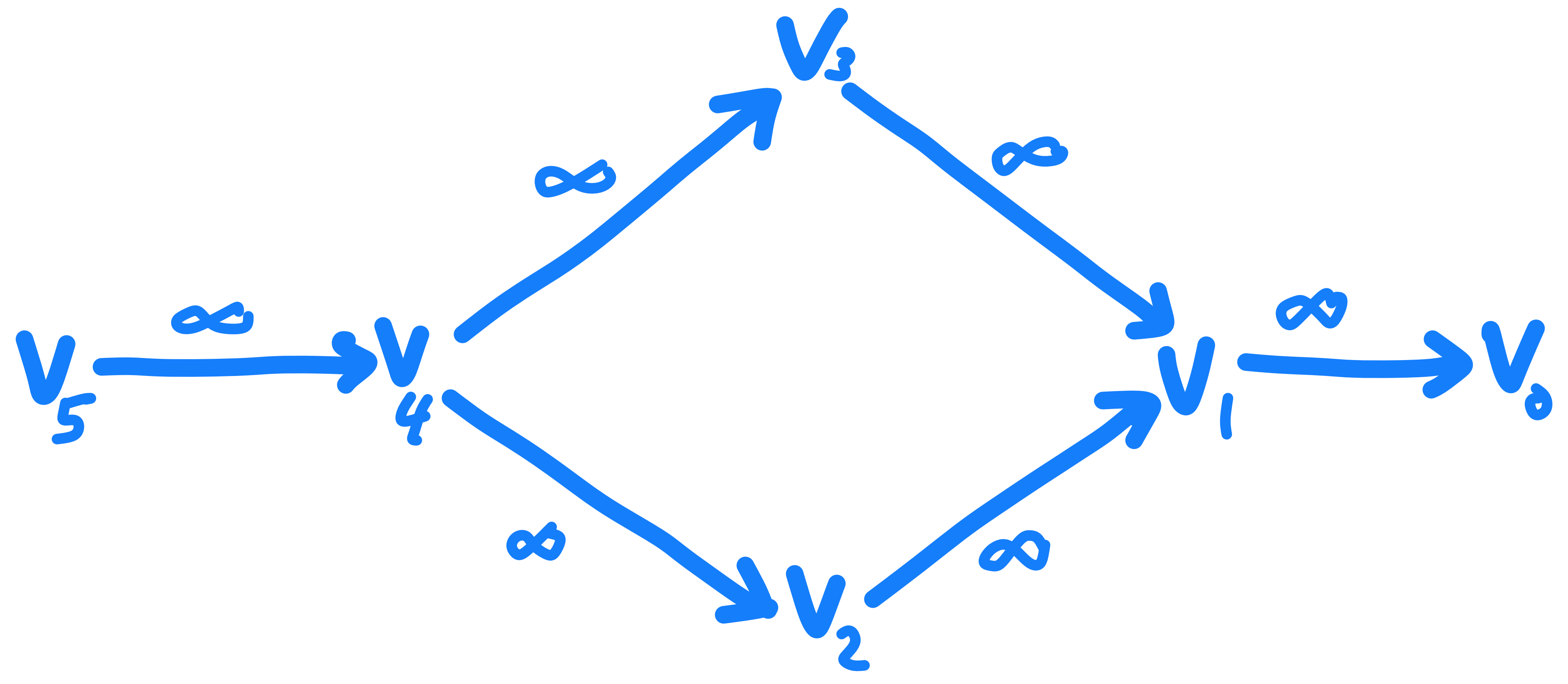
Let $S \subset \{1, \ldots, n\}$ and let $\mathcal{O}_q(SO_{2n+1}) \to \mathcal{O}_q(L_S)$ and $\mathcal{O}_q(Sp_{2n}) \to \mathcal{O}_q(K_S)$ denote the relevant Hopf algebra surjections. Then

$$C_q(SO_{2n+1}/L_S) \cong C_q(Sp_{2n}/K_S),$$

since the Weyl groups are the same.

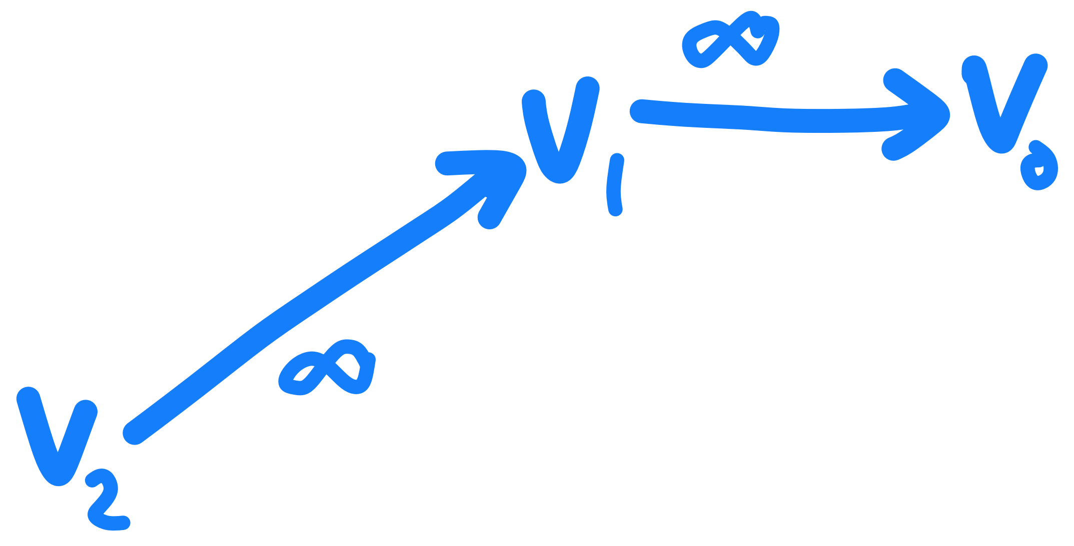
Consequences

Using what we know about quotients of graph C*-algebras, we also see some interesting things.



Quantum (4,2)-Grassmannian

The projections $p_{v_5}, p_{v_4}, p_{v_3}$ generate an ideal. Quotienting by that ideal gives us the graph



Quantum $\mathbb{C}P^2$

Summary

- C*-algebras lie at the intersection of topology and algebra
- Any directed graph gives rise to a C*-algebra
- Many properties of a graph C*-algebra can be read directly from the graph, for example, the ideal structure
- K-theory for C*-algebras is an important invariant which can be used for classification
- K-theory of a graph C*-algebra can be read directly from a graph
- Quantum flag manifolds are q-deformations of classical flag manifolds
- The C*-algebras of quantum flag manifolds are isomorphic to graph C*-algebras
- Consequently, we can see that they have classical K-theory, find interesting isomorphisms, see that they are independent of $q \in (0,1)$, find interesting quotients,...
- What other interesting quantum spaces might we model as graph C*-algebras?

Thanks for listening!
Dziękuję!