Relativistic cosmology from F to Sz

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Contents

24. The quasi-spherical Szekeres (QSS) models

The next bold step in generalising the R-W models (after Lemaître-Tolman) was undertaken by Szekeres [50-51,1] in 1975.

He took the following Ansatz for the metric

$$ds^2 = dt^2 - e^{2\alpha}dz^2 - e^{2\beta}\left(dx^2 + dy^2\right) \qquad (24.1)$$

where α and β are functions of (t, x, y, z) to be determined from the Einstein equations.

The source in the Einstein eqs was assumed to be dust.

Szekeres obtained the full set of solutions of Einstein's equations for (24.1).

One sub-family of his solutions (with $\beta_{,z} = 0$) generalizes the Datt – Ruban model [35], we shall ignore it here.

The other sub-family generalizes the L-T models and their plane- and hyperbolically symmetric counterparts.

In general, the ***Sz models have no symmetry*** (all Killing vectors are zero).

Invariant definitions of the whole Szekeres family are known, but they are somewhat lengthy, see Ref. [1].

We will consider here only the ***quasi-spherical (QSS) Szekeres*** solutions that generalize the proper L-T models (i.e. we omit their quasi-plane and quasi hyperbolic analogues).

[50] P. Szekeres, A class of inhomogeneous cosmological models. *Commun. Math. Phys*. **41**, 55 (1975).
[51] P. Szekeres, Quasispherical gravitational collapse, *Phys. Rev.* **D12**, 2941 (1975).
[1] J. Plebański and A. Krasiński, An introduction to general relativity and cosmology. Cambridge University Press 2006.
[35] V. A. Ruban, Spherically symmetric T-models in the general theory of relativity. *Zh. Eksper. Teor. Fiz.* **56**, 1914 (1969); English translation with comments: *Gen. Relativ. Gravit.* **33**, 375 (2001).

The QSS solutions have the metric

$$
\begin{aligned}
\mathrm{d}s^2 &= \mathrm{d}t^2 - \frac{\mathcal{E}^2 (\Phi/\mathcal{E}),_r{}^2}{1 + 2E(r)} \mathrm{d}r^2 - \frac{\Phi^2}{\mathcal{E}^2} \left(\mathrm{d}x^2 + \mathrm{d}y^2\right), \\
\mathcal{E} &\overset{\text{def}}{=} \frac{(x - P)^2}{2S} + \frac{(y - Q)^2}{2S} + \frac{S}{2},
\end{aligned}
\tag{24.2}
$$

where $E(r)$, $M(r)$, $P(r)$, $Q(r)$ and $S(r)$ are arbitrary functions, and

$$
\Phi,_t{}^2 = 2E(r) + \frac{2M(r)}{\Phi} + \frac{1}{3}\Lambda\Phi^2. \tag{24.3}
$$

The mass density is

$$
\kappa\rho = \frac{2\left(M/\mathcal{E}^3\right),_r}{(\Phi/\mathcal{E})^2 \left(\Phi/\mathcal{E}\right),_r}, \qquad \kappa = \frac{8\pi G}{c^2}. \tag{24.4}
$$

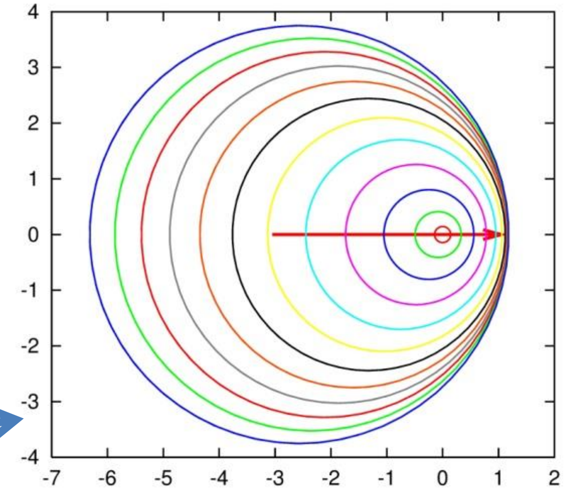
Eq. (25.3) is the same as in the L–T model, and again implies that the bang time is in general position-dependent:

$$
\int_0^\Phi \frac{\mathrm{d}\widetilde{\Phi}}{\sqrt{2E + 2M/\widetilde{\Phi} + \frac{1}{3}\Lambda\widetilde{\Phi}^2}} = t - t_B(r). \tag{24.5}
$$

$$ds^2 = dt^2 - \frac{\mathcal{E}^2 (\Phi/\mathcal{E}),_r{}^2}{1 + 2E(r)} dr^2 - \frac{\Phi^2}{\mathcal{E}^2} \left(dx^2 + dy^2\right), \qquad \mathcal{E} = \frac{(x - P)^2}{2S} + \frac{(y - Q)^2}{2S} + \frac{S}{2}, \tag{24.2}$$

$$\Phi,_t{}^2 = 2E(r) + \frac{2M(r)}{\Phi} + \frac{1}{3}\Lambda\Phi^2, \tag{24.3}$$

$$\int\limits_0^\Phi \frac{d\widetilde{\Phi}}{\sqrt{2E + 2M/\widetilde{\Phi} + \frac{1}{3}\Lambda\widetilde{\Phi}^2}} = t - t_B(r). \tag{24.5}$$



The surfaces of constant t and r in (24.2)

$ds_2{}^2 = \frac{\Phi^2}{\mathcal{E}^2} \left(dx^2 + dy^2\right)$

are ***nonconcentric spheres***, x and y are stereographic coordinates on each sphere.

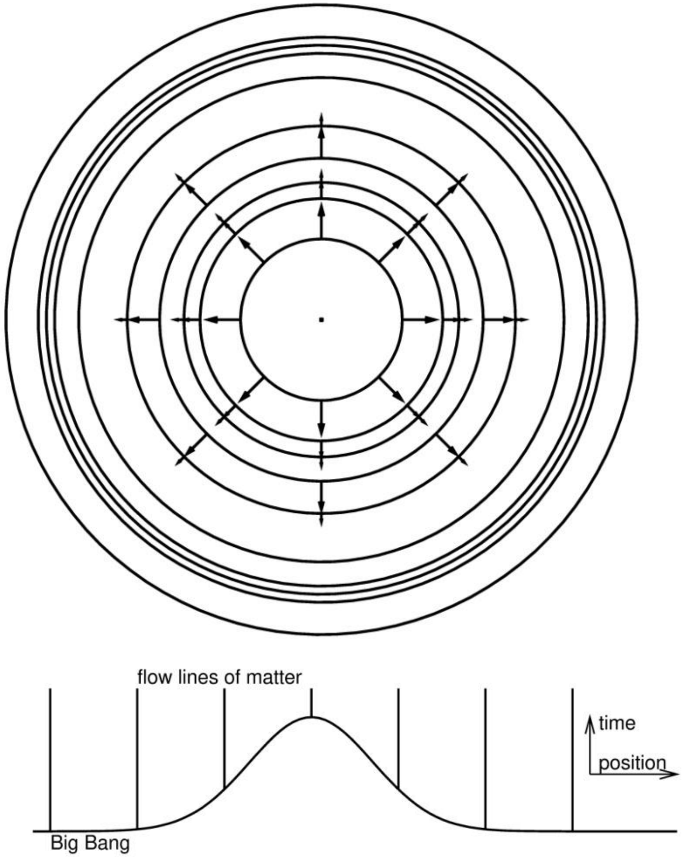
The functions P, Q and S determine how the center of a sphere shifts when the radius is increased.

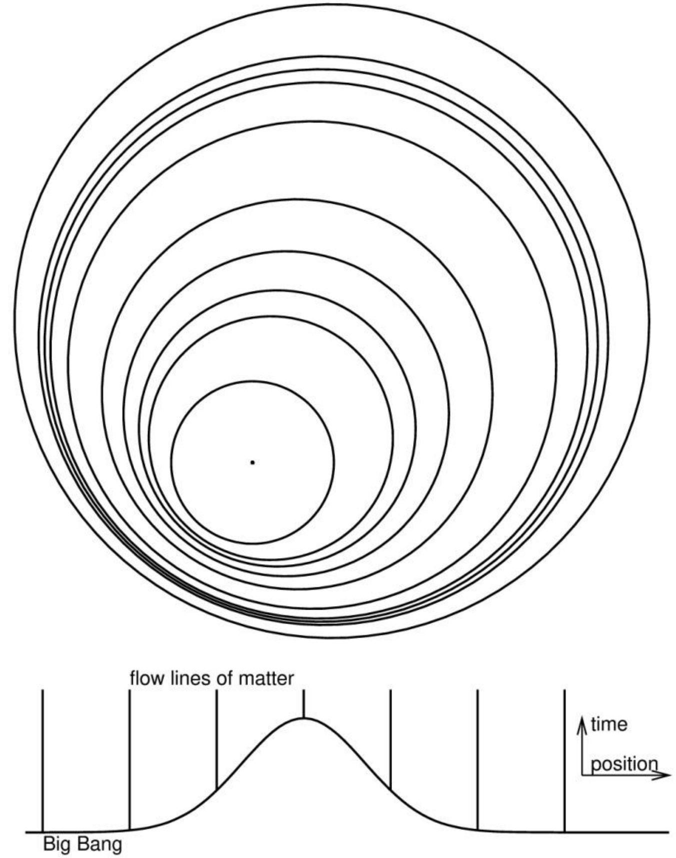
The figure shows an axisymmetric configuration with constant P and Q. Fully nonsymmetric configurations are difficult to show graphically, but such figures exist, see Ref. [54].

The L-T model is contained here as the limit of constant (P, Q, S) – then the spheres become concentric.

The Friedmann limit follows when, in addition, $\Phi(t,r) = rS(t)$, $2E = -kr^2$ where k is the Friedmann curvature index, and t_B is constant.

[52] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).





<u>Expansion in L-T models.</u>

Velocity of expansion is uncorrelated with the radius of a matter shell.

The BB is non-simultaneous

→ the age of matter particles depends on r.

Constant-density shells are concentric.

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{{R,_r}^2}{1+2E(r)}\mathrm{d}r^2 - R^2(t,r)\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta \mathrm{d}\varphi^2\right) \qquad (18.1)$$

<u>Expansion in Szekeres models.</u>

Velocity of expansion is uncorrelated with the radius of a matter shell and ***the shells are not concentric***.

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{\mathcal{E}^2 {(\Phi/\mathcal{E}),_r}^2}{1+2E(r)}\mathrm{d}r^2 - \frac{\Phi^2}{\mathcal{E}^2}\left(\mathrm{d}x^2 + \mathrm{d}y^2\right)$$

$$\mathcal{E} = \frac{(x-P)^2}{2S} + \frac{(y-Q)^2}{2S} + \frac{S}{2} \qquad (24.2)$$

25. Drift of light rays [53]

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{\mathcal{E}^2 (\Phi/\mathcal{E}),_r{}^2}{1 + 2E(r)} \mathrm{d}r^2 - \frac{\Phi^2}{\mathcal{E}^2} \left(\mathrm{d}x^2 + \mathrm{d}y^2\right) \qquad \mathcal{E} = \frac{(x-P)^2}{2S} + \frac{(y-Q)^2}{2S} + \frac{S}{2} \qquad (24.2)$$

In an open segment of a null geodesic $k^r \neq 0$; a line with $k^r = 0$ on an open stretch would be timelike [53] and not necessarily geodesic.

→ r can be used as a (non-affine) parameter on ray segments where $k^r \neq 0$.

Let two rays in a QSS spacetime be sent by the same source, the second one later by τ.

Let the trajectory of the first ray be

$$(t, x, y) = (T(r), X(r), Y(r)). \qquad (25.1)$$

The second ray will in general not proceed through the same values of X and Y as the first one. The same is true for nonradial rays in an L-T model.

→ The equation of the second ray will be

$$(t, x, y) = (T(r) + \tau(r), X(r) + \zeta(r), Y(r) + \psi(r)). \qquad (25.2)$$

Already here, without any calculations, we may conclude the following:

In general, the second ray intersects each hypersurface $r = r_0$ at a different comoving location (r, x, y) than the first one.

[53] A. Krasiński and K. Bolejko, Redshift propagation equations in the β' ≠ 0 Szekeres models. *Phys. Rev*. **D83**, 083503 (2011).

→ The two rays intersect different sequences of cosmic dust worldlines between the source and the observer.

→ The second ray will reach the observer from a different direction in the sky.

→ An observer in a QSS spacetime should see a generic light source drift across the sky. The same is true for nonradial rays in an L-T model.

The drift vanishes only for axial directions in an axially symmetric QSS spacetime and for radial directions in an L-T spacetime.

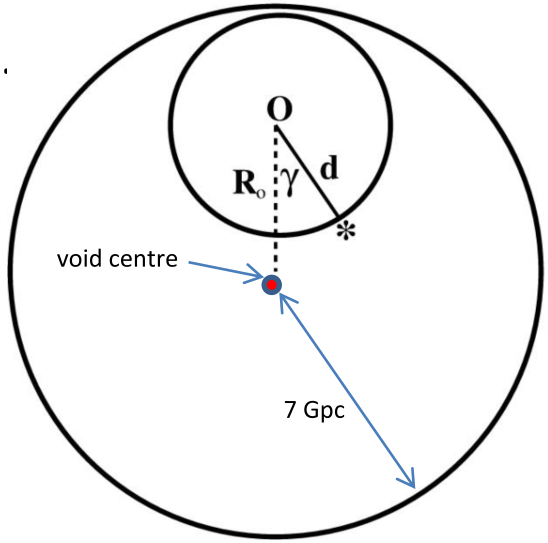
The only spacetimes in the Szekeres family with no drift for all observers and all rays are the Friedmann models [53].

Observational detection of the drift would be evidence of inhomogeneity of the Universe on large scales.

For a geometric description of the drift in a general spacetime see Refs. [54,55].

[53] A. Krasiński and K. Bolejko, Redshift propagation equations in the $\beta' \neq 0$ Szekeres models. *Phys. Rev*. **D83**, 083503 (2011).
[54] M. Korzyński and J. Kopiński, *J. Cosm. Astropart. Phys.* **03**, 012 (2018).
[55] M. Grasso, M. Korzyński and J. Serbenta, *Phys. Rev.* **D99**, 064038 (2019).

26. A numerical example of the drift



The calculations in this section were done in an L-T model [53].

The figure shows an exemplary setup for observing the drift.

The parameters of this setup are only illustrative, they do not reflect the properties of any real object.

The rays propagate through a void of radius ≈7Gpc; the centre of the void is at the centre of symmetry of the L-T region.

The mass density at the centre is $\rho_0 = 0.3\rho_{\Lambda CDM}$,
$\rho_{\Lambda CDM}$ being the average present density in the ΛCDM model.

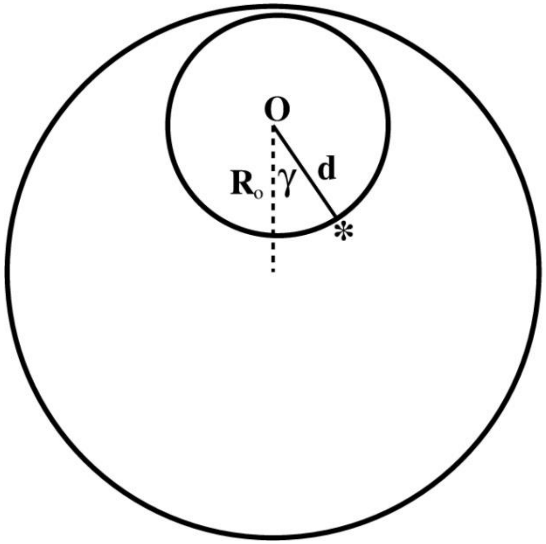
The observer is at R_0 = 3 Gpc or 1 Gpc from the void centre,

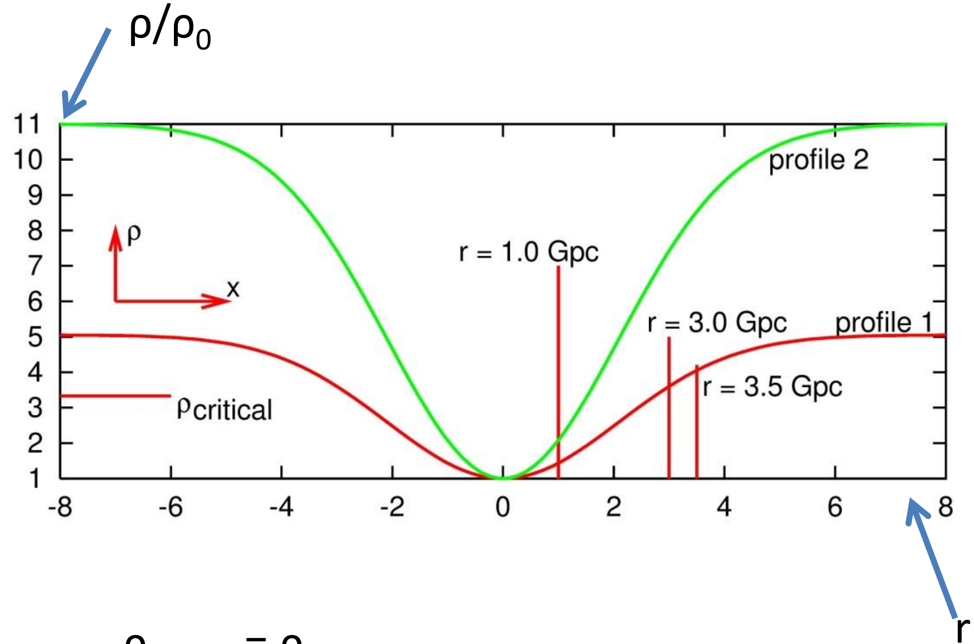
and receives rays from directions at angles γ to the void diameter, $0 \leq \gamma \leq \pi$.

The rays all come from objects at distance d = 1 Gyr ≈ 306.6 Mpc from the observer.

The graphs show $d\gamma/dt$ for three different assumed density profiles in the void.

[53] A. Krasiński and K. Bolejko, Redshift propagation equations in the β' ≠ 0 Szekeres models. *Phys. Rev.* **D83**, 083503 (2011).





Observer O is at the distance R_0 from the void centre; the directions toward the observed galaxy * and toward the void centre are at the angle γ .

3 examples were calculated.

All have d = 1 Gyr ≈ 306.6 Mpc, but different R_0 and different density profiles.

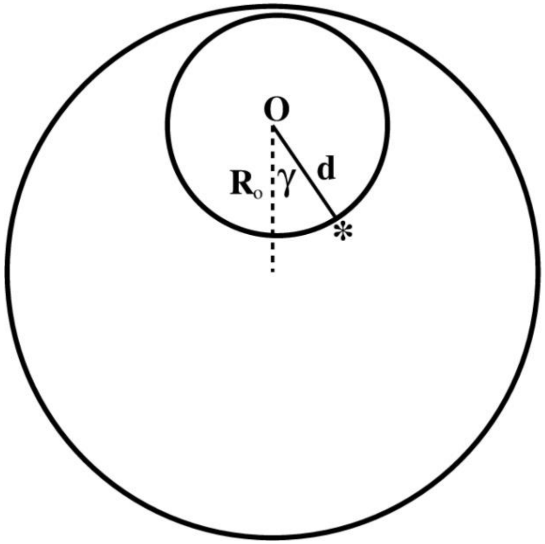
$\rho_{critical} = \rho_{\Lambda CDM}$

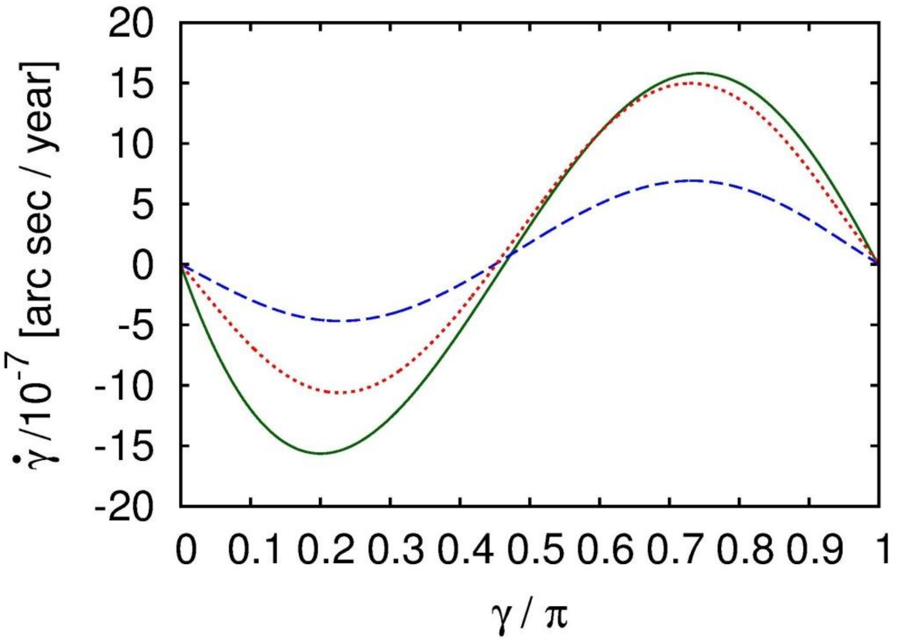
ρ_0 is the density at the void centre.

Example 1: R_0 = 3 Gpc, Profile 1;

Example 2: R_0 = 1 Gpc, Profile1;

Example 3: R_0 = 1 Gpc, Profile 2 (a deeper void in the background of higher density).





d γ/dt as a function of direction, in arc sec/(10^7yr)

continuous line: Example (1),

dashed line: Example (2),

dotted line: Example (3).

The maximum |d γ/dt| is ≈10^{-7} for (2) and ≈10^{-6} for (1) and (3) (extrema are attained at γ ≈ π/4, 3π/4).

With the astrometric accuracy of the Gaia observatory (≈ 10^{-6} arcsec [56]) a few years of monitoring a selected light source would allow us to measure (or rule out) this effect.

But Gaia was designed for a different purpose (precise mapping of star positions in our Galaxy) and this observation is not in its programme.

[56] https://arxiv.org/pdf/2102.11712.pdf

27. The blueshift (a recall)

In the R-W models all observers would see light emitted at the BB with ***infinite redshift***.

$1 + z := \nu_{em}/\nu_{obs} = S_o/S_e$ → if $S_e \to 0$ then $z \to \infty$ and $\nu_{obs} \to 0$

In the L-T and Szekeres models **some rays** from the BB would reach the observers with ***infinite blueshift*** → $\nu_{obs} \to \infty$, $z \to -1$ [57,58].

The necessary conditions for $z = -1$ are:

- $dt_B/dr \neq 0$ at the emission point [58],
- The ray is emitted radially in the L-T model [58], or
- along one of two preferred directions in the Szekeres model [52].

But real observations do not reach back in time beyond the last scattering hypersurface (≈380 000 years after the BB).

→ ***Some*** rays in the L-T and Sz models can have finite blueshift, $\nu_{obs} > \nu_{em}$.

[57] P. Szekeres, Naked singularities. In: *Gravitational Radiation, Collapsed Objects and Exact Solutions*. Edited by C. Edwards. Springer (Lecture Notes in Physics, vol. 124), New York, pp. 477 -- 487 (1980).
[58] C. Hellaby and K. Lake, The redshift structure of the Big Bang in inhomogeneous cosmological models. I. Spherical dust solutions. *Astrophys. J.* **282**, 1 (1984) + erratum *Astrophys. J.* **294**, 702 (1985).
[52] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).

The blueshift is generated when neighbouring shells of constant mass recede from each other with a smaller velocity than the average velocity of expansion of the Universe.

Could blueshifted rays be seen today as ***gamma ray bursts*** (GRBs)?

In principle **yes!**

Models of the GRB sources must reproduce [60]:

(1) The observed frequency range of the GRBs [0.24×10^{19}Hz $\leq \nu \leq 1.25 \times 10^{23}$Hz];

(2) The limited durations of the bursts (up to 30 hours, mostly ≈ 2 minutes [61]);

(3) Existence and durations of ``afterglows'' (mostly a few days, max. n × 100 days [62]);

(4) The (hypothetical) collimation of the GRBs into narrow bundles.

(5) Large distances to their sources ($n \times 10^9$ light years);

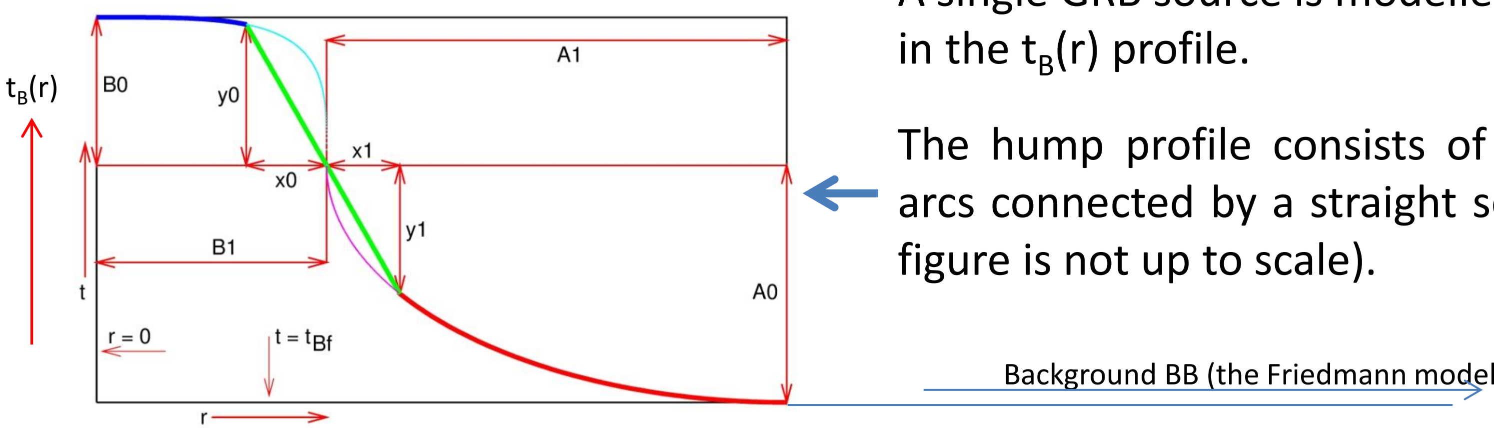
(6) The multitude of the observed bursts (≈ 300/year, one nearly every day).

The mechanism discussed in next section could even more easily produce flashes of e-m radiation of lower frequencies (e.g., X or UV rays).

I concentrated on the gamma rays because they are the most difficult to handle.

[60] Gamma-Ray Bursts, http://swift.sonoma.edu/about_swift/grbs.html
[61] https://imagine.gsfc.nasa.gov/science/objects/bursts1.html
[62] http://astronomy.swin.edu.au/cosmos/G/gamma+ray+burst+afterglow

28. Gamma ray flashes from the last-scattering hypersurface



A single GRB source is modelled by a hump in the $t_B(r)$ profile.

The hump profile consists of two curved arcs connected by a straight segment (the figure is not up to scale).

The upper arc is a segment of the elipse-like curve:

$$\frac{r^n}{{B_1}^n} + \frac{(t - t_{\mathrm{Bf}} - A_0)^n}{{B_0}^n} = 1 \qquad \text{where n = 4 or 6.} \qquad (28.1)$$

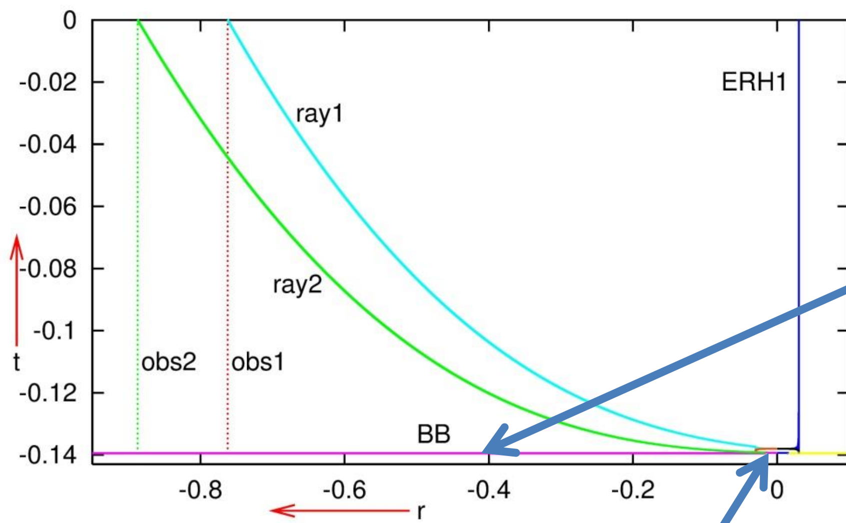
The lower arc is a segment of an ellipse.

The straight segment prevents $dt_B/dr \rightarrow \infty$ at the junction of full arcs.

The free parameters of the model are A_0, A_1, B_0, B_1 and x_0.

This is a proof of existence (through an example) of the blueshifting mechanism, not a model of any real GRB source!

Numerical time unit:
1 NTU =
9.8×10^{10} y =
3×10^{4} Mpc



the present instant

background BB (Friedmann)

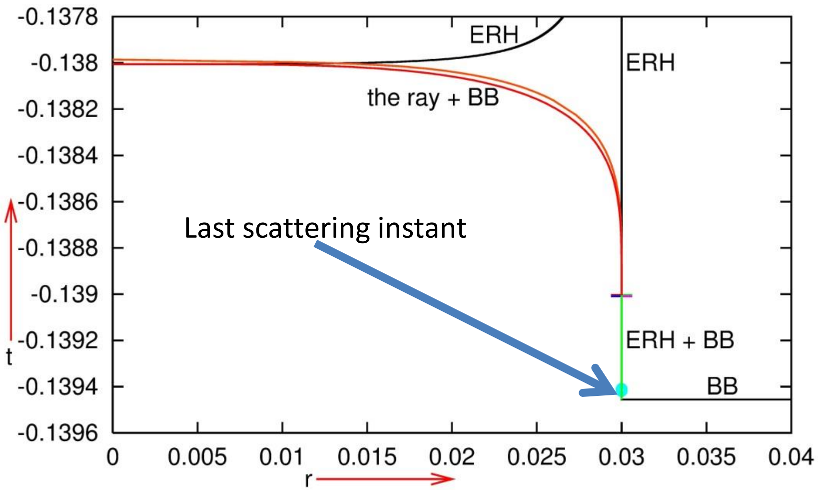
Two humps are drawn here at the right scale relative to the age of the Universe

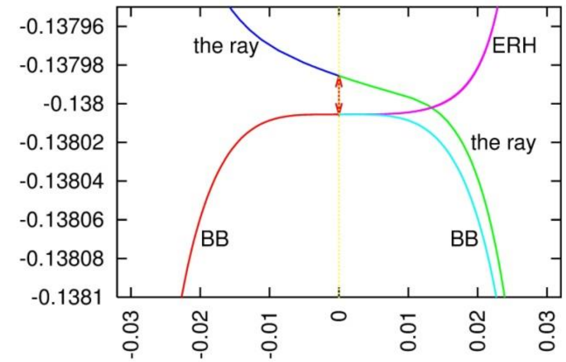
The lower hump (and ray 2) models the GRB source of the lowest observed energy.

Its height is 8.9×10^{-4} × (age of the Universe in the ΛCDM model) ≈ 1.23×10^{7} yrs,

It contains mass ≈ 3.1×10^{6} × (the mass of our Galaxy).

The second hump is 11.5 times higher and 2 times wider, and models a GRB source of the highest observed energy.





Neighbourhood of the hump top, enlarged

The real hump profile and the ``maximally violet'' ray near the BB

Along a ray back in time from the present, z increases up to the intersection of the ray with the ERH (***Extremum Redshift Hypersurface***).

Further toward the past, z decreases up to the intersection with the last scattering hypersurface (LSH). At earlier times, the Sz models do not apply, so the calculations is stopped there.

The hump parameters are chosen so that $2.5 \times 10^{-8} < 1 + z_{\text{observed today}} < 1.7 \times 10^{-5}$.

This moves the light frequency from the hydrogen emission range [63]

$$4.054 \times 10^{13} \text{ Hz } [7400 \text{ nm}] \leq \nu \leq 3.2 \times 10^{15} \text{ Hz } [93.782 \text{ nm}]$$

to the GRB range:

$$0.24 \times 10^{19} < \nu_{GRB} < 1.25 \times 10^{23} \text{ Hz}.$$

[63] Measured Hydrogen Spectrum, http://hyperphysics.hy-astr.gsu.edu/hbase/tables/hydspec.html

L-T models with such a BB profile reproduce [64]:

(1) The observed frequency range of γ rays [0.24×10^{19}Hz $\leq \nu \leq 1.25 \times 10^{23}$Hz];

(5) Large distances to their sources (in this model $\approx 13.6 \times 10^9$ light years);

(6) The multitude of the GRBs (obsered: nearly 1 every day).

The currently best model allows to place ≈330 000 potential sources in the sky [65] (by matching many BB humps into a Friedmann background).

Property **(4)** (the collimation of the GRBs) is not reproduced in the L-T model because of its spherical symmetry, but follows at once from a Szekeres model [52].

Property **(2)** (duration of a GRB – mostly ≈2 min [61]) is accounted for if the ray on the way to the observer flies through another QSS region [66].

The last property from the list:

(3) Existence and duration of afterglows (mostly a few days, max. n×100 days [62]);

is reproduced qualitatively (afterglows exist), but the model implies their too long duration, ≈350 000 y (assuming that the intensity of the radiation is all the time sufficiently large for the detector to see it) → the model needs improvements.

[64] A. Krasiński, Cosmological blueshifting may explain the gamma ray bursts. *Phys. Rev.* **D93**, 043525 (2016).
[65] A. Krasiński, Gamma radiation from areal radius minima in a quasi-spherical Szekeres metric. *Acta Phys. Polon*. **B51**, 483 (2020).
[52] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).
[61] https://imagine.gsfc.nasa.gov/science/objects/bursts1.html
[66] A. Krasiński , Short-lived flashes of gamma radiation in a quasi-spherical Szekeres metric. ArXiv 1803.10101, not to be published.
[62] http://astronomy.swin.edu.au/cosmos/G/gamma+ray+burst+afterglow

29. Expression of hope

Most astronomers see non-R-W models of the Universe as an enemy to elliminate.

Example [67]: The Gaia or E-ELT observatories could ``distinguish a LTB void from an accelerating FRW universe, ***possibly eliminating an exotic alternative explanation to dark energy***''.

E-ELT = European Extremely Large Telescope (this is an official name).

My comment: Are mass condensations in the Universe really more exotic than dark energy?

The L-T and Sz models imply interesting consequences – in the exact Einstein theory.

History of science teaches us that if an otherwise credible theory predicts a new phenomenon, then the prediction must be tested in experiments and observations.

→ The predictions of the L-T and Sz models should also be tested one day without prejudice.

[67] C. Quercellini, L. Amendola, A. Balbi, P. Cabella, M. Quartin, Real-time cosmology. *Phys. Rep.* **521**, 95 - 134 (2012).

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https://www.austms.org.au/wp-content/uploads/Gazette/2005/Sep05/Szekeres.pdf