Relativistic cosmology from F to Sz

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Contents

Lecture 2: The Friedmann – Lemaître (FL) models and their basic implications

5. A continuous medium as a model of the Universe

In theoretical cosmology, the Universe is assumed to be filled with a continuous medium (fluid or gas).

Its state is described by the fields of mass density, pressure and velocity of flow.
And the fields that can be calculated from these.

This medium is assumed to obey the equations of hydrodynamics and thermodynamics.

All these are crude approximations.

The real Universe has a ``granular'' structure – its basic units are stars.

The relevant information is the number of stars or galaxies in a given volume rather than the average mass density in that volume.

The view on what are the `elementary cells' of the cosmic fluid was changing.

In the 1930s these were galaxies. Later, galaxy clusters took over.

In 1978 it turned out [9] that most galaxies occupy edges of large volumes of space, called ***voids***.

The number density of galaxies in voids is < 10% of the large-scale average [10].

Diameters of voids range from 40 Mpc to 600 Mpc [11].

[1Mpc = 10^6 pc, 1 pc = 3.2616 l.y. = 30,857,000,000,000 km]

The largest void is adjacent to our local group of galaxies (! – distance estimates must be not quite precise).

By the current view the elementary units of the Universe are groups of voids.

But as far as observations reach, structures are seen.

[9] S. A. Gregory and L. A. Thompson, The Coma/A1367 supercluster and its environs. *Astrophys. J.* **222**, 784 (1978).
[10] S. G. Patiri, F. Prada, J. Holtzman, A. Klypin, J. Betancort-Rijo, The Properties of Galaxies in Voids. *Mon.Not.Roy.Astron.Soc.* **372,** 1710 (2006).
[11] https://en.wikipedia.org/wiki/List_of_voids

6. The ``cosmological principle''

Observations provide reliable information about a neighbourhood of the Solar System.

To describe the Universe, local observations have to be extrapolated to large volumes.

Extrapolations rely on *assumed* models of spacetime.

The *cosmological principle* is the widest extrapolation.

It originated from the observation of Copernicus that the Earth is not at the centre, but at an unimportant position in the Solar System.

Later, the Earth had been ``degraded'' a few more times:

the Sun is not at the centre of the Universe and many other stars are bigger,

our Galaxy is one of many, not the greatest one and not in any special position.

The cosmological principle is a summary of this line of thinking.

In its weaker form, it says: our position in the Universe is not in any way privileged.

In the extreme form it says: all positions in the Universe are equivalent; the results of observations do not depend on the observer's position.

This is not a summary of observational results, but an ***assumption***.

It was an acceptable working hypothesis in the 1920s when the first-ever models of the Universe were constructed [5, 6, 12] because of lack of observational data.

Today, it is still unverified (cosmic distances are determined with insufficient precision).

Models not obeying this principle are known (examples will be given later).

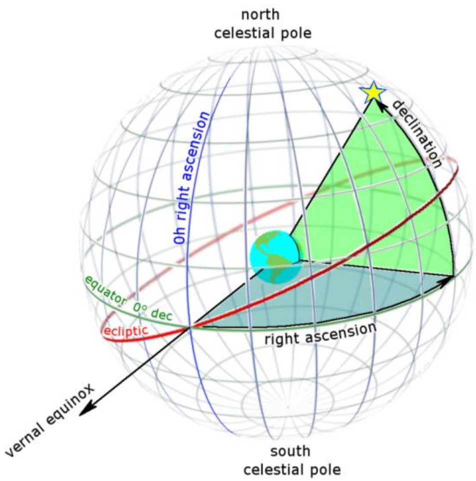
[5] A. A. Friedmann, Über die Krümmung des Raumes. *Z. Physik* **10**, 377 (1922); *Gen. Relativ. Gravit.* **31**, 1991 (1999) + addendum: **32**, 1937 (2000).
[6] A. A. Friedmann, Über die Möglichkeit einer Welt mit konstanter negativer Krümmung des Raumes. *Z. Physik* **21**, 326 (1924); *GRG* **31**, 2001 (1999) ; both reprinted papers with an editorial note by A. Krasiński and G. F. R. Ellis, *Gen. Relativ. Gravit.* **31**, 1985 (1999).
[12] G. Lemaître, *Ann. Soc. Sci. Bruxelles* **A47**, 19 (1927); English translation (somewhat updated): *Mon. Not. Roy. Astr. Soc.* **91**, 483 (1927). Faithful translation: *Gen. Relativ. Gravit*. **45**, 1635 (2013), with an editorial note by J.-P. Luminet,*Gen. Relativ. Gravit*. **45**, 1619 (2013).

Is this distribution homogeneous?

At what scale?

Is this a representative probe of the full 3d distribution?

Copy from:
[13] V. Springel, C. S. Frenk, S. D. M. White, The large-scale structure of the Universe, arXiv:astro-ph/0604561v1 (*Nature* **440**, 1137 (2006)).



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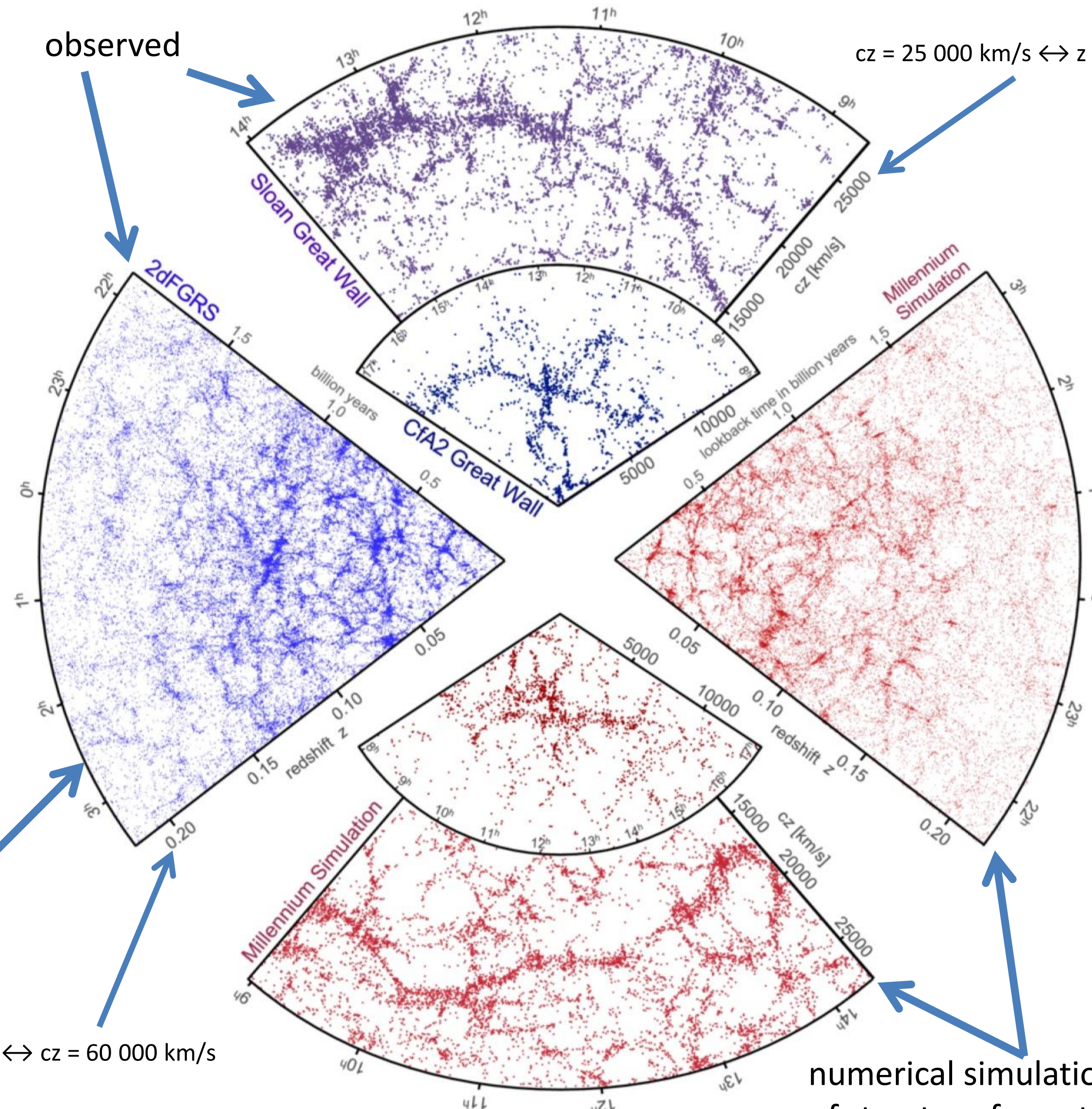
https://en.wikipedia.org/wiki/Right_ascension#/
media/File:Ra_and_dec_on_celestial_sphere.png

observed

cz = 25 000 km/s ↔ z = 0.083



a slice ≈2° around declination 15° south
https://en.wikipedia.org/wiki/2dF_Galaxy_Redshift_Survey

z = 0.2 ↔ cz = 60 000 km/s

numerical simulations of structure formation

Astronomers claim that the space is *approximately* isotropic around us.

The main argument for this statement is the *approximate* isotropy of the measured temperature of the cosmic microwave background (CMB) radiation, $\Delta T/T \approx 10^{-5}$.

But this is the largest anisotropy that the currently existing structures could produce.

→ The 10^{-5} only proves that the interaction between radiation and matter condensations is weak, not that matter is distributed highly isotropically.

Taking the claim of isotropy for granted,

by the cosmological principle the space should be isotropic around every other point.

A space that is isotropic around every point is homogeneous. This points to the FLRW spacetimes introduced in Sec. 4.

7. The Robertson—Walker metrics as models of the Universe

These metrics were presented in Sec. 4:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - S^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta \mathrm{d}\varphi^2\right)\right] \quad (7.1)$$

where S(t) is the ***scale factor*** to be determined from the Einstein equations.

The coordinates of (7.1) are comoving, so the velocity field $u^\alpha = (1, 0, 0, 0)$.

u^α is defined so that $g_{\alpha\beta} u^\alpha u^\beta = 1$.

The ***curvature index*** k, when ≠ 0, can be scaled to +1 or -1 by
$r = \hat{r}/\sqrt{|k|}$, $S(t) = \sqrt{|k|}\hat{S}(t)$,

but this makes the limiting transition $k \to 0$ impossible and creates the illusion that we have here three distinct classes of models. In truth, k is a continuous parameter.

Other representations of the R—W metric are also in use. Let

$$r = \frac{r'}{1 + \frac{1}{4}kr'^2}\ . \quad (7.2)$$

This changes (7.1) to

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{S^2(t)}{\left(1 + \frac{1}{4}kr'^2\right)^2}\left[\mathrm{d}r'^2 + r'^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta \mathrm{d}\varphi^2\right)\right]\ . \quad (7.3)$$

We shall use (7.1) or (7.3), whichever is more convenient.

$$\mathrm{d}s^2 = \mathrm{d}t^2 - S^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta \mathrm{d}\varphi^2\right)\right] \qquad (7.1)$$

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{S^2(t)}{\left(1+\frac{1}{4}kr'^2\right)^2}\left[\mathrm{d}r'^2 + r'^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta \mathrm{d}\varphi^2\right)\right] \qquad (7.3)$$

These spacetimes were first introduced by Aleksandr Aleksandrovich Friedmann in 1922 [5] (the case k > 0) and in 1924 [6] (the case k < 0); in coordinates different from (7.1) and (7.3).

The case k = 0 was first introduced by Robertson in 1929 [7].

Friedmann solved the Einstein equations for these metrics, in the generalised form:

$G_{\alpha\beta} + \Lambda g_{\alpha\beta} = (8\pi G/c^2)\, T_{\alpha\beta}$,

where Λ is a universal ***cosmological constant*** to be determined from observations.

He treated his results as merely a mathematical curiosity.

Both papers were quickly forgotten – astronomers had not yet been ready to accept their implications, while Friedmann was not sufficiently influential to win attention.

In 1925, he died prematurely and had no chance to claim credit for his discovery when the expansion of the Universe was generally accepted as fact in 1929.

[5] A. A. Friedmann, Über die Krümmung des Raumes. *Z. Physik* **10**, 377 (1922); *Gen. Relativ. Gravit.* **31**, 1991 (1999) + addendum: **32**, 1937 (2000).
[6] A. A. Friedmann, Über die Möglichkeit einer Welt mit konstanter negativer Krümmung des Raumes. *Z. Physik* **21**, 326 (1924); *GRG* **31**, 2001 (1999) ; both reprinted papers with an editorial note by A. Krasiński and G. F. R. Ellis, *Gen. Relativ. Gravit.* **31**, 1985 (1999).
[7] H. P. Robertson, Relativistic cosmology. *Rev. Mod. Phys.* **5**, 62 (1933); *Gen. Relativ. Gravit.* **44**, 2115 (2012), with an editorial note by G. F. R. Ellis, *Gen. Relativ. Gravit.* **44**, 2099 (2012).

$$\mathrm{d}s^2 = \mathrm{d}t^2 - S^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2\right)\right] \qquad (7.1)$$

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{S^2(t)}{\left(1+\frac{1}{4}kr'^2\right)^2}\left[\mathrm{d}r'^2 + r'^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2\right)\right] \qquad (7.3)$$

The case $k > 0$ was independently discovered by Georges Lemaître in 1927 [12].

He generalised Friedmann's solution to nonzero pressure, and was aware that it reflects some properties of the real Universe.

The metric (7.1) – (7.3) was derived from symmetry assumptions by Robertson in 1933 [7] and by Walker in 1935 [8]. This is when its geometric definition became clear.

These metrics are often called Friedmann—Lemaître—Robertson—Walker (FLRW) models, but various subsets of this collection of four names are also in use.

It seems appropriate to name the general metric (7.1) or (7.3) Robertson—Walker, and Friedmann or Lemaître when referring to their solutions.

[12] G. Lemaître, *Ann. Soc. Sci. Bruxelles* **A47**, 19 (1927); English translation (somewhat updated): *Mon. Not. Roy. Astr. Soc.* **91**, 483 (1927). Faithful translation: *Gen. Relativ. Gravit*. **45**, 1635 (2013), with an editorial note by J.-P. Luminet,*Gen. Relativ. Gravit*. **45**, 1619 (2013).
[7] H. P. Robertson, Relativistic cosmology. *Rev. Mod. Phys.* **5**, 62 (1933); *Gen. Relativ. Gravit.* **44**, 2115 (2012) , with an editorial note by G. F. R. Ellis, *Gen. Relativ. Gravit.* **44**, 2099 (2012).
[8] A. G. Walker, On Riemannian spaces with spherical symmetry about a line, and the conditions for isotropy in general relativity. *Quart. J. Math. Oxford*, ser. 6, 81 (1935).

8. The redshift

An observer moving with four-velocity u_1^α will find the rate of change of the phase P of an electromagnetic wave to be

$v_p = P_{,\alpha} u_1^\alpha = k_{1\alpha} u_1^\alpha$,

where $k_{1\alpha} = P_{,\alpha}$ is the wave vector field, tangent to the rays.

k^α obeys the geodesic equation $\frac{\mathrm{d}k^\alpha}{\mathrm{d}s} + \left\{ \begin{matrix} \alpha \\ \rho\sigma \end{matrix} \right\} k^\rho k^\sigma = 0$,so is affinely parametrised, and is null, $k_\alpha k^\alpha = 0$ [1].

Within a short time-interval Δs_1, the phase will thus change by

$\Delta P = k_{1\alpha} u_1^\alpha \Delta s_1$.

Another observer, moving with velocity u_2^α and measuring ΔP at another point, where $k_\alpha = k_{2\alpha}$, will find the same ΔP to take the time-interval, Δs_2: $\Delta P = k_{2\alpha} u_2^\alpha \Delta s_2$.

$$\rightarrow \quad \frac{\Delta s_1}{\Delta s_2} = \frac{(k_\alpha u^\alpha)_2}{(k_\alpha u^\alpha)_1} \qquad (8.1)$$

[1] J. Plebański and A. Krasiński, An introduction to general relativity and cosmology. Cambridge University Press 2006.

$$\frac{\Delta s_1}{\Delta s_2} = \frac{(k_\alpha u^\alpha)_2}{(k_\alpha u^\alpha)_1} \qquad (8.1)$$

If the wave is periodic with frequency ν, then $\Delta S = 2\pi\nu\Delta s$.

→ For the same ΔS measured by two observers $\nu_1\Delta s_1 = \nu_2\Delta s_2$, so

$$\frac{\nu_2}{\nu_1} = \frac{\Delta s_1}{\Delta s_2} = \frac{(k_\alpha u^\alpha)_2}{(k_\alpha u^\alpha)_1} \quad . \qquad (8.2)$$

This formula applies in all geometries.

Let the subscripts e and o denote quantities calculated at the point of emission of the light ray and at the point of observation, respectively.

Let λ denote the wavelength of a light wave. Then, by definition

$$z = (\lambda_o - \lambda_e)/\lambda_e = \lambda_o/\lambda_e - 1$$

is the ***redshift*** between the emission point and the observation point.

"Red" because in nearly all cosmological observations $\lambda_o > \lambda_e$, so the observed light is "redder" than at emission.

$z = (\lambda_o - \lambda_e)/\lambda_e = \lambda_o/\lambda_e - 1$

$$\frac{\nu_2}{\nu_1} = \frac{\Delta s_1}{\Delta s_2} = \frac{(k_\alpha u^\alpha)_2}{(k_\alpha u^\alpha)_1} \qquad (8.2)$$

But $\lambda_o/\lambda_e = \nu_e/\nu_o$ because $c = \lambda\nu$ is constant, so

$$1 + z = \frac{\nu_e}{\nu_o} = \frac{(k_\alpha u^\alpha)_e}{(k_\alpha u^\alpha)_o} \ . \qquad (8.3)$$

The direction of the ray at the observer is determined by the unit spacelike vector n^α:

$$n^\alpha = \xi\,(\delta^\alpha{}_\beta - u^\alpha u_\beta)\,k^\beta, \qquad n^\alpha u_\alpha = 0. \qquad (8.4)$$

n^α is collinear with the projection of k^α on the hypersurface element orthogonal to the velocity of the observer.

We calculate ξ from the requirement $n^\alpha n_\alpha = -1$.

Substituting the first of (8.4) in $n^\alpha n_\alpha = -1$ we obtain $\xi^2 = 1/(k_\rho u^\rho)^2$, so

$$n^\alpha = -\,(1/\,k_\rho u^\rho)\,k^\alpha + u^\alpha.$$

The minus is there because n^α points from the observer *toward* the source of light, opposite to the direction of k^α.

$$1 + z = \frac{\nu_e}{\nu_o} = \frac{\left(k_\alpha u^\alpha\right)_e}{\left(k_\alpha u^\alpha\right)_o} \qquad (8.3)$$

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{S^2(t)}{\left(1 + \frac{1}{4}kr'^2\right)^2}\left[\mathrm{d}r'^2 + r'^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2\right)\right] \qquad (7.3)$$

To use (8.3), we have to know k^α along the ray (recall: it must be affinely parametrised).

In R-W spacetimes all points within the same space t = constant are equivalent

→ the result of any calculation is independent of the spatial position of the observer.

Let us assume that the observer is at r = 0.

A ray sent toward r = 0 will keep the radial direction $d\vartheta/dt = d\varphi/dt = 0$ all along.

→ Such a ray obeys

$$0 = \mathrm{d}t^2 - \frac{S^2(t)}{\left(1 + \frac{1}{4}kr^2\right)^2}\mathrm{d}r^2 \qquad (8.5)$$

Let us introduce the parameter v on the ray by

$dt/dv = 1/S(t)$. (8.6)

In this parametrisation, the tangent vector to the ray (8.5) is

$$k^\alpha = \left[\frac{1}{S}\,, -\frac{1}{S^2}\,\left(1 + \frac{1}{4}kr^2\right), 0, 0\right] \qquad (8.7)$$

and it can be verified that v is an affine parameter, so $dk^\alpha/dv + \left\{ {\alpha \atop \rho\sigma} \right\} k^\rho k^\sigma = 0$.

$$1 + z = \frac{\nu_e}{\nu_o} = \frac{(k_\alpha u^\alpha)_e}{(k_\alpha u^\alpha)_o} \qquad (8.3)$$

$$k^\alpha = \left[\frac{1}{S}\,, -\frac{1}{S^2}\ \left(1 + \frac{1}{4}kr^2\right), 0, 0\right] \qquad (8.7)$$

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{S^2(t)}{\left(1+\frac{1}{4}kr'^2\right)^2}\ \left[\mathrm{d}r'^2 + r'^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2\right)\right] \qquad (7.3)$$

The velocity field is $u^\alpha = (1, 0, 0, 0)$, so with k^α given by (8.7) we have

$k_\alpha u^\alpha = 1/S$ and $1 + z = S(t_o)/S(t_e)$. (8.8)

Now note that

$$\ell(t) = S(t)\int_0^{r_e} \frac{\mathrm{d}r}{1+\frac{1}{4}kr^2} \qquad (8.9)$$

is the distance between the light source at $r = r_e$ and the observer at $r = 0$, calculated within the hypersurface of constant t along the radial line $d\vartheta/dt = d\varphi/dt = 0$.

In comoving coordinates r_e is independent of t, and then

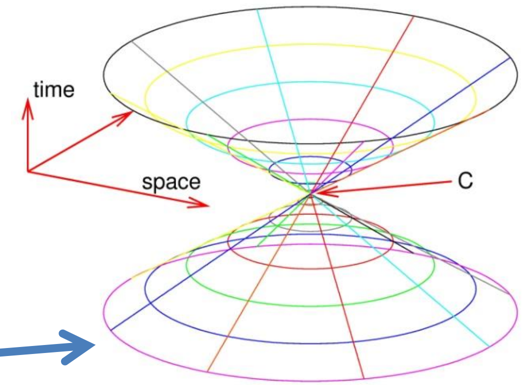
$$\frac{\mathrm{d}\ell}{\mathrm{d}t} = \frac{\dot{S}}{S}\,\ell = H\ell/c. \qquad (8.10)$$

This is the ***Hubble*** (now, officially, ***Hubble-Lemaître***) ***law***, with H called (by tradition) the ***Hubble constant***, although it is a function of the cosmic time t.

Astronomers interpret ℓ as the Euclidean distance and (8.10) as the Euclidean relation (recession velocity) = H × (distance).

$$\ell(t) = S(t) \int_0^{r_e} \frac{\mathrm{d}r}{1 + \frac{1}{4}kr^2} \qquad (8.9)$$

$1 + z = S(t_o)/S(t_e)$. (8.8)



Mathematically speaking, there is no unique way to define distance in cosmology.

Events seen at C lie on C's past light cone, i.e., in a different space.

But not-too-distant objects lie nearly in the Euclidean space tangent to t = constant at C.

Astronomers use a measure of distance called ***luminosity distance***:

$$D_L = \sqrt{\frac{L}{4\pi \mathcal{F}_o}}\,, \qquad (8.11)$$

where the ***luminosity*** L is the total energy per unit time emitted by the light source (assumed known *from theory*), and $\mathcal{F}_o$ is the flux of energy (i.e. energy through unit surface in a unit of time) *measured* by the observer.

D_L is the distance from which the radiating body, if motionless in a Euclidean space, would produce the energy flux equal to $\mathcal{F}_o$ measured by the observer.

$D_L = \sqrt{\frac{L}{4\pi\mathcal{F}_o}}$ (8.11)

$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right)\right] \tag{7.1}$$

D_L is related to the redshift by the ***Mattig formula***

$$D_L = \frac{c}{H}\frac{q_0 z + (q_0 - 1)\left(\sqrt{2q_0 z + 1} - 1\right)}{q_0{}^2(1+z)^2} \tag{8.12}$$

where

$$q_0 \stackrel{\text{def}}{=} -SS_{,tt}\,/S_{,t}{}^2\big|_{\text{now}} \equiv -1 - \frac{c}{H^2}\frac{dH}{dt} \tag{8.13}$$

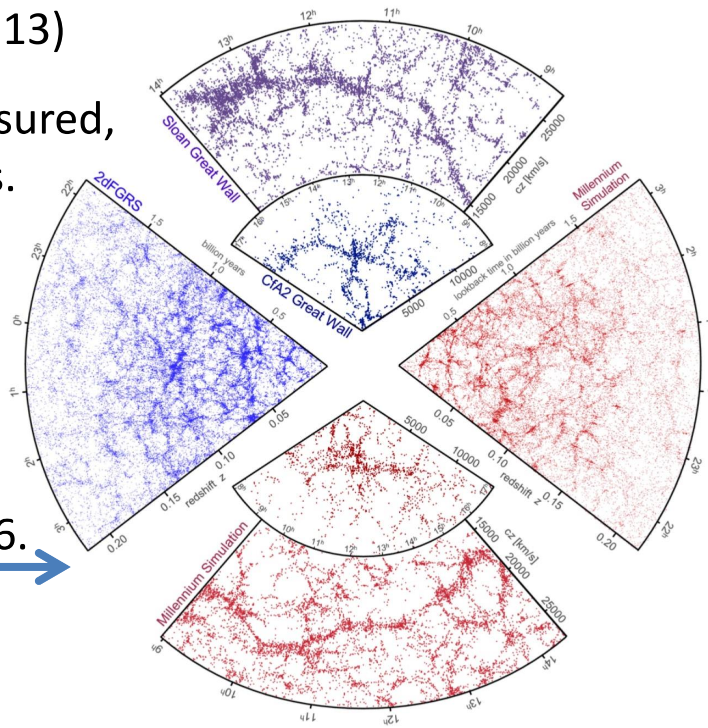
is the ***deceleration parameter***. It cannot be directly measured, but can be calculated from other measureable quantities.

For small z, (8.12) reduces to

$D_L \approx cz/H$

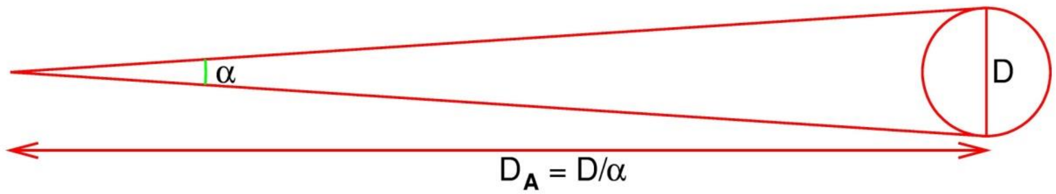
so z is often used as a proxy for distance.

It was used as a measure of distance in the graphs on p. 6.



The ***corrected luminosity distance*** $\check{D}_L$ takes into account the motion of the light source away from the observer:

$\check{D}_L = D_L/(1 + z)^2$.



A still more realistic measure is the ***angular diameter distance*** D_A (also called *area distance*) – see figure.

In the Euclidean space it coincides with the luminosity distance D_L.

It makes sense also in Friedmann spacetimes:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - S^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta \mathrm{d}\varphi^2\right)\right] \rightarrow D_A = rS(t)$$

and happens to coincide with the corrected luminosity distance $\check{D}_L = D_L/(1 + z)^2$.

(The calculation proving it is simple in principle but long and tedious [14,1].)

[14] G. F. R. Ellis: Relativistic cosmology, in: Sachs, R.K. (ed.) Proceedings of the International School of Physics "Enrico Fermi", Course 47: General relativity and cosmology, pp. 104–182. Academic Press, New York and London (1971). Reprinted in *Gen. Relativ. Gravit.* **41**,581 (2009) with an editorial note by W. Stoeger, *Gen. Relativ. Gravit.* **41**,575 (2009) .
[1] J. Plebański and A. Krasiński, An introduction to general relativity and cosmology. Cambridge University Press 2006.

$$\mathrm{d}s^2 = \mathrm{d}t^2 - S^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2\right)\right] \qquad (7.1)$$

9. The Friedmann equations

The present average mass-density in the Universe $\rho < 10^{-28}$ g/cm^3, so pressure does not influence the large-scale motion of matter and p = 0 is an acceptable hypothesis.

So, following Friedmann, we assume p = 0.

But to describe the evolution of the Universe in the early period of high density, pressure must be taken into account. This would change eq. (9.2) below.

The Einstein equations then imply

$\rho S^3 = C =$ constant, $\quad C := 3\mathcal{M}/(4\pi)$, (9.1)

$S_{,t}{}^2 = 2G\mathcal{M}/(c^2 S) - k - (1/3)\Lambda\, S^2$ (9.2)

With $cS_{,t}/S = H$ – the Hubble `constant', eq. (9.2) may be written in another form:

$$\rho = \frac{3H^2}{8\pi G} + \frac{c^2}{8\pi G}\left(\frac{3k}{S^2} + \Lambda\right). \qquad (9.3)$$

$$\rho = \frac{3H^2}{8\pi G} + \frac{c^2}{8\pi G}\left(\frac{3k}{S^2} + \Lambda\right) \qquad (9.3)$$

The quantities ρ and H are *in principle* measurable.

The uncertain element in both ρ and H is measuring the distances.

Direct measurement, by parallax, is possible only for a few nearby galaxies.

For distant galaxies, the Hubble law is *assumed* to hold exactly, and then used as a measure of distance.

Also in principle, the value of Λ may be deduced from observations.

→ Eq. (9.3) gives us a chance to determine the sign of k for the real Universe.

The results of the Planck satellite mission [15] imply $\Lambda \approx 10^{-46}$ km^{-2}.

A sphere of this curvature would have the radius $R_\Lambda \approx 109$ Mpc.

For comparison: our neighbouring group of galaxies, M81, is ≈ 3.5 Mpc from us [16], this is $R_\Lambda/31$.

[15] Planck collaboration, Planck 2013 results. XVI. Cosmological parameters. *Astron. Astrophys.* **571**, A16 (2014).
[16] https://en.wikipedia.org/wiki/List_of_galaxy_groups_and_clusters

$G_{\alpha\beta} + \Lambda\, g_{\alpha\beta} = (8\pi G/c^2)\, T_{\alpha\beta}$ $S,_t{}^2 = 2G\mathcal{M}/(c^2 S) - k - (1/3)\Lambda S^2$ (9.2)

10. The ΛCDM model

The current conventional wisdom in astronomy is that the Universe is expanding at an accelerated rate [17–18].

This conclusion was reached by comparing the observed luminosities of the type Ia supernovae with their absolute peak luminosities calculated from theory and distances to them calculated from z by the Hubble law.

A type Ia supernova explodes when a white dwarf in a double star system accreting matter from its companion reaches a certain limiting mass.

Since these supernovae are created always in the same way, their peak luminosity is *assumed* the same for all – they are called ***standard candles***.

By Refs. [17 – 18] all the type Ia supernovae were too dim as for their distances calculated with $\Lambda = 0$.

A good fit of observations to *models of the R-W class* was achieved when $\Lambda < 0$.

Λ may be replaced by something dependent on t that imitates its action. (This changes the algebraic form of the solution of (9.2).)

All those ``somethings'' are collectively called ***dark energy***.

[17] S.Perlmutter *et al.*, Measurements of Omega and Lambda from 42 High-Redshift Supernovae. *Astrophys. J.* **517**, 565 (1999).
[18] A. G. Riess *et al.*, Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, *Astron. J.* **116**, 1009 (1998).

$S,_t{}^2 = 2G\mathcal{M}/(c^2S) - k - (1/3)\Lambda S^2$ (9.2)

$$\rho = \frac{3H^2}{8\pi G} + \frac{c^2}{8\pi G}\left(\frac{3k}{S^2} + \Lambda\right) \qquad (9.3)$$

Let us differentiate (9.2) by t:

$S,_{tt} = -G\mathcal{M}/(cS)^2 - \Lambda S/3$

If $\Lambda \geq 0$, then $S,_{tt} < 0$ → the expansion of the Universe is decelerated.

With $\Lambda < 0$, the accelerated expansion ($S,_{tt} > 0$) appears at large S. It sets in when

$S = S_i = [3G\mathcal{M}/(-\Lambda c^2)]^{1/3}$.

The time of that inflection point can be calculated once we know the solution of (9.2).

The accelerated expansion is implied only when the observations of supernovae of type Ia are interpreted using the models of the R-W class. With inhomogeneous models, one may account for these observations by inhomogeneities in mass distribution and expansion rate, without introducing Λ – see next lecture.

The ``standard candle'' status of the Ia supernovae is questioned [19] now that more of them were observed.

Even so, the current conventional wisdom in astronomy says still more:

The values of ρ, H and Λ are such that $k = 0$ in (9.3) (the Universe is *spatially flat*).

Such a model is called ΛCDM, the CDM meaning ``cold dark matter''.

[19] R. Mohayaee, M. Rameez and S. Sarkar, Do supernovae indicate an accelerating universe? arXiv:2106.03119.

$S_{,t}{}^2 = 2G\mathcal{M}/(c^2S) - k - (1/3)\Lambda S^2$ (9.2)

$$\rho = \frac{3H^2}{8\pi G} + \frac{c^2}{8\pi G}\left(\frac{3k}{S^2} + \Lambda\right) \qquad (9.3)$$

The solution of (9.2) with k = 0 and $\Lambda < 0$ is

$$S(t) = \left[\frac{6M_0}{(-\Lambda)}\right]^{1/3} \sinh^{2/3}\left[\frac{\sqrt{-3\Lambda}}{2}(t - t_B)\right], \qquad M_0 := G\mathcal{M}/c^2 \qquad (10.1)$$

where and t_B is the Big Bang time at which S = 0, so $H \to \infty$ and $\rho \to \infty$ by (9.3).

The inflection $S_{,tt} = 0$ occurs at such t_i where

$$t_i - t_B = \frac{1}{\sqrt{-3\Lambda}} \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) \qquad (10.2)$$

Taking $(-\Lambda) \approx 10^{-46}$ km^{-2} [15] and remembering that the physical time $\tau = t/c$ we obtain

$(t_i - t_B)/c = 8 \times 10^9$ years.

The current estimate of the age of the Universe is $T_U \approx 13.67 \times 10^9$ years [15].

→ The acceleration began somewhat earlier than at $(2/3)T_U$.

[15] Planck collaboration, Planck 2013 results. XVI. Cosmological parameters. *Astron. Astrophys.* **571**, A16 (2014).

11. The redshift drift – a test for accelerated expansion

$H = cS_{,t}/S = H(t)$

Since H depends on the cosmic time t, what about dH/dt?

A. Sandage [20] in 1962 was the first to consider measuring the ***redshift drift*** dH/dt,.

With the 1962 technology, $\approx 10^7$ years of monitoring the value of H would be required to detect its changes [21].

By a more recent estimate, a few decades would be sufficient [22].

The hypothesis of accelerated expansion of the Universe can be tested by detecting just the sign of dH/dt.

[20] A. Sandage, *Astrophys. J.* **136**, 319 (1962).
[21] R. Lazkoz, I. Leanizbarrutia and V. Salzano, *European Phys. J. C*, **78**, 11 (2018).
[22] A. Loeb, *Astrophys. J.* **499**, L111 (1998).

$$\frac{\nu_2}{\nu_1} = \frac{\Delta s_1}{\Delta s_2} = \frac{(k_\alpha u^\alpha)_2}{(k_\alpha u^\alpha)_1} \qquad (8.2)$$

$$1 + z = \frac{\nu_e}{\nu_o} = \frac{(k_\alpha u^\alpha)_e}{(k_\alpha u^\alpha)_o} \qquad (8.3)$$

$$1 + z = S(t_o)/S(t_e) \qquad (8.8)$$

For galaxies, t_e increases when t_o increases.

From (8.2) and (8.3),

$dt_o/dt_e = 1 + z$.

Then

$$\frac{dS(t_e)}{dt_o} = \frac{dS(t_e)}{dt_e} \times \frac{dt_e}{dt_o} = \frac{1}{1+z} \times \frac{dS(t_e)}{dt_e},$$

and so, from (8.8)

$$\frac{dz}{dt_o} = \frac{1}{S(t_e)} \left[\frac{dS(t_o)}{dt_o} - \frac{dS(t_e)}{dt_e} \right]$$

So, $dz/dt_o > 0$ when dS/dt has increased between t_e and t_o, and $dz/dt_o < 0$ otherwise.

→ If $dz/dt_o > 0$ for some observed objects, then the expansion of the Universe had been accelerating for some time,

but if $dz/dt_o < 0$ for all objects, then no accelerated expansion has ever occurred.

Let us wait and see: is dark energy real or was it a hutzpah?

$$\mathrm{d}s^2 = \mathrm{d}t^2 - S^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta \mathrm{d}\varphi^2\right)\right] \qquad (7.1)$$

$$S_{,t}{}^2 = 2G\mathcal{M}/(c^2 S) - k - (1/3)\Lambda S^2 \qquad (9.2)$$

12. The Friedmann solutions with Λ = 0

Let $\Lambda = 0$ and $\alpha := G\mathcal{M}/c^2$. The solutions of (9.2) are best represented parametrically.

For $k < 0$:

$$S = -\frac{\alpha}{k}(\cosh\omega - 1), \qquad t - t_B = \frac{\alpha}{(-k)^{3/2}}(\sinh\omega - \omega) \qquad (12.2)$$

where $0 \le \omega < \infty$ is the parameter, and t_B is an arbitrary constant.

Eqs. (12.2) can be solved for $t = t(S)$, but $S(t)$ is not an elementary function.

For $k = 0$ the solution is

$$S = \left[\frac{9}{2}\alpha\left(t - t_B\right)^2\right]^{1/3} \qquad (12.3)$$

Here, α can be scaled by transforming r, so its actual value has no physical meaning.
(It defines the unit of distance.)

Finally, for $k > 0$, the solution of (12.1) is

$$S = \frac{\alpha}{k}(1 - \cos\omega), \qquad t - t_B = \frac{\alpha}{k^{3/2}}(\omega - \sin\omega),\ 0 \le \omega \le 2\pi. \qquad (12.4)$$

In all three cases we have taken into account the observed fact that at present $S_{,t} > 0$.

Each solution has a singularity at $t \to t_B$, where $S \to 0$ and $\rho \to \infty$.

At $t = t_B$ matter would be squeezed to a point (but the models do not apply at t near t_B).

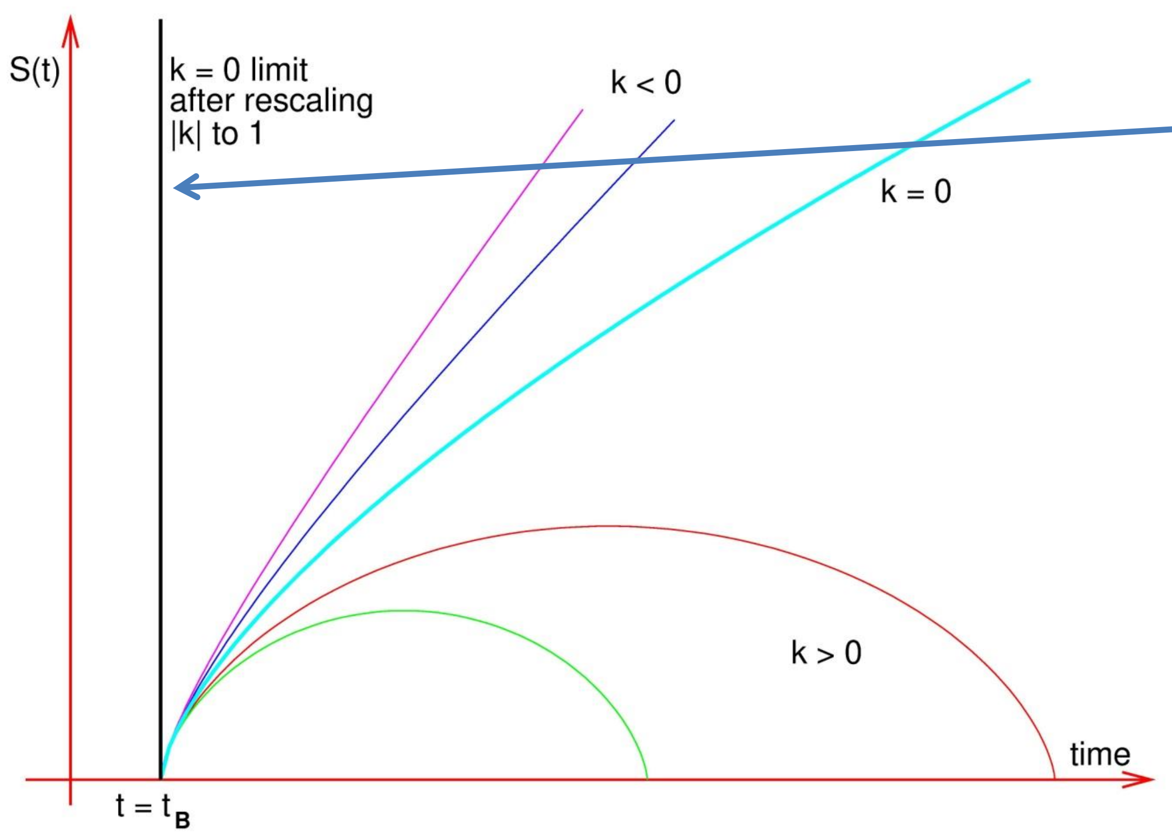
$$S = \frac{\alpha}{k}(1 - \cos\omega), \qquad t - t_B = \frac{\alpha}{k^{3/2}}(\omega - \sin\omega) \qquad (12.4)$$

The model (12.4) has a second singularity at $\omega = 2\pi$, where

$$t = t_{FS} = t_B + \frac{2\pi\alpha}{k^{3/2}} = t_B + \frac{2\pi G\mathcal{M}}{k^{3/2}c^2} \qquad (12.5)$$

It has a finite lifetime $t_{FS} - t_B = 2\pi\alpha/k^{3/2}$; at $t = t_{FS}$ its existence is terminated.

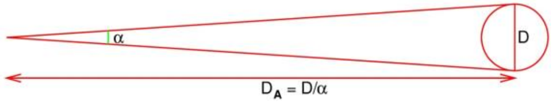
The other two models expand forever.



$S(t) = \sqrt{|k|}\hat{S}(t)$
in the limit $k \to 0$
all values of $\hat{S}(t) \to \infty$

$$\rho = \frac{3H^2}{8\pi G} + \frac{c^2}{8\pi G}\left(\frac{3k}{S^2} + \Lambda\right) \quad (9.3)$$

$$\mathrm{d}s^2 = \mathrm{d}t^2 - S^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta \mathrm{d}\varphi^2\right)\right] \quad (7.1)$$



$D_A = rS(t) = D_L/(1 + z)^2$

13. The redshift—distance relation in the Λ ≠ 0 Friedmann models

Let us define the following three dimensionless parameters:

$$(\Omega_m, \Omega_k, \Omega_\Lambda) \stackrel{\text{def}}{=} \frac{c^2}{3{H_0}^2}\left(\frac{8\pi G\rho_0}{c^2}, -\frac{3k}{{S_0}^2}, -\Lambda\right)\Bigg|_{t=t_o} \quad (13.1)$$

where t_o is the current instant,
$\rho_o = \rho(t_o)$ is the current mean mass density in the Universe, $S = S(t_o)$,
H_o is the current value of the Hubble parameter, $H_o = c(S,_t/S)_{t=to}$.

They are the ***density-***, ***curvature-*** and ***cosmological constant parameters***.

With (13.1), the transformed Friedmann equation (9.3) becomes

$$\Omega_m + \Omega_k + \Omega_\Lambda = 1. \quad (13.2)$$

We assume the observer at r = 0, so all rays reaching her are radial.

Now we need to solve the equation of radial null geodesics in the metric (7.1).

We take the initial point of the geodesic at r = 0, and follow it to the past. Then

$$\frac{1}{S}\frac{\mathrm{d}t}{\mathrm{d}r} = -\frac{1}{\sqrt{1-kr^2}} \quad (13.3)$$

$$\frac{1}{S}\,\frac{\mathrm{d}t}{\mathrm{d}r} = -\frac{1}{\sqrt{1-kr^2}} \qquad (13.3)$$

take k < 0 for the beginning

$$S_{,t}^{\;2} = 2G\mathcal{M}/(c^2S) - k - (1/3)\Lambda\, S^2 \qquad (9.2)$$

$$(\Omega_m, \Omega_k, \Omega_\Lambda) \stackrel{\mathrm{def}}{=} \frac{c^2}{3{H_0}^2}\left(\frac{8\pi G\rho_0}{c^2}, -\frac{3k}{{S_0}^2}, -\Lambda\right)\Bigg|_{t=t_o} \qquad (13.1)$$

$$D_A = rS(t) = D_L/(1+z)^2 \qquad (*)$$

Integrating (13.3) from $(t, r) = (t_o,0)$ to the light source at (t_e, r_e) we obtain

$$\int_{t_o}^{t_e}\frac{\mathrm{d}t}{S(t)} = -\int_0^{r_e}\frac{\mathrm{d}r}{\sqrt{1-kr^2}} = -\frac{1}{\sqrt{-k}}\,\mathrm{arsinh}\left(\sqrt{-k}r_e\right) \qquad (13.4)$$

On the l.h.s we transform the variable by $dt/S(t) \equiv dS/(SS_{,t})$, then use (9.2) for $S_{,t} > 0$.

In integrating backward from the observer t_o= constant and the integration variable is:

$$S = S(t_e) = S(t_o)/(1 + z). \qquad (13.5)$$

So, we change the ingegration variable to z by (13.5) and solve (13.4) for r_e:

$$r_e = \frac{1}{\sqrt{-k}}\sinh\left[\int_0^z\frac{\sqrt{\Omega_k}\mathrm{d}z'}{\sqrt{\Omega_m(1+z')^3+\Omega_k(1+z')^2+\Omega_\Lambda}}\right] \qquad (13.6)$$

Then, from (*)

$$\boxed{D_L(z) = \frac{c(1+z)}{H_0\sqrt{\Omega_k}}\sinh\left[\int_0^z\frac{\sqrt{\Omega_k}\mathrm{d}z'}{\sqrt{\Omega_m(1+z')^3+\Omega_k(1+z')^2+\Omega_\Lambda}}\right]} \qquad (13.7)$$

This is the ``canonical'' (uncorrected) luminosity distance vs. redshift relation.

$$(\Omega_m, \Omega_k, \Omega_\Lambda) \stackrel{\text{def}}{=} \frac{c^2}{3{H_0}^2}\left(\frac{8\pi G\rho_0}{c^2}, -\frac{3k}{{S_0}^2}, -\Lambda\right)\Big|_{t=t_o} \qquad q_0 \stackrel{\text{def}}{=} -SS_{,tt}/{S_{,t}}^2\big|_{\text{now}} \equiv -1 - \frac{c}{H^2}\frac{\mathrm{d}H}{\mathrm{d}t}$$

$$\boxed{D_L(z) = \frac{c(1+z)}{H_0\sqrt{\Omega_k}} \sinh\left[\int_0^z \frac{\sqrt{\Omega_k}\mathrm{d}z'}{\sqrt{\Omega_m(1+z')^3 + \Omega_k(1+z')^2 + \Omega_\Lambda}}\right]} \qquad (13.7)$$

Equation (13.7) holds also for $k > 0$ with

$(\sinh, \sqrt{\Omega_k}) \longrightarrow (\sin, \sqrt{-\Omega_k})$

which is consistent with the identity $\sinh(ix) \equiv i \sin x$.

The now-standard ΛCDM model has $k = 0 = \Omega_k$. The limit of (13.7) at $\Omega_k \to 0$ is

$$D_L(z) = \frac{c(1+z)}{H_0}\int_0^z \frac{\mathrm{d}z'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}} \qquad (13.8)$$

When $\Lambda = 0 = \Omega_\Lambda$ the integral in (13.7) can be explicitly calculated; then (13.7) reduces to the Mattig formula:

$$D_L(z) = \frac{c}{H}\,\frac{q_0 z + (q_0 - 1)\left(\sqrt{2q_0 z + 1} - 1\right)}{{q_0}^2(1+z)^2} \qquad (13.9)$$

To verify this, one must observe that $q_0 = \Omega_m/6 - \Omega_\Lambda/3$. The calculation is simple but long.

We will come back to (13.8) in the third lecture and show that it can be reproduced in the Lemaître – Tolman model with $\Lambda = 0$.

14. Horizons in the Robertson—Walker models

Observers in the Universe receive information about distant objects via light rays.

In most R-W models there exist objects from which the present observer has not yet received any ray.

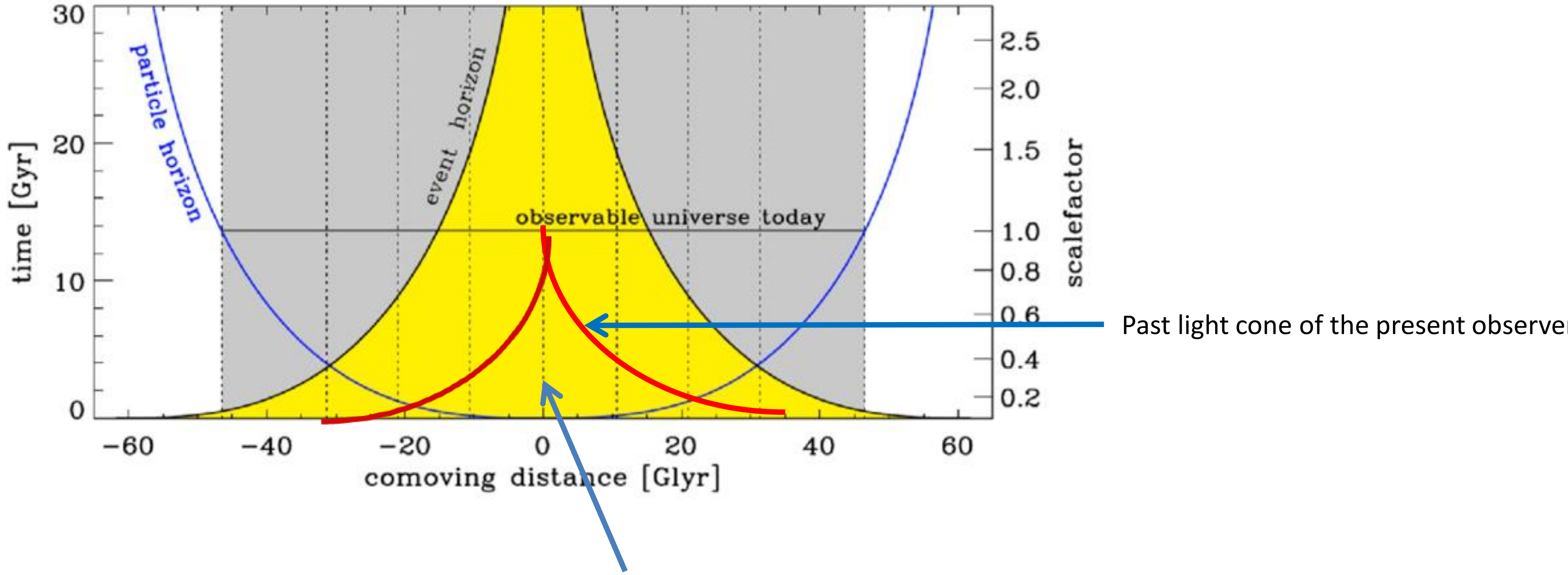
In R-W models expanding with acceleration objects exist from which the present observer has not received and will never receive any ray.

The boundaries separating objects already observed from those not yet observed, and objects observable from unobservable are called ***horizons***.

The definitions of horizons were introduced by Wolfgang Rindler [23] in 1956.

Wolfgang Rindler
18 May 1924, Vienna, Austria – 8 February 2019, Dallas, Texas, USA
Pictured in December 2013

[23] W. Rindler, *Mon. Not. Roy. Astr. Soc.* 116, 662 (1956); reprinted in Gen. Relativ. Gravit. **34**, 133 (2002), with an editorial note by A. Krasiński, *Gen. Relativ. Gravit.* **34**, 131 (2002).



The ***event horizon*** (EH) for observer A is the hypersurface in spacetime that divides all events into two nonempty classes:

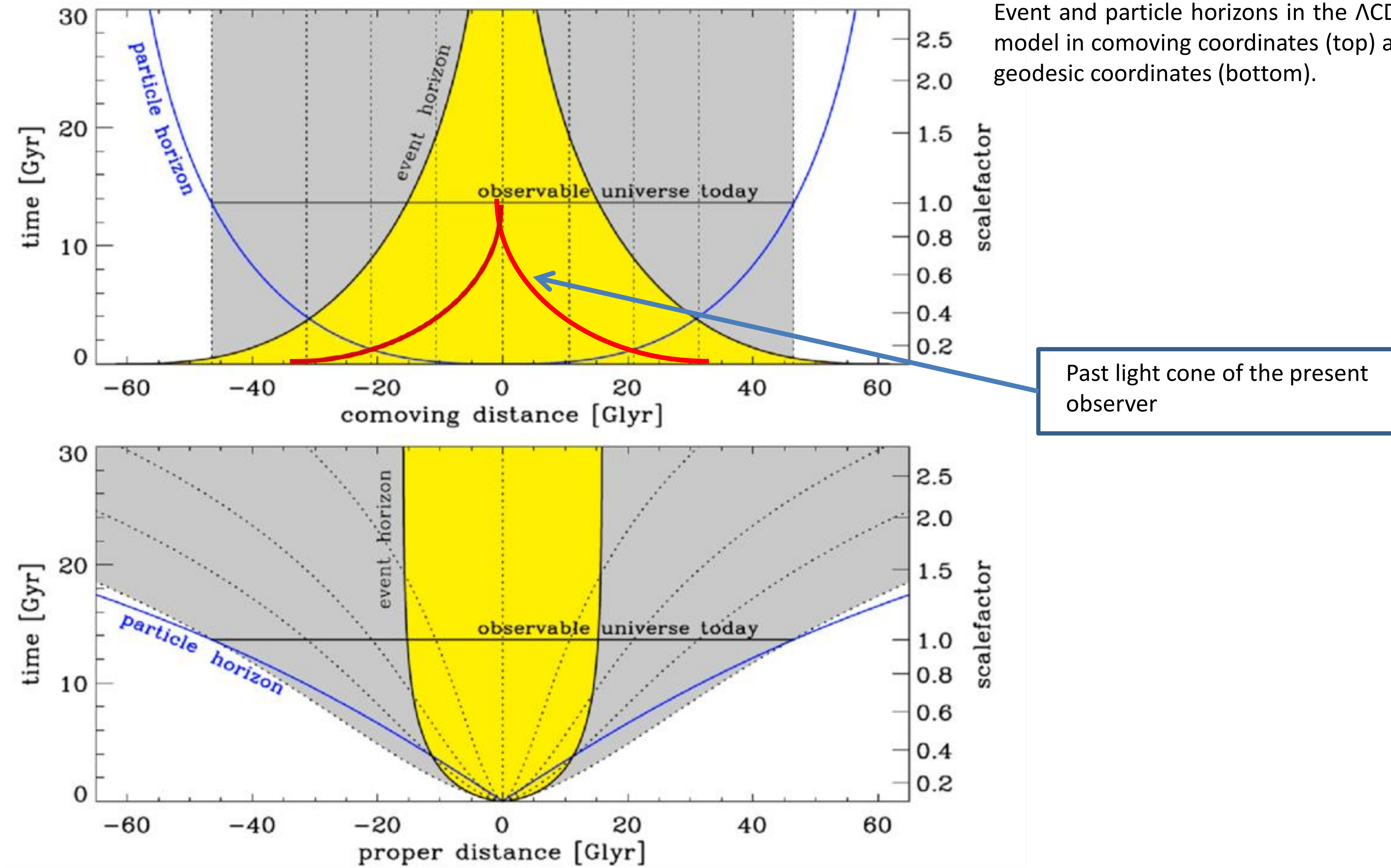
Those that have been, are or will be seen by A,
and those that A has never seen and will never be able to see.

Not every R-W spacetime has event horizons.

The ***particle horizon*** (PH) for observer A at t_0 is a 2-dimensional surface in the space $t = t_0$ that divides all world lines of matter into two nonempty classes: those that A has seen up to $t = t_0$ and those that A has not yet seen.

All the Friedmann solutions with $\Lambda = 0$ have particle horizons.

From
https://www.researchgate.net/figure/The-size-of-the-cosmic-event-horizon-depends-on-whether-one-is-using-comoving-distances
_fig2_301455097, by Charley Lineweaver



Event and particle horizons in the ΛCDM model in comoving coordinates (top) and geodesic coordinates (bottom).

$$\mathrm{d}s^2 = \mathrm{d}t^2 - S^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\,\mathrm{d}\varphi^2\right)\right] \quad (7.1)$$

Define

$$\sigma(r) \overset{\mathrm{def}}{=} \int_0^r \frac{\mathrm{d}r'}{\sqrt{1-kr'^2}} = \begin{cases} \frac{1}{\sqrt{k}}\arcsin\left(\sqrt{k}r\right) & \text{for} \quad k>0, \\ r & \text{for} \quad k=0, \\ \frac{1}{\sqrt{-k}}\,\mathrm{arsinh}\left(\sqrt{-k}r\right) & \text{for} \quad k<0. \end{cases} \qquad (14.1)$$

$$T(t_1, t) \overset{\mathrm{def}}{=} \int_{t_1}^{t} \frac{\mathrm{d}\tau}{S(\tau)} \qquad (14.2)$$

A photon emitted at (t, r) = (t_1, r_1) and proceeding *toward* the observer at r = 0 obeys

$\sigma(r) = \sigma(r_1) — T(t_1,t)$. (14.3)

When k ≤ 0, the necessary and sufficient condition for the existence of an EH is the convergence of $T(t_1,t)$ to a finite limit at t → ∞.

Then, $\sigma(r_1)$ differs from σ(r) by a finite quantity σ_H even at infinite future.

→ A photon emitted at $r \geq r(\sigma_H)$ at any t will never reach the observer at r = 0 = σ(0).

Below, the most important properties of horizons are given without proof.
For details see [23] and [1].

[23] W. Rindler, *Mon. Not. Roy. Astr. Soc.* **116**, 662 (1956); reprinted in *Gen. Relativ. Gravit.* **34**, 133 (2002).
[1] J. Plebański and A. Krasiński, An introduction to general relativity and cosmology. Cambridge University Press 2006.

$$\sigma(r) \stackrel{\text{def}}{=} \int_0^r \frac{\mathrm{d}r'}{\sqrt{1-kr'^2}} = \begin{cases} \frac{1}{\sqrt{k}} \arcsin\left(\sqrt{k}r\right) & \text{for} \quad k>0, \\ r & \text{for} \quad k=0, \\ \frac{1}{\sqrt{-k}} \operatorname{arsinh}\left(\sqrt{-k}r\right) & \text{for} \quad k<0. \end{cases} \tag{14.1}$$

$$T(t_1, t) \stackrel{\text{def}}{=} \int_{t_1}^{t} \frac{\mathrm{d}\tau}{S(\tau)} \tag{14.2}$$

$$\sigma(r) = \sigma(r_1) — T(t_1,t) \tag{14.3}$$

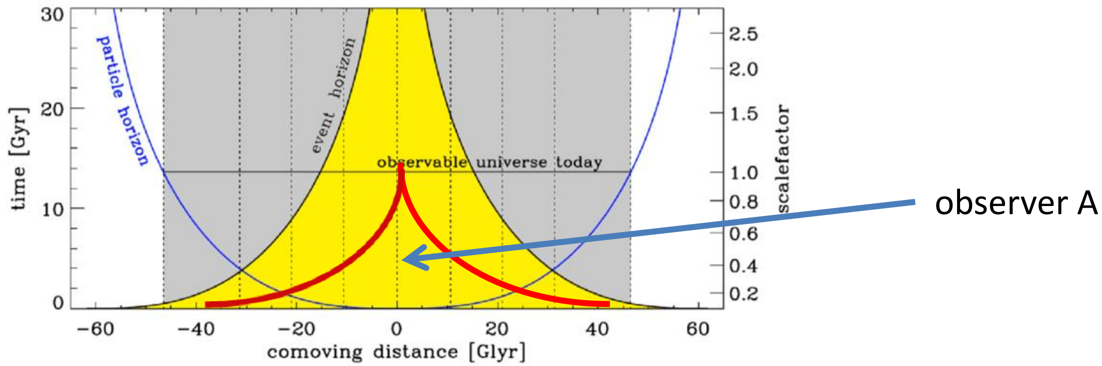
Because of homogeneity and isotropy of the R-W models, if an EH exists for one observer, then EHs exist for all observers.

When an EH exists, some events will never be seen by the observer even though light from them will keep going toward the observer.

How can this be understood intuitively?

If S(t) increases sufficiently rapidly, then the spatial distance $\sigma(r_1)S(t)$ between the observer at $r = 0$ and the light source at $r = r_1$ increases so fast that light stays behind.

Eddington once explained this as follows: imagine someone who runs on an expanding race track, but the finish line moves away faster than he can run.



$$T(t_1, t) \stackrel{\text{def}}{=} \int_{t_1}^{t} \frac{d\tau}{S(\tau)} \qquad (14.2)$$

In the Friedmann models with $\Lambda = 0$ and $k \leq 0$ EHs do not exist: every event will eventually become visible for every observer because $T(t_1,\infty) = \infty$.

If a particle was initially inside the EH for observer A, then it will remain visible for A for ever – see figure.

But A will see the particle only up to a certain instant $t = t_{out}$ on the particle's clock.

The signal sent from $r = r_1$ at $t = t_{out}$ will reach A at $t \rightarrow \infty$, i.e. never.

A $t_{out} < \infty$ exists for every particle that was ever visible to A, see figure.

Each particle visible for A will remain visible only for a finite period of its history and then will escape from A's view through the EH.

$T(t_1, t) \stackrel{\text{def}}{=} \int_{t_1}^{t} \frac{d\tau}{S(\tau)}$ (14.2) $\sigma(r) = \sigma(r_1) - T(t_1,t)$ (14.3) $1 + z = S(t_o)/S(t_e)$ (8.8)

The necessary and sufficient condition for the existence of a PH is the convergence of $T(t_1,t)$ to a finite limit at $t_1 \to t_B$ (the Big Bang time).

If $T(t_B,t) < \infty$, then $T(t_B,t)$ determines, via (14.3), the farthest particles from which the observer at r could have received a light signal up to the instant t.

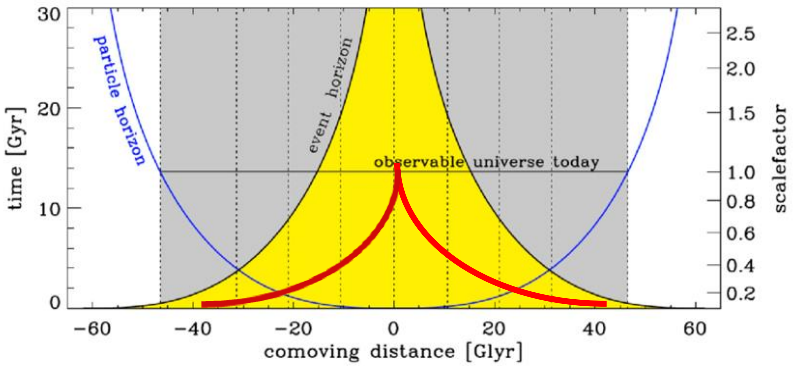
If a PH exists, then still more particles come within it (see figure).

All Friedmann models with $\Lambda = 0$ do have PHs.

The first ray received by observer A from each particle is the one emitted at the initial singularity $t = t_B$. But $S(t_B) = 0$, so by eq. (8.8), that ray comes with infinite redshift.

→ ***In the R-W models the initial singularity is not observable***.

In the real Universe no direct light rays from the Big Bang can be received for another reason: for some time after the BB matter in the Universe is nontransparent because photons keep interacting with other elementary particles.



$$1 + z = \frac{\nu_e}{\nu_o} = \frac{(k_\alpha u^\alpha)_e}{(k_\alpha u^\alpha)_o} \qquad (8.3)$$

15. The `history of the Universe'

The idea that the Universe might have a history followed the discovery of its expansion and the realisation that the Friedmann and Lemaître models account for it.

Since the Universe had been denser in the past, it must have been hotter as well.

→ Some time ago it should have been sufficiently hot that all atoms were ionised.

At the ionisation temperature, in the cooling Universe radiation decoupled from atoms.

Expansion should have cooled the radiation still further.

With the black-body spectrum preserved all the time, the evolution of temperature can be calculated. The intensity I of the black-body radiation as a function of frequency ν is

$$I(\nu) = \frac{2h\nu^3}{c^2 \left\{ \exp\left[h\nu/(kT)\right] - 1 \right\}} \qquad (15.1)$$

where *h* and *k* are the Planck and Boltzmann constants and *T* is the temperature.

The received frequencies ν_o obey (8.3), so $\nu_o(1 + z) = \nu_e$ = constant along each ray.

$$I(\nu) = \frac{2h\nu^3}{c^2 \{\exp[h\nu/(kT)] - 1\}} \qquad (15.1)$$

$\nu_o(1 + z) = \nu_e = \text{constant}$

To keep the form of I(ν) in (15.1) unchanged, $T_o(1 + z)$ = constant must hold.

That ***cosmic microwave background*** (CMB) radiation should still be with us.

This idea first occurred to Gamow [24] and collaborators [25] in 1948.

The CMB radiation was detected in 1965: today it has the temperature 2.73 K [26,27].

The extrapolation back in time went on.

All the known atomic nuclei could, in principle, be built by adding protons and neutrons one by one to the nucleus of hydrogen (the proton).

Before emitting the CMB, the Universe must have been hot enough to crash all heavier nuclei; only protons, neutrons and loose electrons could survive.

Is it possible that matter originally consisted only of these particles, and heavier nuclei were created in collisions between them?

This idea again occurred to Gamow [24] and Alpher and Herman [25].

[24] G. Gamow, The evolution of the Universe, *Nature* **162**, 680 (1948).
[25] R. A. Alpher and R. C. Herman, Evolution of the Universe, *Nature* **162**, 774 (1948).
[26] R. H. Dicke, P. J. E. Peebles, P. G. Roll and D. T. Wilkinson, Cosmic black-body radiation, *Astrophys. J.* **142**, 414 (1965).
[27] A. A. Penzias and R. W. Wilson, A measurement of excess antenna temperature at 4080 Mc/s, *Astrophys. J.* **142**, 419 (1965).

Computer simulations showed that only a few nuclei could be created in this way:

- ≈25% of the mass of the Universe would be converted into helium ^{4}He,
- tiny traces of deuterium, helium ^{3}He, lithium, beryllium and boron would appear,

but falling temperature would stop any further synthesis [28] and ≈75% of the mass of the Universe should remain in the form of protons.

Heavier elements are synthesised later, in the stars [29].

These calculated proportions of nuclides were confirmed by observations [30].

Thinking along these lines, cosmologists reconstructed the possible sequence of events in the evolution of the Universe [31].

The conclusions listed above were based on phenomena known from laboratory and verified observationally, others are speculations.

[28] R. V. Wagoner, W. A. Fowler and F. Hoyle, On the synthesis of elements at very high temperatures, *Astrophys. J.* **148**, 3 (1967).
[29] W. A. Fowler, *Nuclear Astrophysics*. American Philosophical Rociety, Philadelphia 1967.
[30] A. M. Boesgaard and G. Steigman, Big bang nucleosynthesis – theories and observations, *Ann. Rev. Astron. Astrophys.* **23**, 319 (1985).
[31] T. Padmanabhan, *Structure Formation in the Universe.* Cambridge University Press 1993.

The leading motive of the speculations was this:

As we go backward in time, density and temperature of matter increase without limits.

→ Whatever processes we know that should take place at high temperatures, must have really occurred in the early Universe.

Why the BB explosion occurred and what preceded it cannot be explained by means of the currently existing physics or mathematics. Thus, we take the BB as a given thing.

At precisely what times did the different stages of evolution take place?

The numbers vary between sources.

The references given below are random; with no claim to being precise and up to date.

- The current number for the time of the BB explosion is $\approx 13.67 \times 10^9$ years ago [15].

- During the first 10^{-34} s, the Universe had a temperature $>10^{27}$ K and was supposedly described by a `Grand Unified Theory' (GUT, still in the making) that unites the strong nuclear, weak nuclear and electromagnetic interactions [32, vol. II].

The elementary particles known today had not necessarily existed then, and matter might have been composed of free quarks and gluons.

[15] Planck collaboration, Planck 2013 results. XVI. Cosmological parameters. *Astron. Astrophys.* **571**, A16 (2014).
[32] K. Lang, Astrophysical Formulae. Vol. I: Radiation, Gas Processes and High Energy Astrophysics; Vol. II: Space, Time, Matter and Cosmology. Springer, Berlin – Heidelberg -- New York 1999.

- Between 10^{-34} s and 10^{-32} s after the BB, ***inflation*** took place.

This is a hypothetical period when the Universe expanded at an exponential rate.

At the end of it, the elementary particles known today should already exist, together with seeds for structure formation.

The mechanism and instant of creation of the seeds are still under debate [32,vol. II].

- About 1 s after the BB, neutrinos decoupled and thereafter propagated freely.

- During the next few seconds, protons, neutrons, electrons, positrons and photons existed in thermal equilibrium, at temperatures $T \geq 10^{10}$ K [33].

- Light atomic nuclei formed between 2 and 1000 s after the BB [33].

At the end of this period, the temperature dropped to $\approx 10^9$ K.

- Later, the Universe continued to be a radiation-dominated plasma, but the radiation mass-density ρ_r (which obeys $\rho_r S^4$ = constant with $S \sim 1/T$) was decreasing faster than the mass-density of massive particles, ρ_m (obeying $\rho_m S^3$ = constant).

[32] K. Lang, Astrophysical Formulae. Vol. I: Radiation, Gas Processes and High Energy Astrophysics; Vol. II: Space, Time, Matter and Cosmology. Springer, Berlin – Heidelberg -- New York 1999.
[33] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation*. Freeman, San Francisco 1973.

● $\approx 3 \times 10^5$ years after the BB [32, Vol.II] and at $T \approx 10^3$ K, ρ_r became $< \rho_m$ and radiation decoupled from matter, being unable to ionise the atoms.

It evolved into the CMB radiation observed today.

● Still later, structures (galaxies, galaxy clusters, voids) must have formed.

Even though the process of structure formation should be well within the domain of classical gravitation theory, it is still poorly understood.

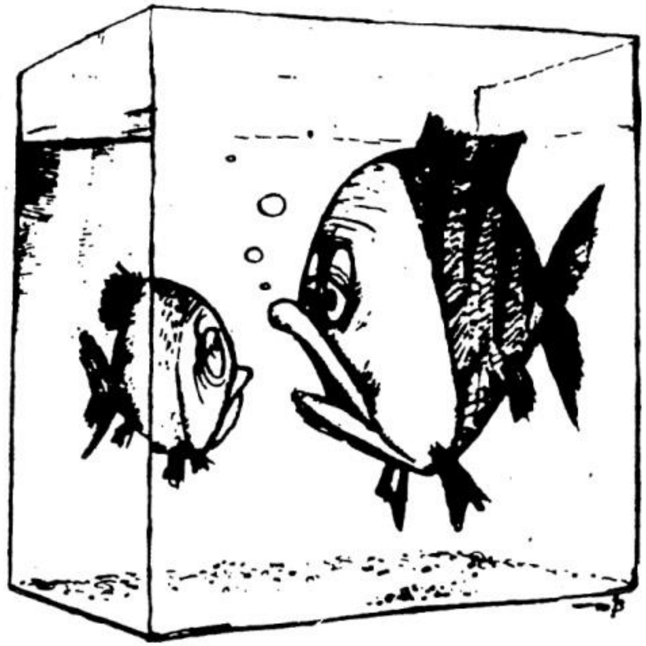
There exists no quantitative account of the emergence of structures, only a general firm belief that structures came about by gravitational magnification of density fluctuations created very early.

Models of the R-W class are unable to describe structure formation – perturbations of them must be considered.

This is done by linearizing the Einstein equations around the Minkowski metric $\eta_{\alpha\beta}$: the full metric $g_{\alpha\beta}$ is assumed to have the form $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$ and terms involving $(h_{\alpha\beta})^p$ with $p \geq 2$ are neglected.

[34] W. Hu (2004), http://background.uchicago.edu/\~whu/physics/tourpage.html

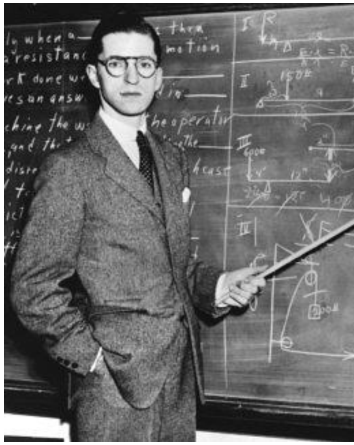
16. Summary



'The universe, my son, is a large tank full of water'

Fig. 25.1. The cosmological principle.

Copied from:
H. Stephani, *General Relativity*. Second edition. Cambridge University Press (1990).

Georgiy Antonovich Gamov (Георгий Антонович Гамов)
4 March 1904 (O.S. 10 February 1904), Odessa, Ukraine
-- 19 August 1968, Boulder, Colorado, USA
Copied from: https://www.tekportal.net/gamow/

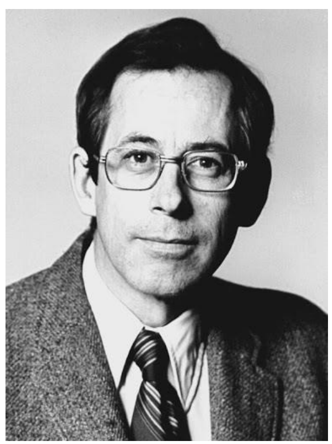
http://www.timeone.ca/cast/ralph-alpher/#sthash.CgnF7ioS.dpbs

https://www.researchgate.net/figure/Robert-C-Herman-as-photographed-by-Ralph-A-Alpher-in-1948-Credit-Alpher-Papers_fig2_257315887

Ralph A. Alpher
3 February 1921, Washington, USA
-- 12 August 2007, Austin, Texas, USA

Robert C. Herman
29 August 1914, New York
-- 13 February 1997, Austin, Texas

Robert Henry Dicke
6 May 1916, St. Louis, Missouri
-- 4 March 1997, Princeton, New Jersey
https://en.wikipedia.org/wiki/Robert_H._Dicke



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