Quantum correlations, non-local games and operator algebras

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Abstract

There are several mathematical models to describe the conditional probability densities/correlations that can occur when two labs in entangled state conduct quantum measurements. It has been fundamental research done to study whether these models give rise to the same sets of correlations. The celebrated Bell theorem demonstrates that the set of classical correlations are strictly smaller than the quantum ones, while the Tsirelsson problems are related to differences between "physically realizable" bipartite probability distributions. Operator systems, operator algebras and their tensor products have been an important tool to study such distributions. One of the Tsirelson problems is related to finite approximability in operator algebras and is equivalent to the Connes' embedding problem in von Neumann algebras, the solution of which was recently announced. Many results on correlations have come from the study of non-local games and their winning strategies. They witness the differences between classes of correlations and provide ways of constructing new interesting classes of operator algebras.

In these lectures I will give an introduction of non-local games. After going over the basic theory of operator systems I will highlight the role C^*-algebras and operator systems play in mathematical understanding of quantum correlations and perfect strategies of non-local games. Synchronous games as for example graph homomorphism/isomorphism games are of particular interests as their perfect strategies can be described through traces of affiliated C^*-algebras. I will discuss differences between classes of quantum correlations.

The last lecture will be devoted to some recent results on quantum no-signaling correlations which appear as strategies of non-local games with quantum inputs and outputs. If time allows I will also talk about quantum graphs and quantum homomorphism/isomorphism games.