Dynamics of Jellet's Egg. Asymptotic solutions revisited

Jellett's egg (JE) is a rigid body having a shape of rotational ellipsoid with half-axes (a, b) where b is a symmetry axis. The center of the mass (CM) is shifted with respect to the geometric center O by distance $d = \varkappa b < b$, $0 \le \varkappa \le 1$ along b. The axes of main moment of inertia I_1, I_3 are aligned along the geometrical axes a, b. JE has an egg-like form for $a < b$, it looks like en elongated cigar when $a \ll b$ and for $b \ll a$ the JE looks like a button which becomes a flat disc when $b \to 0$.

It ***rolls and slides*** on the plane under the action of force of gravity, friction force and the reaction force of the supporting plane. When the JE is spun sufficiently fast, it displays behaviour similar to the tippe top (TT), its lower and heavier part may turn upside down so that the center of mass CM rises above the geometrical center O. Then JE spins for some time on the long edge before it falls down due to rotational friction and dissipation of kinetic energy. It is an effect that may be noted when one plays with a boiled egg on a kitchen table.

Physical parameters of both TT and JE have to satisfy certain geometrical and inertial conditions in order to display inversion under fast spinning initial conditions.

The main tool for studying the dynamical behaviour of a JE ***rolling and sliding i***n the plane is analysis of asymptotic/stationary solutions of equations of motion with friction and the conditions of reality for the asymptotic values of angular velocities $\dot{\phi}_{JE}, \omega_{3JE}$.

The answer is that fast spinning JE inverts when parameters $\alpha=a/b$, $\gamma=I_1/I_3$ satisfy a) geometric condition $1 - \varkappa < \alpha^2 < 1 + \varkappa$ and b) inertial condition $1 - \varkappa < \alpha^2 \gamma < 1 + \varkappa$. For TT the a) condition is always satisfied as a=b so $\alpha=a/b=1$ and b) condition becomes the known inequality $1 - \varkappa < \gamma < 1 + \varkappa$ when the whole range of tumbling solutions with $\theta \in (0, \pi)$ is admissible and the inversion is observed. This result is illustrated by numerical simulations of JE launched almost vertically with $\theta(0) = 1/10, 1/100$.

Reference "*On Dynamics of Jellet's Egg. Asymptotic Solutions Revisited*" S.Rauch-Wojciechowski and M.Przybylska, Regular and Chaotic Dynamics, 2020, Vol. 25, No. 1, pp. 40–58