

Kronecker webs and nonlinear PDEs

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A web on a manifold is a collection of foliations of constant dimension in a general position. Theory of classical webs, i.e. finite collection of foliations, was developed in the first half of XX century by the school of W. Blaschke. In the end of that century I. Gelfand and I. Zakharevich introduced Kronecker webs [GZ91], collections of foliations parametrized by one-dimensional projective space with a particular dependence on the parameter, as objects encoding the local geometry of finite-dimensional bi-hamiltonian structures, i.e. pairs of compatible Poisson structures. Later Kronecker webs appeared as independent object of investigations in relation with integrable nonlinear systems of PDEs [Zak00], [DK14], [KP17], [Pan19].

In this series of lectures I will give an outline of this relation based on the last two references and also explain recent ideas putting preceding results into the context of the so-called heavenly PDEs describing vacuum self-dual Einstein metrics in neutral signature [PS22].

References

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