

# BERGMAN KERNELS, FEFFERMAN'S METRIC, AND QUANTIZATION OF COMPLEX MANIFOLDS

SORIN DRAGOMIR

## 1. LECTURE I

1. Bergman kernel of a domain  $\Omega \subset \mathbb{C}^n$ . 2. Explicit calculation of Bergman kernels (for balls, ellipsoids, etc.). 3. Differentiability up to the boundary of the Bergman kernel of a smoothly bounded strictly pseudoconvex domain (Kerzman's theorem). 4. Fefferman's asymptotic formula. 5. Graham-Lee connection. 6. Smoothness up to the boundary of the solution to the Dirichlet problem for the Bergman Laplacian.

## 2. LECTURE II

1. Fefferman's metric. 2. Problems in Cauchy-Riemann geometry springing from Lorentzian versions of classical problems (CR Yamabe problem, theory of subelliptic harmonic maps, etc.). 3. Schmidt boundaries of Fefferman space-times. 4. Boundaries of worm domains  $\Omega \subset \mathbb{C}^2$ : intrinsic circles running into curvature singularities. 5. Quantum mechanical resolution of singular points in Fefferman space-times.

## 3. LECTURE III

1. Weighted Bergman kernels. 2. Infinite dimensional Banach manifolds of weights. 3. Forelli-Rudin-Ligočka-Peloso asymptotic formula for a weighted Bergman kernel on a strictly pseudoconvex domain  $\Omega \subset \mathbb{C}^n$ . 4. Quantization of complex domains  $\Omega \subset \mathbb{C}^n$ . 5. Infinite dimensional complex projective spaces associated to complex Hilbert spaces. 6. Odziejewicz's quantization maps  $\mathcal{K} : \Omega \rightarrow \mathbb{C}\mathbb{P}(L^2H(\Omega, \gamma))$  within harmonic map theory (Lichnerowicz's theorem). 7. Boundary behavior of quantization maps.

---

Università degli Studi della Basilicata, Dipartimento di Matematica, Informatica ed Economia, Via dell'Ateneo Lucano 10, 85100 Potenza, Italy, e-mail [sorin.dragomir@unibas.it](mailto:sorin.dragomir@unibas.it)

Lectures at "Geometry and Physics" [satellite school of Workshop on Geometric Physics (<https://wgmp.uwb.edu.pl/>)] Białystok (June 27 - July 1, 2022).

## REFERENCES

- [1] E. Barletta & S. Dragomir & K.L. Duggal, *Foliations in Cauchy-Riemann geometry*, Mathematical Surveys and Monographs, Vol. 140, American Mathematical Society, 2007.
- [2] S. Dragomir & G. Tomassini, *Differential Geometry and Analysis on CR Manifolds*, Progress in Mathematics, Vol. 246, Birkhäuser, Boston-Basel-Berlin, 2006.
- [3] G.B. Folland & J.J. Kohn, *The Neumann Problem for the Cauchy-Riemann Complex*, Annals of Mathematical Studies, Vol. 75, Princeton University Press, 1972.
- [4] S.G. Krantz, *Function theory of several complex variables*, John Wiley and Sons, Inc., New York, 1982.