KOSTANT-SOURIAU-ODZIJEWICZ QUANTIZATION OF A MECHANICAL SYSTEM WHOSE CLASSICAL PHASE SPACE IS A SIEGEL DOMAIN

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Abstract. We adopt the Kostant-Souriau-Odzijewicz quantization scheme for quantizing both the quantizable observables and the classical states of a mechanical system whose classical phase space is the Siegel domain $\Omega_n = \left\{\zeta \in \mathbb{C}^n \,:\, \mathrm{Im}(\zeta_1) > |\zeta'|^2\right\}$. We compute the transition probability amplitude $a_{0\overline{0}}(\zeta,\, z)$ from the state $z \in \Omega_n$ to the state $\zeta \in \Omega_n$. When the system interacts with weak external fields $\epsilon\, B$, $B \in L^\infty(\Omega)$, $0 < \epsilon << 1$, we show that the corresponding transition probability amplitudes are $a_{0\overline{0}}(\zeta,\, z) + \mathrm{O}(\epsilon)$. We discuss A. Odzijewicz's assumption that the measure on phase space [associated to the reproducing kernel of $L^2H(\Omega_n\,,\,\gamma)$] should coincide, up to a multiplicative constant, with the Liouville measure.

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