

QUANTUM DIRICHLET FORMS AND THEIR RECENT APPLICATIONS

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ABSTRACT. We will discuss the notion of classical Dirichlet forms, quadratic forms giving rise to Markov semigroups on the spaces of the form $L^2(X, \mu)$, and its quantum generalizations, defined in terms of von Neumann algebras. Some very recent applications of such quantum Dirichlet forms will be presented and further directions of research outlined.

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PLAN OF THE LECTURES

- Lecture 1 **C_0 -semigroups of operators and classical Dirichlet forms:** C_0 -semigroups of operators and their generators; quadratic forms; Choquet-Deny conditions; some examples.
- Lecture 2 **Quantum Dirichlet forms:** noncommutative L^p -spaces (tracial and non-tracial case); quantum Markov semigroups; noncommutative Choquet-Deny conditions.
- Lecture 3 **Recent applications and perspectives:** Haagerup property for von Neumann algebras; quantum convolution semigroups; open problems.

The lectures should be accessible to the audience having a general functional analytic background and some knowledge of operator algebras.

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