Conic sub-Hilbert-Finsler structure on a Banach manifold

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Abstract

A Hilbert-Finsler metric $\mathcal{F}$ on a Banach bundle $\pi : E \to M$ is a classical Finsler metric on E whose fundamental tensor is definite-positive. A conic Hilbert-Finsler metric $\mathcal{F}$ on E is a Hilbert-Finsler metric which is defined on an open conic submanifold of E. In the particular case where we have a (strong) Riemannian metric g on E then $\sqrt{g}$ is a natural example Hilbert-Finsler metric on E. According to [1], if moreover we have an anchor $\rho : E \to TM$ we get a sub-Riemannian structure on M that is g induces a "singular" Riemannian metric on the distribution $\mathcal{D} = \rho(E)$ on M. By analogy, a sub-Hilbert-Finsler structure on M is the data of conic Hilbert-Finsler metric $\mathcal{F}$ on a Banach bundle $\pi : E \to M$ and an anchor $\rho : E \to TM$. Of course we get a "singular" conic Hilbert-Finsler metric on $\mathcal{D} = \rho(E)$.
In the finite dimensional sub-Riemanniann framework, it is well known that "normal extremals" are projection of Hamitonian trajectories and any such an extremal is locally minimizing (relative to the associated distance). Analogous results in the context of sub-Riemannian Banach manifold were obtained in [1]. By an adaption of the same arguments we generalize these properties to the sub-Hilbert-Finsler framework.

References

[1] S. Arguillère, *Sub-Riemannian Geometry and Geodesics in Banach Manifolds.* arXiv:1601.00827v1

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