Universal Markov Kernels for Quantum Observables

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Positive operator valued measures play a crucial role in the modern formulation of quantum mechanics where they represent quantum observables as well as in quantum information theory since they are important for a range of quantum information processing protocols where classical post-processing plays a role. The spectral measures (also known as Projection Valued Measures or PVMs) are particular examples of POVMs and are able to describe only a very limited class of observables. Furthermore they are not able to give a clear mathematical representation of the joint measurability of two observables. For example, the non-commutativity of the position and momentum operators Q and P forbids a mathematical description of the joint measurement of position and momentum. Things go differently if position and momentum are represented by POVMs. Indeed, there are two commutative POVMs F^Q and F^P that are informationally equivalent to Q and P respectively and are the marginals of a joint POVM. Moreover F^Q and F^P are noisy versions of Q and P respectively. This is at the root of the formulation of quantum mechanics on phase space [1–3] where POVMs allow a rigorous quantization procedure. All that underlines the relevance of POVMs in general and of commutative POVMs in particular.

There are three characterizations of commutative POVMs [4–14] and one of them shows that every commutative POVM is the randomization of a spectral measure by means of a Markov kernel (or transition probability). That is exactly what happens in the phase space formulation of quantum mechanics when the symmetry group is the Heisenberg group, i.e., position and momentum are jointly described by a non commutative POVM whose marginals are randomization (through Markov kernels) of the position and momentum operators.

In the present work we analyze more in details the randomization procedure. In particular, we show that there is a universal Markov kernel μ. It is called universal because every commutative POVM F is the randomization of a particular spectral measure E^F by means of μ. Formally, we prove that there is a Markov kernel μ such that, for any real commutative POVM F,

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$$F(\Delta) = \int \mu_{\Delta}(\lambda)\, dE^F_{\lambda}$$

where E^F is a spectral measure which depends on F.

The talk will include a brief introduction to the relevance of POVMs in phase space quantum mechanics.

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