Non-periodic one-gap potentials of the Schrödinger operator

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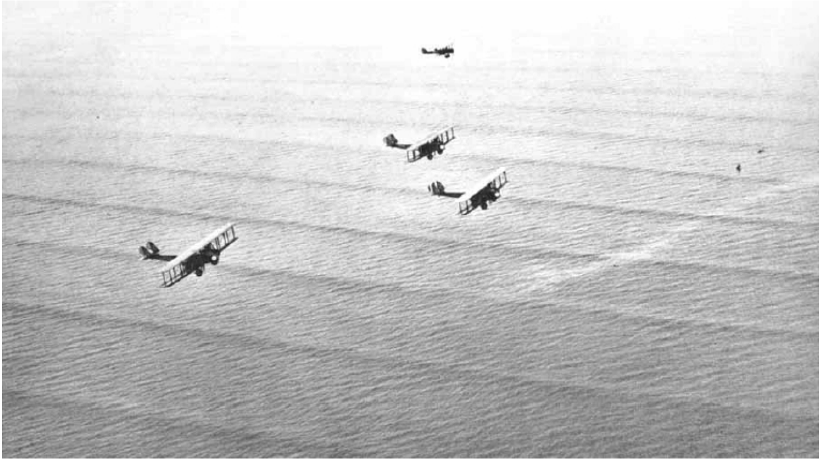
April 9, 2016

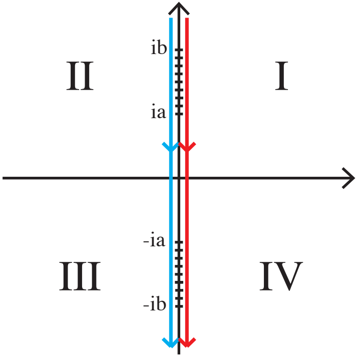
Ergodic Theory and Statistical Mechanics Seminar
Princeton University

Cnoidal wave

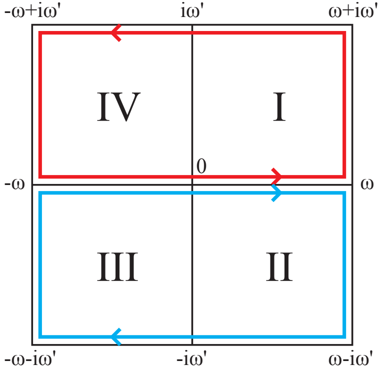


$$u(x,t) = 2\wp(x + i\omega' - ct) + \text{const}$$

One-gap potentials via the Riemann–Hilbert problem



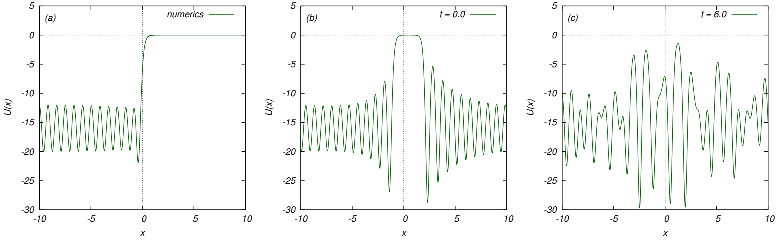
The k-plane



The z-plane

A potential $U(x)$

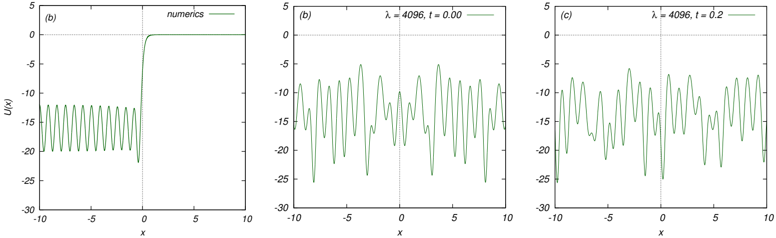
[ht]



A potential $U(x)$ that appears as a result of dressing with: (a) $R_1 = 1/\pi$ and $R_2 = 0$ decaying for $x > 0$; (b) $R_1 = \frac{1}{\pi} \times 10^{-3}$ and $R_2 = \frac{1}{\pi} \times 10^{-6}$; (c) $R_1 = \frac{1}{\pi} \times 10^{-3}$ and $R_2 = \frac{1}{\pi} \times 10^{-6}$ at moment of time $t = 6$ under KdV flow

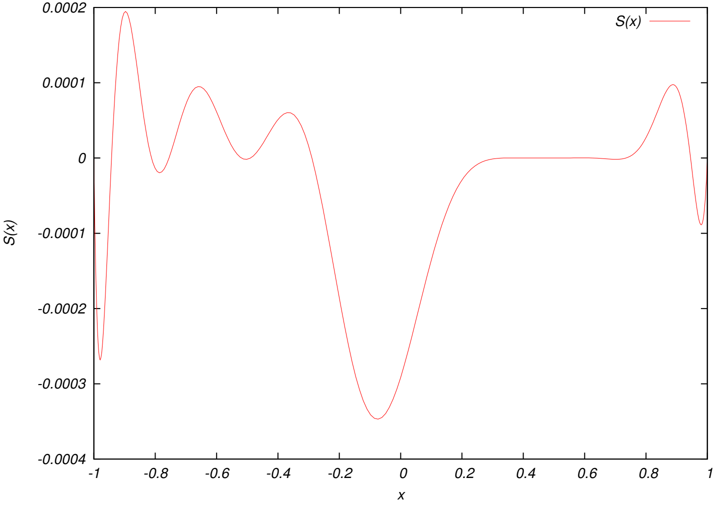
A potential $U(x)$

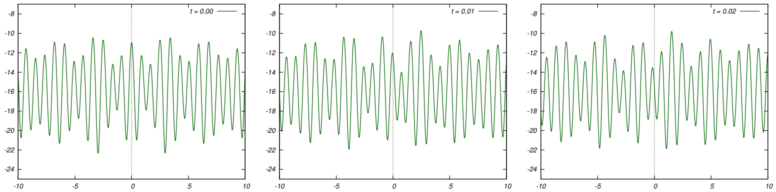
[ht]

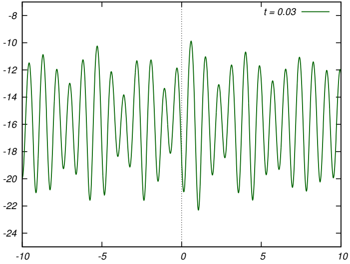


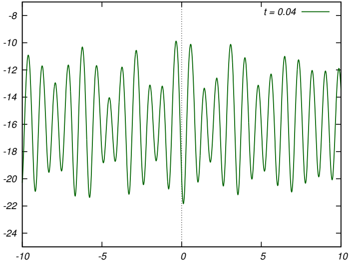
$R_1 = 1/\pi$ and $R_2 = 0(a)$; $\tilde{R}_1(p) = \frac{1}{\pi}e^{\lambda S(p)+S_0(pt)}$ for λ=4096; t=0.0 (b) and t=0.2 (c). Here $S(p) = \lambda \prod_{n=1}^{N_{max}}(p - r_n)$, where $r_1 = -1$, $r_{N_{max}} = 1$ and r_n is a sequence of randomly generated real numbers in the interval $-1 \leq r_n \leq 1$ for $n = 2...N_{max} - 1$. In this simulation $N_{max} = 10$. The constant λ is a variable parameter and $S_0(p) = 8[(p+3)^3 - 12(p+3)]$. The potential $u(x,t)$ satisfies the KdV equation in the moving frame.

Fresh results




t = 0.00
t = 0.01
t = 0.02
-8
-10
-12
-14
-16
-18
-20
-22
-24
-10
-5
0
5
10


t = 0.03
-8
-10
-12
-14
-16
-18
-20
-22
-24
-10
-5
0
5
10


t = 0.04
-8
-10
-12
-14
-16
-18
-20
-22
-24
-10
-5
0
5
10

The end

THANK YOU!