The puzzle of empty bottle in quantum theory. (Are quantum states real?)

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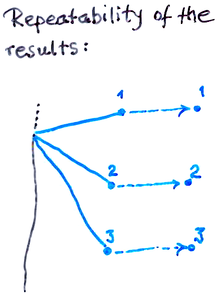
Abstract

A short review of doubts concerning the traditional interpretation of quantum states and their evolution is presented.

1. INTRO: From the very beginning in 1926 the customary interpretation of Quantum Mechanics shows some repetitive aspects which may frustrate the deeper efforts of understanding.

- the pure states are represented always by vectors in linear spaces in which they always navigate linearly except if the process is interrupted by a measurement.
- the results of measurements are statistical and can be certainly predicted only for some special states.



In the first instants, or in the repetition of the measurement, the initial state changes into the one which no longer changes under the repetition or continuation of the measurement!

Otherwise, the measurements practically could not be performed, because of the instability (dancing) of the apparatus needle during the measurement. Almost all physicists accepted the argument...

2. BUT NOT EVERYBODY... [28]

This antagonized E. Schrödinger:
"Diese Verdammte Quantenspringerei"! Indeed,
is there any fundamental difference between
the measuring devices and the rest of physical
bodies? Moreover, is there any essential difference
between the microscopic and macroscopic
bodies?

The problem is that the indeterminacy originally restricted to the atomic domain can be transformed into macroscopic indeterminacy.

In ample and fundamental paper of E.
Schrödinger Naturwiss 23, 807 (1935)
appear about 10 lines of hesitantly written
idea of the Shrödinger's CAT

a semi
-humorous story...

This can even set up quite ridiculous cases, A cat is penned up in a steel chamber, along with the following device (which must be secured against the direct intervence by the cat): in a Geiger counter there is a tiny bit of radioactive substance, so small that perhaps in the course of the hour one of the radioactive atoms will decay, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through relay releases the hammer which shatters a small flask of hydrocyanic acid. If one has left the entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.

Schrödinger, 1935

The whole fragment counts less than 1/300 part of an ample and interesting 22 page article. Yet, almost nobody remembers the profound work of E. Schrödinger [27], but almost everybody heard about his humorous, and provocative fragment upon the cat in a superposed state of being dead and alive. However, almost everybody thinks it is just an anecdote of the past. Yet, it might be not!

3. THE HAWKING'S GUN

The situation antagonized strongly Stephen Hawking:

"When I hear of Schrödinger's cat, I reach for my gun"

An auto-ironical story?

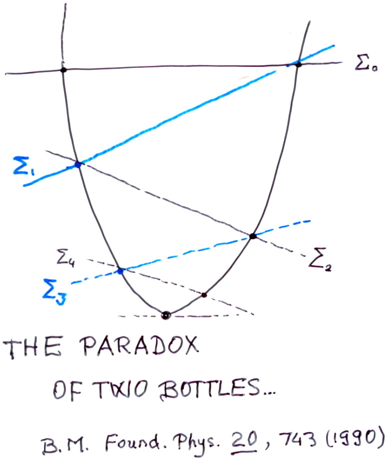
However, the Schrödinger's CAT will probably survive the Hawking's gun. The point is that until now we cannot indeed construct a truly consistent picture of quantum state reduction. A part of difficulty with 'Diese verdammte Quentenspringerei' occurs in the relativistic measurement theories.

4. THE QUANTUM BI-LOCALIZATION.

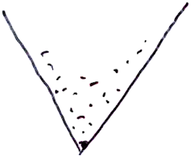
If one forgets about the cat, some historical, but not completely concluded discussions arised about the entangled states (EPR [9]) and teleportation [4], the idea which is still experimentally examined [15, 16, 12] with hopes to develop the quantum computing techniques in the near future.

Curiously, even without the entangled states, a serious difficulty appears if one tries to define a unique localization probability of a particle on the space-like hyperplanes Σ defined by the simultaneity conditions $t = const$ of various Lorentz frames.

This can be illustrated by considering two closed containers, 'bottles', with two coherent parts of the same particle, (1/2 and 1/2 probabilities), initially almost at the same localization. Then suppose the bottles travel to two distant spacetime areas. Now, if any inertial observer examines the bottle 1 in his time moment t_0 (on the hyperplane Σ_0) and detects the particle, then the particle state is reduced on his hyperplane Σ_0. and by the same, the probability of finding the particle in the second, distant bottle 2 becomes 0. However, this is not the end of the story. If now the second observer, moving with a different velocity checks immediately after the contents of the bottle 1, he must also find the particle, and so, he reduces the state, verifying the particle presence in bottle 1 and therefore, its absence in bottle 2 on a different spacelike hyperplane Σ_1. Henceforth, the third observer checking the presence of particle in bottle 2 immediately afterwards must also find the bottle 2 empty, meaning now that the particle was certainly in the bottle 1 on his simultaneity hyperplane Σ_2, i.e, still before the original measurement on Σ_0 was performed. By induction, it implies that the packet was reduced from the very beginning, the particle was always in the bottle 1, or else, that the whole 'reduction phenomenon' is a literary fiction.



In my paper [20], I was in favor of this last option, suggesting some error in the reduction doctrine. However, I ignored the earlier works of Aharonov and Albert [1, 2], who tried to save both proposals of linearity and of reduction. In some panic I have published the "Corrigendum" [21]. Yet, the idea was soon discussed by J. Finkelstein [11] and S.N. Mosley [24], proposing two different visions.



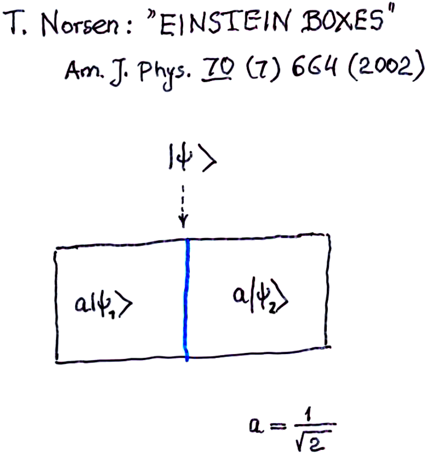
J. Finkelstein, "reduction only inside of the future light cone". Found. Phys. Lett. 5, 383 (1992)

S.N. Mosley: the quantum state is an information which the local observer accumulates from its past cone. Its correction ("reduction") therefore affects the past cone

Phys. Lett. A. 182, 1 (1993)

Let us notice, however, that if the quantum states were described only on either future [11] or past cones [24], then the evolution equations of all quantum theories dealing typically with the states (and state reductions) on the t-dependent spacelike hyperplanes, would lose their sense.

In autumn 2015 I happened to discuss this problem with David Albert (coauthor of Aharonov). Oh! said Albert, then you are the follower of 'Einstein boxes'. In fact, if not the remark of Albert, I would still ignore the subject.



By developing the doubts already expressed in the EPR paper [9], Einstein asked whether it was still possible that two distant boxes contained the coherent parts of one wave packet? The answer of de Broglie was cathegoric [25]:

> *Suppose, a particle is enclosed in a box B with impermeable walls. The associated wave* $|\psi>$ *is confined in the box and cannot leave it. (...) Let us suppose that by some process or other, for example, by inserting a partition into the box, the box B is divided into two separate parts* B_1 *and* B_2 *and that* B_1 *and* B_2 *are then transported to two very distant places, for example to Paris and Tokyo. (...) The particle which has not yet appeared thus remains potentially in the assembly of the two boxes and its wave function* $|\psi>$ *consists of two parts, one of which* ψ_1 *is located in* B_1 *and the other* ψ_2 *in* B_2*. (...) According to the usual interpretation (...) the particle (...) would be immediately localized in box* B_1 *in the case of a positive result in Paris. This does not seem to me to be acceptable. If we show that the particle is in box* B_1 *it implies that it was already there prior to localization.*

The opinion of de Broglie basically referred to the existence of the trapped states, without even entering into the relativistic aspects. The situation is even more dramatic in the "delayed choice measurements" of J.A. Wheeler [32, 26], considered to be almost a dark anecdote, to be resolved later, while the scientific community is still cultivating obligatory trends. No easier is the situation in the later designed "interaction free measurement" [10, 30, 31].

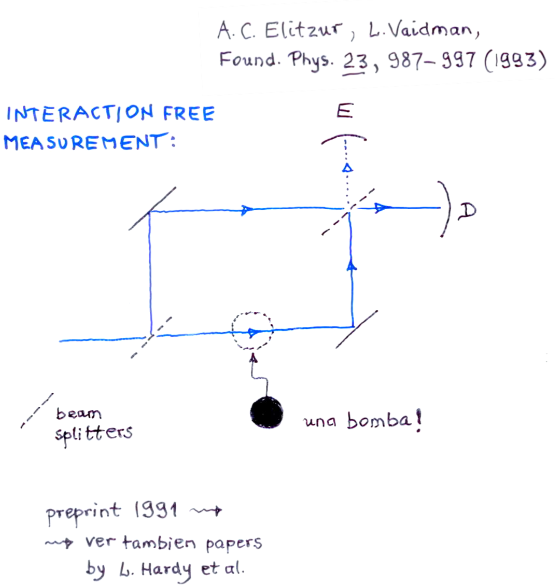


Figure 1: Interaction free measurement

In their idealized experiment Elitzur and Vaidman imagine a photon in a system of optical fibers and mirrors of the Mach-Zehnder interferometer (see Figure 1). The photon wave function is divided by the first beam splitter into two parts, reflected then by two mirrors. If there is no obstacle they meet again at the second splitter, recovering their original motion. So, the photon ends up in the detector $\mathbf{D}_1$. However, if in one of the branches (e.g. the right vertical one) there is a perfectly absorbing obstacle, then it performs the first state reduction. Either it detects (by absorbing) the photon, which therefore can arrive neither to $\mathbf{D}_1$ nor to $\mathbf{D}_2$. Or the state will be reduced to the upper trajectory. The second splitter will then divide it into the superposition of two parts, arriving either to $\mathbf{D}_1$ or $\mathbf{D}_2$. The choice of one of them will be the next packet reduction.

The challenge is that the first reduction performed by the absorbing obstacle in 1/2 of cases eliminates completely just one of the alternative trajectories, so the effect can be checked by $\mathbf{D}_2$, though the photon never approached the obstacle. Elitzur and Vaidman choose a striking formulation of this fact, assuming that the obstacle is a supersensitive bomb, which would explode immediately if only it had any contact with the photon. So, if $\mathbf{D}_2$ responds, the bomb is discovered (but not exploded) in an "interaction free measurement", by a photon which could pass hundred kilometers away [30, 31].
To make the challenge even more extreme we permit ourselves to imagine the same EV design with the pair of horizontal fibers very long (see Fig.1). (I beg the reader to forgive me this element of S/F story: we all know that there are no interstellar fibers!). Assume, however, they are just quite long! The so obtained rectangular design leads to the second splitter opening the way to $\mathbf{D}_1$ or $\mathbf{D}_2$. If there is no obstacle in one of fibers, the photon state still conserves two coherent parts, which will again join at the second splitter, activating only $\mathbf{D}_1$.

All this is easy to imagine if the photon runs as a very short pulse. However, can the photon propagate only as a pulse? Or perhaps, it can also form a long, narrow wave, divided by the first splitter into two still weaker but very long components, which laboriously recover their initial forms at the second splitter, falling then gradually into the detector $\mathbf{D}_1$? If so, when exactly the detector responds to the *single photon*? At the beginning or at the end of the process?

Worse, since if one of the EV fibers is blocked by the bomb, then after what time the bomb explodes? If it doesn't, then after what time the long (but incomplete) photon component *which would cross the bomb* is mysteriously annihilated contributing (again mysteriously) to the other component to create the complete 1 photon state, which (crawling laboriously) might finally arrive to the alternative detector $\mathbf{D}_2$? We can only conclude that our story is incomplete. Hence, let me remind an important point made by Sudbery [29]:

"It is often stated that however puzzling some of its features may be, quantum mechanics does constitute a well defined algorithm for calculating physical quantities. Unless some form of continuous projection postulate is included as a part of the algorithm, this is <u>not</u> <u>true</u>."

A. Sudbery, in *Quantum Concepts in Space and Time*, Ed. R. Penrose, C.J. Isham, OUP (1986)

The question still is, what is indeed the photon state in the fiber? Can it be an extended creature, crawling laboriously over thousands kilometers? It seems that the answer is positive. The description of photon waves in optical fibers was already found by I. Białynicki-Birula [5], given not by plane waves but by Bessel functions, eq. (55) (a significant progress comparing with the peculiar vision of Quarks as the plane waves running inside of the nucleon surfaces - without any credible attempts to explain the quark confinement!)
In the distinct problems of the freely propagating entangled states of two or more particles, described e.g. in the EPR, and the following works [9], the idea of a *benign* teleportation is assumed, which accepts that the measurement performed on one particle affect the states of all others, but the 'message' is unreadable without the an additional information which cannot propagate faster than light. Anyhow, if the state reduction occurs on the past cone of the local measurement (Mosley [24]), the uniqueness of the particle states and probabilities on the space like planes is in danger. In his provocative essay Seth Lloyd finds that his operation on Wednesday could affect the Tuesday state [19]. The imperfect versions of the the Elitzur-Vaidman bomb are still studied in [19, 7]. To avoid too complex problems, I will report below some equally uneasy questions without the entangled states and without non-locality troubles.

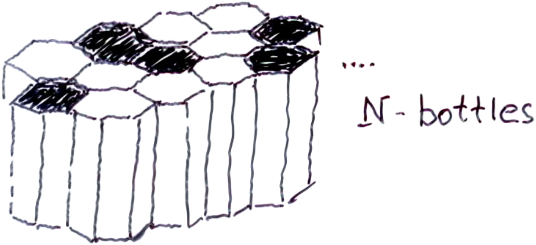


Figure 2: N-bottles

4. HALF FULL HALF EMPTY.

Our story concerns a quantum system in a superposed energy state - which will be reduced - though not when the experimentalist decides, but when the system itself decides by emitting a photon (compare with the 'time of arrival' [17]). As a simplified model, we consider a bottle containing an atom in a state *superposed* of two lowest energy eigenstates, ground state ϕ_0 and an excited state ϕ_1. We assume the bottle is great enough to neglect the influence of its surface onto the atom behaviour. If the atom is in the excited state, we say that "the bottle is full", but if in the ground state, "the bottle is empty". The bottle here is to assure that the atom is not externally perturbed.

In the times of Bohr-Sommerfeld model, the physicist always imagined an atom in one of definite energy states, but since almost a century the picture changed. The existence of the superposed (but pure) energy states is unavoidable if one takes seriously the quantum mechanical formalism. Basically, the only important thing our atom can do is to radiate, settling itself in the ground state (perhaps, with the center slightly affected by the macroscopic velocity due to the momentum conservation in the atom - photon system). We shall also assume, that the bottle is ended by some sensitive screen, prepared to detect the photon, should the atom radiate (See Fig. 2).

By reading works on the radiative decays you can always see the description of the process starting from the excited state alone, but never from a superposition (e.g. fifty-fifty of the excited and ground states).

True, the excited state itself is typically described as some narrow superposition of different energy states with the average lifetime τ inverse to the energy width δE, in agreement with the time-energy uncertainty, (even though the last principle awakes a lot of unfinished discussions [17, 18, 22]). However, I have never seen a study of a decay starting from a superposition of two very distant energy states. Perhaps, the difference is superfluous, but anyhow, why nobody considered the *coherent superposition* of two distant levels as a starting point of the decay?

To fix the attention, let us therefore assume that the initial state ϕ is an equitative superposition $\phi = a_0\phi_0 + a_1\phi_1$, where $|a_0|^2 = |a_1|^2 = 1/2$ (bottle half full half empty). From the credible phenomenology we know the behaviour of the ground state ϕ_0: if unperturbed, it can only show the phase dependence: $\phi_0(t) = e^{-itE_0}\phi_0$. We know also something about $\phi_1(t)$. On the level of pure quantum mechanical description it would be as stationary:

$$\phi_1(t) = e^{-itE_1}$$

At the first sight, it may seem that there is hardly any problem. The evolution of any atom must must simply obey the linear law:

$$\phi(t) = a_0 e^{-iE_0 t}\phi_0 + a_1 e^{-iE_1 t}\phi_1$$

granted by the linearity principle, except if it suddenly radiates, emitting a photon of energy $\Delta E = E_1 - E_0$ and falling into the ground state ϕ_0. However, this plausible picture contains certain disquieting gaps.

We are accustomed to think, that the quantum system in presence of measuring device performs always a unitary evolution process, in a certain Hilbert space (an extreme linear picture?), until suddenly, BAM!! it is interrupted by the wave packet reduction (an extreme non-linear event?). Since the result is anyhow indeterministic, in most cases (or at least for the free propagation ended up by a particle localization), it is legitimate to reconstruct the future evolution knowing the result of the measurement but unproductive to retrospect, trying to reconstruct the past, (if one accepts *bona fide* the orthodox QM doctrine). Hence, the warning about the "post-selection!".

This last rule deserves some more comments. If the freely propagating particle (e.g. represented by the de Broglie wave) marks a point on a sensitive screen, it can be misleading to retrospect, finding the exact solution of the propagation law which at the given moment centered precisely at the detection point. (As well known, the quantum evolution equations together with the future of any wave function permit to read its past). Supposing that the screen is a plane with a continuum of points, the initial plane wave can be decomposed into a continuous superposition of the "conic" solutions, a kind of "eigenfunctions" focusing on the different screen points, their analytical reconstruction in the past neither more nor less difficult than the development in the future. Yet, the particle detection at one of these points does not prove that its past was a conic wave converging to the detection point: this extrapolation would be wrong, because the natural evolution of the wavepacket was reduced by the sudden detection act. Hence, the warning against the *postselection* (or retrospection).

For an initial ensemble of purely excited states, the number $N(t)$ of the ones surviving in ϕ_1 will decrease in (approximately) exponential way. However, what happens for atoms in the superposed energy state, $\phi = a_0\phi_0 + a_1\phi_1$ (the bottle *neither full nor empty*?) Indeed, the idea about the linear navigation interrupted by a state reduction in this case evades some details important for the orthodox doctrine. The average energy of the superposed initial state is only $\frac{1}{2}(E_1 - E_0)$ and... nobody observed photons emitted with partial energies smaller than $E_1 - E_0$). Should the atom perform first of all a spontaneous (*introspective*?) state reduction? Or, must it ask for some energy credit from the detector? Or perhaps, one should assume some influence of the detector due to its very existence, even if the measurement is still not performed? [13, 12, 6]

It is curious to imagine a population of N atoms in the same initial state $\psi = a_0\Psi_0 + a_1\psi_1$, with $|a_0|^2 = |a_1|^2 = 1/2$. Each one in its own, mesoscopic cell Fig.4 (I ask your tolerance for painting them hexagonal like in bee hive. Bees are now also in problems). By calculating the number of cells which turned dark, we can see how many photons incubated. If somebody performed the measurement upon all bottles at $t = 0$, reducing the states, he would find 1/2 of them exactly in the excited state ϕ_1, and 1/2 in the ground state ϕ_0, hence unable to radiate. However, if no initial measurement was performed, then for $t \to +\infty$ anyhow all atoms must end up in the ground state ϕ_0, though for different reasons: 1/2 of them since they radiated and settled down in ϕ_0, but the remaining 1/2 since they didn't radiate and never will.

Obviously the idea that the state of a microobject evolves linearly in agreement with a certain unitary transformations until the measurement is performed is in trouble. One of customary answers is *ensemble and only ensemble* (the single microobjects have no states!). For the ensemble of all bottles, there is no total energy problem. Moreover, since the photon detections in different bottles (the state reductions) might happen in different moments, the whole ensemble evolves into a state mixture... That would be nice, but in 1/2 of the bottles the photon was never emitted (so, the measurement was not performed?)

THE VANISHING HOPE?

In fact, even if the global energy balance is not affected, the situation looks strange. While the atoms which radiated cause some problem [3], the ones which didn't contain a puzzle. Their superposed energy state vanished, giving place to ϕ_0, but no measurement was performed. Indeed, the only detection process was our *vanishing hope* (take it as the rhetorical figure if you dislike!). Supposing that it was a "measurement" with the power to "reduce the wave packet", then in the first place, it was quite extended in time (remember Sudbery [29]?). Worse, the bottle was *half full, half empty*, nothing escaped, and the bottle is *empty*. The warning against "post-selection" would make sense if the "vanishing hope" was an orthodox quantum measurement. But if not, then to forbid it is just one of "don't think principles" which you can find in various areas of quantum theory. To break it by retrospecting, is then an interesting experiment. It will tell that, at least in this case, we have no credible description of what happened with any *single atom state* in any *single*, isolated bottle. How did it evolve from the *fifty-fifty* superposed to the final ground state? Was it some nonlinear process [23, 8] or certain type of *shadowing* [13, 12, 6])?

Note that the difficulty will vanish if one simply assumes that in some cases no coherent superposition of two *different bound states* can be created. Our supposed *superposition* of two eigenstates of different energies could be from the very beginning just a mixture (comparable to Einstein boxes?) The energy values could be deduced from some generalized case of Bohr-Sommerfeld theory. The well known victories of quantum theory could be limited to measurements on freely navigating particles. Thus, the de Broglie waves would be only certain mental pictures guessed without well understanding why they tell the truth - even if we believe that the linear propagation in Hilbert spaces leads to the ”theory of everything“.

If so, it might mean that our consciousness observes only some images similar to shadows in Platonic caves, but it may be premature to construct out of them a universal theory. Just forget it, if you dislike!

OUR TREE OF KNOWLEDGE:

IS IT CONSISTENT?

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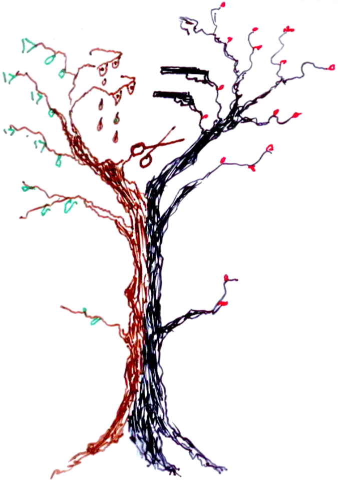

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