Integrability of distributions on infinite dimensional manifolds and applications

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In differential geometry, a *distribution* on a smooth manifold M is an assignment

$$\mathcal{D} : x \mapsto \mathcal{D}_x \subset T_xM$$

on M, where $\mathcal{D}_x$ is a subspace of T_xM. The distribution is *integrable* if, for any $x \in M$ there exists an immersed submanifold $f : L \to M$ such that x belongs to $f(L)$ and for any $z \in L$ we have $Tf(T_zL) = \mathcal{D}_{f(z)}$. On the other hand, $\mathcal{D}$ is called *involutive* if, for any vector fields X and Y on M which are tangent to $\mathcal{D}$, the Lie bracket $[X, Y]$ is also tangent to $\mathcal{D}$. The distribution is *invariant* if for any vector field X tangent to $\mathcal{D}$, the flow ϕ_t^X leaves $\mathcal{D}$ invariant. On finite dimensional manifold, when $\mathcal{D}$ is a sub-bundle of TM, the classical Frobenius Theorem gives an equivalence between integrability and involutivity. In the other case, the distribution is "singular" and even under assumptions of smoothness on $\mathcal{D}$, in general, the involutivity is not a sufficient condition for integrability (we need some more additional local conditions). These problems were clarified and resolved essentially in [Su], [St].

We will explain how these results in finite dimension can be extended to the context of Banach manifolds and also in the frame work of direct limit of Banach manifolds. We will end by application of these results in the framework of Banach Poisson manifolds and of direct limits of Banach Lie algebroids.

References

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