

III School on Geometry and Physics

7 July – 12 July 2014

LIST OF COURSES

1. **Pierre BIELIAVSKY** – *Université Catholique de Louvain, Belgium*
Non-formal deformation quantization and locally compact quantum groups
2. **Kirill MACKENZIE** – *University of Sheffield, United Kingdom*
Duality for multiple structures
3. **Bogdan MIELNIK** – *CINVESTAV, Mexico*
Quantum control: are we omnipotent or omniimpotent?
4. **Yurii NERETIN** – *Institute for Theoretical and Experimental Physics, Russia*
Infinite-dimensional groups and stochastic processes
5. **Andreas RUFFING** – *Technische Universität München, Germany*
Title to be announced
6. **Theodore VORONOV** – *University of Manchester, United Kingdom*
Q-manifolds and geometric structures
7. **Wojciech WOJTYŃSKI** – *Uniwersytet w Białymstoku, Poland*
Towards Lie theory of diffeomorphism groups – an introduction to string Lie theory

III SCHOOL ON GEOMETRY AND PHYSICS

Białowieża, POLAND, 7 July – 12 July 2014

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DUALITY FOR DOUBLE STRUCTURES

Kirill Mackenzie (Sheffield)

Double vector bundles are implicit in the connection theory of vector bundles. A connection in a vector bundle $E \rightarrow M$ gives a lifting of vector fields on M to vector fields on E ; the latter are *linear* in the sense that they are morphisms of vector bundles from E to TE with the vector bundle structure on base TM obtained by applying the tangent functor to all the vector bundle operations; this is the *tangent prolongation of E* . Connections can also be formulated as linear maps $E \times_M TM \rightarrow TE$ which combines right-inverses to both natural maps $TE \rightarrow E$ and $TE \rightarrow TM$ shown in Figure 1(a) below.

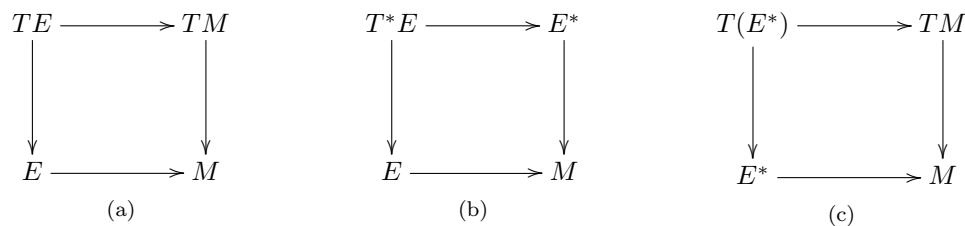
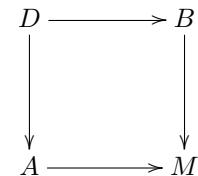


Figure 1.

Figure 1(a) shows the two vector bundle structures on TE ; the standard structure with base E and the tangent prolongation with base TM . Each of these can be dualized in the usual way and they lead to the double vector bundles in (b) and (c) respectively. The double vector bundle in (b) arises in Poisson geometry: there is a canonical diffeomorphism $T^*(E^*) \rightarrow T^*(E)$ and if E (say) has a Lie algebroid structure, then E^* has a Lie-Poisson structure and $T^*(E^*) \rightarrow E^*$ is a Lie algebroid.

In a general double vector bundle D , as on the right, the manifold D has two vector bundle structures, one with base A and one with base B (subject to compatibility conditions). Each structure has its dualization operation; let us call them X and Y . It turns out that $XYX = YXY$, up to canonical isomorphism.



Taking the dual of a (finite rank) vector bundle is reflexive: the dual of the dual is canonically isomorphic to the original vector bundle, and one may say that duality for vector bundles ‘has group C_2 ’. In particular, in a double vector bundle $X^2 = I$ and $Y^2 = I$, and together with $XYX = YXY$, this shows that the duality of double vector bundles ‘has group S_3 .’

The lectures will describe these processes and will sketch the triple and 4-fold cases, where new phenomena arise.

References

- [1] K. C. H. Mackenzie. Duality and triple structures. In *The breadth of symplectic and Poisson geometry*, volume 232 of *Progr. Math.*, pages 455–481. Birkhäuser Boston, Boston, MA, 2005.
- [2] A. Gracia-Saz and K. Mackenzie. Duality functors for triple vector bundles. *Lett. Math. Phys.*, 90(1-3):175–200, 2009.
- [3] A. Gracia-Saz and K. C. H. Mackenzie. Duality functors for n -fold vector bundles. [arXiv:1209.0027](https://arxiv.org/abs/1209.0027), .

Monday, July 7

LECTURES 10:00–13:10

- 10:00–10:50 *Duality for multiple structures*
Kirill MACKENZIE, University of Sheffield, United Kingdom
- 11:00–11:50 *Towards Lie theory of diffeomorphism groups — an introduction to string Lie theory*
Wojciech WOJTYŃSKI, Instytut Matematyki, Uniwersytet w Białymstoku, Poland
- 11:50–12:20 Coffee break
- 12:20–13:10 *Non-formal deformation quantization and locally compact quantum groups*
Pierre BIELIAVSKY, Université Catholique de Louvain, Belgium

Tuesday, July 8

LECTURES 10:00–13:10

- 10:00–10:50 *Towards Lie theory of diffeomorphism groups — an introduction to string Lie theory*
Wojciech WOJTYŃSKI, Instytut Matematyki, Uniwersytet w Białymstoku, Poland
- 11:00–11:50 *Non-formal deformation quantization and locally compact quantum groups*
Pierre BIELIAVSKY, Université Catholique de Louvain, Belgium
- 11:50–12:20 Coffee break
- 12:20–13:10 *Quantum harmonic oscillators in the continuum and on lattices*
Andreas RUFFING, Technische Universität München, Germany

AFTERNOON LECTURE 17:00–18:15

- 17:00–18:15 *Quantum control: are we omnipotent or omniimpotent?*
Bogdan MIELNIK, CINVESTAV, Mexico

Wednesday, July 9

LECTURES 10:00–13:10

- 10:00–10:50 *Duality for multiple structures*
Kirill MACKENZIE, University of Sheffield, United Kingdom
- 11:00–11:50 *Towards Lie theory of diffeomorphism groups — an introduction to string Lie theory*
Wojciech WOJTYŃSKI, Instytut Matematyki, Uniwersytet w Białymstoku, Poland
- 11:50–12:20 Coffee break
- 12:20–13:10 *Towards Lie theory of diffeomorphism groups — an introduction to string Lie theory*
Wojciech WOJTYŃSKI, Instytut Matematyki, Uniwersytet w Białymstoku, Poland

Thursday, July 10

LECTURES 10:00–13:10

- 10:00–10:50** *Q-manifolds and geometric structures*
Theodore VORONOV, University of Manchester, United Kingdom
- 11:00–11:50** *Infinite-dimensional groups and stochastic processes*
Yurii NERETIN, Institute for Theoretical and Experimental Physics, Russia
- 11:50–12:20** Coffee break
- 12:20–13:10** *Non-formal deformation quantization and locally compact quantum groups*
Pierre BIELIAVSKY, Université Catholique de Louvain, Belgium

Friday, July 11

LECTURES 10:00–13:10

- 10:00–10:50** *Q-manifolds and geometric structures*
Theodore VORONOV, University of Manchester, United Kingdom
- 11:00–11:50** *Infinite-dimensional groups and stochastic processes*
Yurii NERETIN, Institute for Theoretical and Experimental Physics, Russia
- 11:50–12:20** Coffee break
- 12:20–13:10** *Duality for multiple structures*
Kirill MACKENZIE, University of Sheffield, United Kingdom

Saturday, July 12

LECTURES 10:00–13:10

- 10:00–10:50** *Infinite-dimensional groups and stochastic processes*
Yurii NERETIN, Institute for Theoretical and Experimental Physics, Russia
- 10:50–11:20** Coffee break
- 11:20–12:10** *Q-manifolds and geometric structures*
Theodore VORONOV, University of Manchester, United Kingdom