Holography, Unfolding and Higher-Spin Theories

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Relativistic Field Theory

Particles: h**-invariant linear equations of motion**

$$L\varphi^{\alpha}(x) = 0\,.$$

$M^d = ISO(d)/SO(d)$ **Minkowski**

$M^d = SO(d-1,2)/SO(d)$ $\mathrm{AdS_d}$

$M^d = SO(d,2)/P$ **compactified Minkowski**

Space of states of an elementary particle: **irreducible** h**-module**

- **Unitarity:**
- **Lowest weight: Energy boundness**

Noncompact h: **infinite-dimensional** h**-modules**

$h = so(d,2)$: a particle $\sim h$-module $D(E,\mathbf{s})$:

$\mathbf{s} = s_1 \dots s_{[d/2]}$: spin(s) = weights of $o(d)$

E : lowest energy (mass) = weight of $o(2)$

Depending on E and $\mathbf{s}$:

- either a conformal field in M^d
- or a field in AdS_{d+1}

Unitarity: $E \geq E_0(\mathbf{s})$

At $E = E_0(\mathbf{s})$ null states appear that form a h-submodule $D(E', \mathbf{s}')$:

Factorization implies gauge symmetry

The gauge fields are called massless fields $(E = E_0(\mathbf{s}))$

Gauge symmetries

Gauge Symmetries: parameters $\varepsilon^{\Omega}(x)$

Examples:

• $s = 1$, $m = 0$: $A(x) = dx^n A_n(x)$

Maxwell field strength $F(x) = dA(x)$, $\quad d = dx^n \frac{\partial}{\partial x^n}$

Field equations: $d\,{}^*F(x) = J(x)$

Gauge transformation: $\delta A(x) = d\varepsilon(x)$

• $s = 2$, $m = 0$: $g_{nm} = \eta_{nm} + \kappa h_{nm}$

h_{nm} **is a metric fluctuation** $(\kappa \to 0)$.

Linearized diffeomorphism $\delta h_{nm}(x) = \partial_{(n}\varepsilon_{m)}(x)$

Higher-spin fields

Any spin s, $m=0$

$\varphi_{n_1\ldots n_s}(x)$ - **rank** s **double traceless:** $\varphi_n{}^n{}_m{}^m{}_{k_5\ldots k_s}(x)=0$ C.Fronsdal (1978)

Gauge transformation:

$$\delta\varphi_{k_1\ldots k_s}(x)=\partial_{(k_1}\varepsilon_{k_2\ldots k_s)}(x)\,,\qquad \varepsilon^n{}_{nk_3\ldots k_{s-1}}=0$$

$\varepsilon_{k_1\ldots k_{s-1}}$- **symmetric traceless:** $\delta\varphi_n{}^n{}_m{}^m{}_{k_5\ldots k_s}=0$

Fronsdal action

$$S=\int_{M^d}\left(\frac{1}{2}\varphi^{m_1\ldots m_s}G_{m_1\ldots m_s}(\varphi)-\frac{1}{8}s(s-1)\varphi_n{}^{n\,m_3\ldots m_s}G^p{}_{p\,m_3\ldots m_s}(\varphi)\right)$$

$$G_{k_1\ldots k_s}(\varphi)=\Box\varphi_{k_1\ldots k_s}(x)-s\partial_{(k_1}\partial^n\varphi_{k_2\ldots k_s n)}(x)+\frac{s(s-1)}{2}\partial_{(k_1}\partial_{k_2}\varphi^n{}_{k_3\ldots k_s n)}(x)$$

Field equations: $G(\varphi)=0$

Nonlinear Gauge Theories

Interactions are described by nonlinear PDE

That the interacting theory should have as many (may be deformed) gauge symmetries as its free limit imposes severe restrictions on the sets of fields involved and on the form of nonlinearities

$s = 1$ Yang-Mills: connection 1-forms take values in compact Lie algebras

- $s = 2$ Einstein gravity: single metric tensor
- $s = 3/2$ Supergravity: supermultiplets of fields of spins $0 \le s \le 2$ with single spin 2 graviton

HS gauge theories contain spin $s > 2$ gauge fields

HS theory

Higher derivatives in interactions

A.Bengtsson, I.Bengtsson, Brink (1983), Berends, Burgers, van Dam (1984)

$$S = S^2 + S^3 + \dots, \qquad S^3 = \sum_{p,q,r} (D^p\varphi)(D^q\varphi)(D^r\varphi)\rho^{p+q+r+\frac{1}{2}d-3}$$

HS Gauge Theories $(m=0)$:

Fradkin, M.V. (1987)

$$AdS_4 : \qquad [D_n, D_m] \sim \rho^{-2} = \lambda^2$$

AdS/CFT:

$(3d, m=0) \otimes (3d, m=0) = \sum_{s=0}^{\infty}(4d, m=0)$ Flato, Fronsdal (1978);

Sundborg (2001), Sezgin, Sundell (2002,2003), Klebanov, Polyakov (2002),

Giombi, Yin (2009)...

Maldacena-Zhiboedov Thm. (2011,2012)

Plan

I Unfolded dynamics and holographic duality

II Relation with Fedosov quantization

III AdS_4/CFT_3 duality

IV Conclusion

Results

CFT_3 dual of AdS_4 HS theory: 3d conformal HS theory

Holography: Unfolding

Unfolded dynamics

First-order form of differential equations

$$\dot{q}^i(t) = \varphi^i(q(t)) \qquad \text{initial values:} \quad q^i(t_0)$$

Unfolded dynamics: multidimensional covariant generalization

$$\frac{\partial}{\partial t} \to d\,, \qquad q^i(t) \to W^\Omega(x) = dx^{n_1} \wedge \ldots \wedge dx^{n_p}$$

$$\mathbf{dW^\Omega(x) = G^\Omega(W(x))}\,, \qquad \mathbf{d = dx^n \partial_n} \qquad \text{MV (1988)}$$

$G^\Omega(W)$: **function of "supercoordinates"** W^Φ

$$G^\Omega(W) = \sum_{n=1}^{\infty} f^\Omega{}_{\Phi_1 \ldots \Phi_n} W^{\Phi_1} \wedge \ldots \wedge W^{\Phi_n}$$

$d > 1$: **Nontrivial compatibility conditions**

$$G^\Phi(W) \wedge \frac{\partial G^\Omega(W)}{\partial W^\Phi} \equiv 0$$

Any solution: FDA Sullivan (1968); D'Auria and Fre (1982)

The unfolded equation is invariant under the gauge transformation

$$\delta W^\Omega(x) = d\varepsilon^\Omega(x) + \varepsilon^\Phi(x) \frac{\partial G^\Omega(W(x))}{\partial W^\Phi(x)}\,,$$

Vacuum geometry

a Lie algebra. $\omega = \omega^{\alpha} T_{\alpha}$: h **valued 1-form.**

$$G(\omega) = -\omega \wedge \omega \equiv -\frac{1}{2}\omega^{\alpha} \wedge \omega^{\beta}[T_{\alpha}, T_{\beta}]$$

the unfolded equation with $W = \omega$ **has the zero-curvature form**

$$d\omega + \omega \wedge \omega = 0\,.$$

Compatibility condition: Jacobi identity for h.

FDA: usual gauge transformation of the connection ω.

Zero-curvature equations: background geometry in a coordinate independent way.

If h **is Poincare or anti-de Sitter algebra it describes Minkowski or** AdS_d **space-time**

$$\omega = e^{n} P_{n} + \omega^{nm} M_{nm}\;: \qquad R^{n} = 0\,, \quad R^{nm} = 0\,.$$

Free fields unfolded

Let W^{Ω} contain p-forms $\mathcal{C}^i$ (e.g. 0-forms) and G^i be linear in ω and $\mathcal{C}$

$$G^i = -\omega^{\alpha}(T_{\alpha})^i{}_j \wedge \mathcal{C}^j \, .$$

The compatibility condition implies that $(T_{\alpha})^i{}_j$ form some representation T of h, acting in a carrier space V of $\mathcal{C}^i$. The unfolded equation is

$$D_{\omega}\mathcal{C} = 0$$

$D_{\omega} \equiv d + \omega$: covariant derivative in the h-module V.

Covariant constancy equation : linear equations in a chosen background

h: global symmetry

Properties

- General applicability
- Manifest (HS) gauge invariance
- Invariance under diffeomorphisms

  Exterior algebra formalism
- Interactions: nonlinear deformation of $G^{\Omega}(W)$
- Local degrees of freedom are in 0-forms $C^i(x_0)$ at any $x = x_0$ (as $q(t_0)$)

  infinite-dimensional module dual to the space of single-particle states
- Lie algebra cohomology interpretation
- Covariant twistor transform
- Emergent ambient space-time

  Geometry is encoded by $G^{\Omega}(W)$

  Hierarchies: commuting flows

  Holography

Fedosov quantization

$$W(y|x) = dx^n W_n(y|x) = \sum_{n=0}^{\infty} W_{a_1 \dots a_n}(x) y^{a_1} \dots y^{a_n}$$

$$\Phi(y|x) = \sum_{n=0}^{\infty} \Phi_{a_1 \dots a_n}(x) y^{a_1} \dots y^{a_n}$$

Sections of Weyl bundle

Star product

$$(f * g)(y) = \int ds dt f(y+s) g(y+t) \exp -i s_a t^a$$

$$[y_a, y_a]_* = 2iC_{ab}\,,$$

Non-Abelian HS curvature

$$R(y|x) = dW(y|x) + W(y|x) * \wedge W(y|x)$$

Adjoint covariant derivative

$$D\Phi(y|x) = d\Phi(y|x) + W(y|x) * \Phi(y|x) - \Phi(y|x) * W(y|x)$$

Fedosov theory

$$R(y|x) := dW(y|x) + W(y|x) * W(y|x) = 0$$

$$D\Phi(y|x) := d\Phi(y|x) + W(y|x) * \Phi(y|x) - \Phi(y|x) * W(y|x) = 0\,.$$

$$W(y|x) = W_0(y|x) + W_1(y|x)\,, \qquad W_0(y|x) = w_{ab}(x)y^a y^b + e_a y^a$$

$W_0(y|x)$: **connection of** $sp(d) \oplus\!\!\!\!\!\!\supset h(d)$.

e_a **satisfies**

$$de_a(x) + w_{ab}(x) \wedge e_c(x) C^{bc} = 0\,, \qquad C^{bc} = -C^{cb}$$

where C^{ba} **is a constant symplectic form.**

$$\omega(x) = e_a(x) \wedge e_b(x) C^{ab}\,, \qquad d\omega(x) = 0\,.$$

ω **is a nondegenerate closed symplectic two-form**

Fedosov Quantization

$$\Phi_1(x) \star \Phi_2(x) := (\Phi_1(y|x) * \Phi_2(y|x))\Big|_{y=0}.$$

$$\Phi(x) := \Phi(y|x)\Big|_{y=0}$$

Fedosov system is off-shell: no differential conditions on $\Phi(x)$.

σ_-**-cohomology language**

$$\sigma_- = e^a \frac{\partial}{\partial y^a}$$

$H^0(\sigma, A) = \mathbb{K} : \Phi(0, x)$ **the only dynamically independent object**

$H^1(\sigma, A) = 0$: **no restriction on** $\Phi(0, x)$

$h = Lie(A_d)/\mathbb{K}$ $\qquad Lie(A) :$ $\quad [a, b]_* = a * b - b * a$

Associativity

$$(\Phi_1(x)\star\Phi_2(x))\star\Phi_3(x) = \Phi_1(x)\star(\Phi_2(x)\star\Phi_3(x)) = (\Phi_1(y|x)*\Phi_2(y|x)*\Phi_3(y|x))\Big|_{y=}$$

Unfolding as twistor transform

Twistor transform

$$
\begin{tikzcd}
 & \mathbf{C}(y|x) \arrow[dl, "\eta"'] \arrow[dr, "\nu"] & \\
\mathbf{M}(x) & & \mathbf{T}(y)\,.
\end{tikzcd}
$$

$W^{\Omega}(y|x)$ are functions on the “correspondence space” C.

Space-time $\mathbf{M}$: coordinates x. Twistor space $\mathbf{T}$: coordinates y.

Unfolded equations describe the Penrose transform by mapping functions on $\mathbf{T}$ to solutions of field equations in $\mathbf{M}$.

Being simple in terms of unfolded dynamics and the corresponding twistor space $\mathbf{T}$, holographic duality in terms of usual space-time may be complicated requiring solution of at least one of the two unfolded systems: a nontrivial nonlinear integral map.

Unfolding and holographic duality

Unfolded formulation unifies various dual versions of the same system.

Duality in the same space-time:

ambiguity in what is chosen to be dynamical or auxiliary fields.

Holographic duality between theories in different dimensions:

universal unfolded system admits different space-time interpretations.

Extension of space-time without changing dynamics by letting the differential d and differential forms W to live in a larger space

$$d = dX^n \frac{\partial}{\partial X^n} \to \tilde{d} = dX^n \frac{\partial}{\partial X^n} + d\widehat{X}^{\widehat{n}} \frac{\partial}{\partial \widehat{X}^{\widehat{n}}}\,, \qquad dX^n W_n \to dX^n W_n + d\widehat{X}^{\widehat{n}} \widehat{W}_{\widehat{n}}\,,$$

$\widehat{X}^{\widehat{n}}$ are additional coordinates

$$\tilde{d} W^{\Omega}(X, \widehat{X}) = G^{\Omega}(W(X, \widehat{X}))$$

Particular space-time interpretation of a universal unfolded system, e.g, whether a system is on-shell or off-shell, depends not only on $G^{\Omega}(W)$ but, in the first place, on space-time M^d and chosen vacuum solution $W_0(X)$.

Two unfolded systems in different space-times are equivalent (dual) if they have the same unfolded form.

Direct way to establish holographic duality between two theories: unfold both to see whether their unfolded formulations coincide.

Given unfolded system generates a class of holographically dual theories in different dimensions.

Invariant functionals via Q–cohomology

Equivalent form of compatibility condition

$$Q^2 = 0\,, \qquad Q = G^{\Omega}(W)\frac{\partial}{\partial W^{\Omega}}$$

Q-manifolds

Hamiltonian-like form of the unfolded equations

$$dF(W(x)) = Q(F(W(x))\,, \qquad \forall F(W)\,.$$

Invariant functionals

$$S = \int L(W(x))\,, \qquad QL = 0 \qquad \textbf{(2005)}$$

$L = QM$: **total derivatives**

Actions and conserved charges: Q cohomology

for off-shell and on-shell unfolded systems, respectively

Free massless fields in AdS_4

Infinite set of spins $s = 0, 1/2, 1, 3/2, 2\ldots$

$\omega(y,\bar{y}\mid x)\,,\qquad C(y,\bar{y}\mid x)\,,$ **two-component spinors:** $\alpha,\beta = 1,2,\ \dot{\alpha},\dot{\beta} = 1,2.$

$$\bar{\omega}(y,\bar{y}\mid x) = \omega(\bar{y},y\mid x)\,,\qquad \bar{C}(y,\bar{y}\mid x) = C(\bar{y},y\mid x)\,.$$

$$A(y,\bar{y}\mid x) = i\sum_{n,m=0}^{\infty}\frac{1}{n!m!}y_{\alpha_1}\ldots y_{\alpha_n}\bar{y}_{\dot{\beta}_1}\ldots\bar{y}_{\dot{\beta}_m}A^{\alpha_1\ldots\alpha_n}{}_{,}{}^{\dot{\beta}_1\ldots\dot{\beta}_m}(x)$$

The unfolded system for free massless fields is

$\star\qquad R_1(y,\bar{y}\mid x) = \eta\,\overline{H}^{\dot{\alpha}\dot{\beta}}\dfrac{\partial^2}{\partial\bar{y}^{\dot{\alpha}}\partial\bar{y}^{\dot{\beta}}}\;C(0,\bar{y}\mid x) + \bar{\eta}\,H^{\alpha\beta}\dfrac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\;C(y,0\mid x)$

$\star\qquad \tilde{D}_0 C(y,\bar{y}\mid x) = 0$

$$R_1(y,\bar{y}\mid x) = D_0^{ad}\omega(y,\bar{y}\mid x)\qquad H^{\alpha\beta} = e^{\alpha}{}_{\dot{\alpha}}\wedge e^{\beta\dot{\alpha}}\,,\quad \overline{H}^{\dot{\alpha}\dot{\beta}} = e_{\alpha}{}^{\dot{\alpha}}\wedge e^{\alpha\dot{\beta}}\;,$$

$$D_0^{ad}\omega = D^L - \lambda e^{\alpha\dot{\beta}}\left(y_\alpha\frac{\partial}{\partial\bar{y}^{\dot{\beta}}} + \frac{\partial}{\partial y^{\alpha}}\bar{y}_{\dot{\beta}}\right)\,,\qquad \tilde{D}_0 = D^L + \lambda e^{\alpha\dot{\beta}}\left(y_\alpha\bar{y}_{\dot{\beta}} + \frac{\partial^2}{\partial y^{\alpha}\partial\bar{y}^{\dot{\beta}}}\right)\,,$$

$$D^L = d_x - \left(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^{\beta}} + \bar{\omega}^{\dot{\alpha}\dot{\beta}}\bar{y}_{\dot{\alpha}}\frac{\partial}{\partial\bar{y}^{\dot{\beta}}}\right)\,.$$

Examples

Different spins: subsystems with

$$(N_y + N_{\bar{y}})\omega(y,\bar{y}|x) = 2(s-1)\omega(y,\bar{y}|x)\,, \qquad (N_y - N_{\bar{y}})C(y,\bar{y}|x) = \pm 2sC(y,\bar{y}|x)\,.$$

$$N_y = y^\alpha \frac{\partial}{\partial y^\alpha}\,, \qquad N_{\bar{y}} = \bar{y}^{\dot\alpha} \frac{\partial}{\partial \bar{y}^{\dot\alpha}}\,.$$

$s = 0$:

$$dC(y,\bar{y}|x) + dx^{\alpha\dot\alpha}\frac{\partial^2}{\partial y^\alpha \partial \bar{y}^{\dot\alpha}} C(y,\bar{y}|x) = 0\,.$$

$$dC_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_n}(x) + dx^{\gamma\dot\gamma} C_{\gamma\alpha_1\ldots\alpha_n\,,\dot\gamma\dot\alpha_1\ldots\dot\alpha_n}(x) = 0$$

Consequences:

$$C_{\alpha_1\ldots\alpha_n\,,\dot\alpha_1\ldots\dot\alpha_n}(x) = \frac{\partial}{\partial x^{\alpha_1\dot\alpha_1}} \ldots \frac{\partial}{\partial x^{\alpha_n\dot\alpha_n}} C(x)\,,$$

$$\left(\frac{\partial^2}{\partial x^{\alpha_1\dot\alpha_1}\partial x^{\alpha_2\dot\alpha_2}} - \frac{\partial^2}{\partial x^{\alpha_1\dot\alpha_2}\partial x^{\alpha_2\dot\alpha_1}}\right) C(x) = 0$$

Equivalent to

$$\varepsilon^{\alpha_1\alpha_2}\varepsilon^{\alpha_1\dot\alpha_2}\frac{\partial^2}{\partial x^{\alpha_1\dot\alpha_1}\partial x^{\alpha_2\dot\alpha_2}} C(x) = 0 \;: \qquad \Box C(x) = 0\,.$$

$s = 1$:

$$d\omega(x) = e_{\alpha}{}^{\dot\alpha} \wedge e^{\alpha\dot\beta}\bar{C}_{\dot\alpha\dot\beta} + e^{\alpha}{}_{\dot\alpha} \wedge e^{\beta\dot\alpha}C_{\alpha\beta}$$

$$dC(y,\bar{y}|x) + dx^{\alpha\dot\alpha}\frac{\partial^2}{\partial y^{\alpha}\partial\bar{y}^{\dot\alpha}}C(y,\bar{y}|x) = 0\,.$$

$$(N_y - N_{\bar{y}})C(y,\bar{y}|x) = \pm 2C(y,\bar{y}|x)\,.$$

Non-Abelian HS algebra

Star product

$$(f*g)(Y) = \int dSdTf(Y+S)g(Y+T)\,\mathrm{exp}\, -iS_A T^A$$

$$[Y_A, Y_B]_* = 2iC_{AB}\,, \qquad C_{\alpha\beta} = \epsilon_{\alpha\beta}\,, \qquad C_{\dot\alpha\dot\beta} = \epsilon_{\dot\alpha\dot\beta}$$

Non-Abelian HS curvature

$$R_1(y,\bar y|x) \to R(y,\bar y|x) = d\omega(y,\bar y|x) + \omega(y,\bar y|x) * \omega(y,\bar y|x)$$

$$\tilde D_0 C(y,\bar y|x) \to \tilde D C(y,\bar y|x) = dC(y,\bar y|x) + \omega(y,\bar y|x) * C(y,\bar y|x) - C(y,\bar y|x) * \omega(y,-\bar y|x)$$

That

$$\star\, R_1(y,\overline{y}\mid x) = \eta\,\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\overline{y}^{\dot\alpha}\partial\overline{y}^{\dot\beta}}\, C(0,\overline{y}\mid x) + \bar\eta\, H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\, C(y,0\mid x)$$

makes the system more complicated than in the pure Fedosov case

Weyl fiber with spinor generating elements

Non-zero curvature

Full nonlinear HS equations 1990,1992

Riemann θ functions solve conformal equations and Riemann θ functions

Conformal invariant massless equations

$$dX^{AB}\left(\frac{\partial}{\partial X^{AB}} \pm \frac{\partial^2}{\partial Y^A \partial Y^B}\right) C(Y|x) = 0\,, \qquad A,B = 1,\dots,2M$$

Shaynkman, MV (2001

Riemann θ–functions solve unfolded massless equations in $\mathcal{M}_M$

Gelfond, MV, arXiv:0801.2191

$$C(Y|Z) = \theta(Y,\,Z) = \sum_{n^A \in \mathbb{Z}^M} \exp i\pi (Z^{AB} n_A n_B + 2 n_A Y^A)$$

$$dZ^{AB}\left(\frac{\partial}{\partial Z^{AB}} + \frac{i}{4\pi} \frac{\partial^2}{\partial Y^A \partial Y^B}\right) C(Z|X) = 0$$

Higher-rank equations and conserved currents

Rank r unfolded equations: tensoring of Fock modules Gelfond, MV (2003)

$$dx^{AB}\Big(\frac{\partial}{\partial x^{AB}}+\eta_{ij}\frac{\partial^2}{\partial Y_i^A\partial Y_j^B}\Big)C(Y|x)=0\,,\qquad i,j=1,\dots r\,.$$

For diagonal η^{ij} higher-rank equations are satisfied by

$$C(Y_i|x)=C_1(Y_1|x)C_2(Y_2|x)\dots C_r(Y_r|x)\,.$$

Rank-two equations: conserved currents

$$\left\{\frac{\partial}{\partial x^{AB}}-\frac{\partial^2}{\partial Y^{(A}\partial u^{B)}}\right\}T(U,\,Y|x)=0$$

$T(U,\,Y|x)$: **generalized stress tensor. Rank-two equation is obeyed by**

$$T(U,\,Y\,|x)=\sum_{i=1}^{N}C_{+\,i}(Y-U|x)\,C_{-\,i}(U+Y|x)$$

Rank-two fields: bilocal fields in the twistor space.

Dynamical currents (primaries)

$$J(u|x) = T(u,0\,|x)\,, \qquad \tilde{J}(y|x) = T(0,y|x)$$

Gelfond, MV (2003)

$$J^{asym}(u,y|x) = u_\alpha y^\alpha \Big(\frac{\partial^2}{\partial u^\beta \partial y_\beta} T(u,y|x)\Big|_{u=y=0} \Big)$$

$J(u|x)$ **generates** $3d$ **currents of all integer and half-integer spins**

$$J(u|x) = \sum_{2s=0}^{\infty} u^{\alpha_1} \dots u^{\alpha_{2s}} J_{\alpha_1 \dots \alpha_{2s}}(x)\,, \qquad \tilde{J}(u|x) = \sum_{2s=0}^{\infty} u^{\alpha_1} \dots u^{\alpha_{2s}} \tilde{J}_{\alpha_1 \dots \alpha_{2s}}(x)\,.$$

$$J^{asym}(u,y|x) = u_\alpha y^\alpha J^{asym}(x)$$

$$\Delta J_{\alpha_1 \dots \alpha_{2s}}(x) = \Delta \tilde{J}_{\alpha_1 \dots \alpha_{2s}}(x) = s+1 \qquad \Delta J^{asym}(x) = 2$$

Differential equations: conventional conservation condition

$$\frac{\partial}{\partial x^{\alpha\beta}} \frac{\partial^2}{\partial u_\alpha \partial u_\beta} J(u|x) = 0\,, \qquad \frac{\partial}{\partial x^{\alpha\beta}} \frac{\partial^2}{\partial y_\alpha \partial y_\beta} \tilde{J}(y|x) = 0$$

$3d$ conformal setup in AdS_4 HS theory

For manifest conformal invariance introduce

$$y^+_\alpha = \frac{1}{2}(y_\alpha - i\bar{y}_\alpha)\,, \qquad y^-_\alpha = \frac{1}{2}(\bar{y}_\alpha - iy_\alpha)\,, \qquad [y^-_\alpha\,, y^{+\beta}]_* = \delta^\beta_\alpha$$

$3d$ conformal realization of the algebra $sp(4;\mathbb{R}) \sim o(3,2)$

$$L^\alpha{}_\beta = y^{+\alpha}y^-_\beta - \frac{1}{2}\delta^\alpha_\beta y^{+\gamma}y^-_\gamma\,, \qquad D = \frac{1}{2}y^{+\alpha}y^-_\alpha$$

$$P_{\alpha\beta} = iy^-_\alpha y^-_\beta\,, \qquad K^{\alpha\beta} = -iy^{+\alpha}y^{+\beta}$$

Conformal weight of HS gauge fields

$$[D, \omega(y^\pm|X)] = \frac{1}{2}\left(y^{+\alpha}\frac{\partial}{\partial y^{+\alpha}} - y^-_\alpha\frac{\partial}{\partial y^-_\alpha}\right)\omega(y^\pm|X)\,.$$

Pullback $\widehat{\omega}(y^\pm|x)$ of $\omega(y^\pm|x)$ to Σ: $3d$ conformal HS gauge fields

Holography at infinity

AdS_4 **foliation:** $x^n = (\mathbf{x}^a, z)$: $\mathbf{x}^a$ **are coordinates of leafs** $(a = 0, 1, 2,)$ z **is a foliation parameter**

Poincaré coordinates

$$W = \frac{i}{z} d\mathbf{x}^{\alpha\beta} y^-_\alpha y^-_\beta - \frac{dz}{2z} y^-_\alpha y^{+\alpha}$$

$$e^{\alpha\dot\alpha} = \frac{1}{2z} dx^{\alpha\dot\alpha}\,, \qquad \omega^{\alpha\beta} = -\frac{i}{4z} d\mathbf{x}^{\alpha\beta}\,, \qquad \bar\omega^{\dot\alpha\dot\beta} = \frac{i}{4z} d\mathbf{x}^{\dot\alpha\dot\beta}$$

$$\left[d_{\mathbf{x}} + \frac{i}{z} d\mathbf{x}^{\alpha\beta} \left(y_\alpha \frac{\partial}{\partial y^\beta} - \bar y_\alpha \frac{\partial}{\partial \bar y^\beta} + y_\alpha \bar y_\beta - \frac{\partial^2}{\partial y^\alpha \partial \bar y^\beta} \right) \right] C(y, \bar y | \mathbf{x}, z) = 0$$

Rescaling y^α and $\bar y^{\dot\alpha}$ **via**

$$C(y, \bar y | \mathbf{x}, z) = z \exp(y_\alpha \bar y^\alpha) T(w, \bar w | \mathbf{x}, z)\,,$$

$$w^\alpha = z^{1/2} y^\alpha\,, \qquad \bar w^\alpha = z^{1/2} \bar y^\alpha$$

$T(w, \bar w | \mathbf{x}, z)$ **satisfies the** $3d$ **conformal invariant current equation**

$$\left[d_{\mathbf{x}} - i d\mathbf{x}^{\alpha\beta} \frac{\partial^2}{\partial w^\alpha \partial \bar w^\beta}\right] T(w, \bar w | \mathbf{x}, z) = 0$$

Connections

Setting

$$W(y^{\pm}|\mathbf{x},z)=\Omega(v^{-},w^{+}|\mathbf{x},z)$$

$$\mathbf{v^{\pm}=z^{-1/2}y^{\pm}}\,,\qquad \mathbf{w^{\pm}=z^{1/2}y^{\pm}}$$

manifest z–dependence disappears

$$D_{\mathbf{x}}\Omega(v^{-},w^{+}|\mathbf{x},z)=\left(d_{\mathbf{x}}+2id\mathbf{x}^{\alpha\beta}v^{-}_{\alpha}\frac{\partial}{\partial w^{+\beta}}\right)\Omega(v^{-},w^{+}|\mathbf{x},z)$$

Using

$$w_{\alpha}=w^{+}_{\alpha}+izv^{-}_{\alpha}\,,\qquad \bar{w}_{\alpha}=iw^{+}_{\alpha}+zv^{-}_{\alpha}$$

in the limit $z\to 0$ free HS equations take the form

$$\star\quad D_{\mathbf{x}}\Omega_{\mathbf{x}}(v^{-},w^{+}|\mathbf{x},0)=d\mathbf{x}_{\alpha}{}^{\gamma}d\mathbf{x}_{\beta\gamma}\frac{\partial^{2}}{\partial w^{+\alpha}\partial w^{+\beta}}\mathcal{T}(w^{+},0\mid\mathbf{x},0)\,,$$

$$\star\quad \left[d_{\mathbf{x}}-id\mathbf{x}^{\alpha\beta}\frac{\partial^{2}}{\partial w^{+\alpha}\partial w^{-\beta}}\right]\mathcal{T}(w^{+},w^{-}|\mathbf{x},0)=0\,.$$

Towards nonlinear $3d$ conformal HS theory

Conformal HS theory is nonlinear since conformal HS curvatures inherited from the AdS_4 HS theory are non-Abelian Fradkin, Linetsky (1990)

$$R_{\mathbf{x}\mathbf{x}}(v^-, w^+ \mid \mathbf{x}) = d_{\mathbf{x}}\Omega_{\mathbf{x}}(v^-, w^+ \mid \mathbf{x}) + \Omega_{\mathbf{x}}(v^-, w^+ \mid \mathbf{x}) \star \Omega_{\mathbf{x}}(v^-, w^+ \mid \mathbf{x})$$

It is important

$$[v^-_\alpha, w^{+\beta}]_\star = \delta^\beta_\alpha$$

The equation on 0-forms deforms to nonlinear twisted adjoint representation

$$dT(w^\pm|x) + \Omega(\frac{\partial}{\partial w^{+\beta}}, w^+_\alpha) \circ T(w^\pm|x) - T(w^\pm|x) \circ \Omega(-i\eta\frac{\partial}{\partial w^{-\alpha}}, -i\eta w^-|x) = O(T^2)\,.$$

Matter fields can be added via the Fock module

$$(d + \Omega_0(v^-, w^+|\mathbf{x})) \star C^i(w^+|\mathbf{x}) \star F = 0$$

Reduction to free CFT_3

The unfolded equation

$$D_{\mathbf{x}}\Omega_{\mathbf{x}}(v^-, w^+|\mathbf{x}, 0) = \mathcal{H}^{\alpha\beta}\frac{\partial^2}{\partial w^{+\alpha}\partial w^{+\beta}}\mathcal{T}(w^+, 0 \mid \mathbf{x}, 0)$$

remains free if

$$\mathcal{T} = 0 \qquad \longrightarrow J^{asym} = 0 \qquad \text{or} \qquad J^{sym} = 0$$

depending on whether A-model or B-model is considered. For these cases the model remains free in accordance with the Klebanov-Polyakov Sezgin-Sundell conjecture.

Free models are equivalent to the reductions of the HS theory with respect to P-involution $y \leftrightarrow \bar{y}$ which is possible for the A and B models.

For HS theory with general phase η parameter such reduction is not possible: no realization as a free conformal theory.

Non-Abelian contribution of conformal HS connections has to be taken into account.

Conclusions

Holographic duality relates theories that have equivalent unfolded formulation: equivalent twistor space description.

AdS_4 HS theory is dual to nonlinear $3d$ conformal HS gauge theory

Beyond $1/N$

Both of holographically dual theories are HS theories of gravity

To do

Nonlinear $3d$ conformal HS theory

Correlators

AdS_3/CFT_2 and Gaberdiel-Gopakumar conjecture

To unfold String Theory