Lecture on Hilbert, Banach and Fréchet manifolds

Abstract

This lecture is an introduction to differential geometry with hightlights on the infinite-dimensional case. It will be divided into 5 sections :

1. Notion of manifolds with model spaces Hilbert, Banach or Fréchet spaces. What should be the tangent vector bundle and the co-tangent vector bundle?
2. Inverse Function Theorems : the Banach version and the Nash-Moser version. Some applications to submanifolds.
3. Riemannian Manifolds : discussion around the existence of geodesics.
4. Complex Manifolds : discussion around the integrability of an almost complex structure.
5. Symplectic structure : discussion around Darboux's Theorem. Examples of canonical symplectic manifolds.

During the lecture the notions introduced will be illustrated with examples related to projective spaces, grassmannians, diffeomorphisms groups, spaces of sections, spaces of curves...

References

[1] P.R. Halmos, *A Hilbert Space Problem Book*, second Edition, Graduate Texts in Mathematics, Springer-Verlag, 1982.

[2] R.S. Hamilton, *The inverse function Theorem of Nash and Moser*, Bulletin (New Series) of the American Mathematical Society, Volume 7, Number1, 1982.

[3] W. Klingenberg, *Riemannian Geometry*, Walter de Gruyter, New York, 1982.

[4] S. Lang, *Fondamentals of Differential Geometry*, Graduate Texts in Mathematics, Springer-Verlag, 1999.

[5] S. Lang, *Differential and Riemannian Manifolds*, Graduate Texts in Mathematics, Springer-Verlag, 1995.