On Spherically Symmetric Finsler metrics

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Abstract

In this paper, we study spherically symmetric Finsler metrics in R^n with quadratic curvatures. We find equations that characterize the metrics of R-quadratic, and Ricci-quadratic types. We give a lot of new Finsler metrics of R-quadratic type which are non-trivial.

Keywords: Spherically symmetric Finsler metric in R^n, Weakly Berwald metrics, R-quadratic Finsler metric, Ricci quadratic Finsler metric
53B40, 53C60, 51F25

1. Introduction

Spherically symmetric space-times are so important to study of general relativity. The Schwarzschild metric in four dimensional space-time is a solution of the vacuum Einstein field equations that describes the spherically symmetric gravitational field. In Riemannian geometry these special spaces have been deeply studied by many authors for example, [6] and [14]. The base of general relativity is (pseudo-) Riemannian geometry, it is natural to consider its generalizations based on Finsler geometry. In fact Finsler geometry have applications in physics, too [1].
Similarly with the definition in general relativity, a spherically symmetric Finsler metric is invariant under any rotations in R^n. In other words, the vector fields generated by rotations are the Killing fields of the Finsler metric. In calculation point of view the Finsler metrics with certain symmetry would greatly simplify the computation. Recently many papers has been published investigating the properties of these metrics, for example [16], [17] and [5].
Riemann curvature is a central concept in Riemannian geometry and was introduced by Riemann in 1854. Berwald generalized it to Finsler metrics. A Finsler metric is said to be R-quadratic if its Riemann curvature is quadratic

[4]. R-quadratic metrics were first introduced by Báscó and Matsumoto [2]. They form a rich class in Finsler geometry. There are many interesting works related to this subject [12], [15]. In this paper we are going to study R-quadratic spherically symmetric Finsler metrics in R^n. The necessary and sufficient conditions which the metrics be R-quadratic are considered.
Ricci-quadratic spherically symmetric Finsler metrics in R^n are studied, too. In particular, we show that non-Riemannian R-quadratic spherically symmetric Finsler metrics in R^n are R-flat ($R^i_k = 0$).

2. References

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